

Power System Voltage Harmonic Identification Using Kalman Filter

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Abstract —This paper presents an approach for the detection of voltage and current harmonics. The demanding use of non-linear loads and power converters in the industry has distorted the voltage waveform profile. The presence of harmonic distortion in transmission network, results in increasing power loss and also creates interference problems in communication systems, so the active filters are used for harmonic elimination. For designing of the harmonic filter, it is necessary to obtain the perfect measurement of harmonic level. It represents a robust algorithm based on Kalman filtering. Extended Kalman filter algorithm is used to design optimal filter for measuring magnitudes and frequencies of harmonic components present in non-sinusoidal voltage with existence of random noise in power system and distortions again taking into account the measurement noise and compare with inbuilt function FFT analysis in Matlab.

Keywords- Extended Kalman Filter(EKF), Harmonic analysis, Amplitude and phase estimation, FFT analysis, Gaussian random noise.

I. INTRODUCTION

The objectives of power quality are to meet a regulated, pure sinusoidal and uninterrupted supply in electrical systems. The problem of estimating Amplitude and frequency of the sinusoidal signal in white noise in power network have been extensively studied. There are several methods for amplitude and frequency of non-stationary signals; Fast Fourier Transform (FFT) and Discrete Fourier Transform (DFT) are widely used, but both the methods suffer from picket fence effects, aliasing, and leakage. So need adaptive window width and error compensation.

There are some famous signal processing techniques like supervised Gauss-Newton algorithm, artificial neural networks, linear prediction technique adaptive filter, least- error square and its variants, extended Kalman filters, have been used for time-varying signal parameter estimation. These algorithms require a heavy computational outlay and suffer from inaccuracies in the presence of noise with low signal to noise ratio (SNR).

The estimation, detection, and tracking of signals play a significant role in many aspects of military and civilian operations. The behavior of the Extended Kalman filter is studied because most of the real world problems are non-linear. Voltage waveform such as fundamental phase and frequency, phases and frequencies of voltage harmonics are considered unknown and are estimated by the Extended Kalman filter algorithm.

Many electric power system applications Kalman filtering extensively use as a digital signal processing tool. Kalman filter successfully used to measure power system frequency, voltage flicker, harmonic distortion, voltage dips, voltage unbalance, Voltage and current phasors, and other power system magnitudes.

II. EXTENDED KALMAN FILTER

This filter linearizes the nonlinear system and the Kalman filter estimate is based on the linearized system. This is the idea of the Extended Kalman filter Linearize Process.

2.1 Linearize Process:

$$\begin{aligned} x_k &= f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \\ y_k &= h_k(x_k, v_k) \\ W_k &\sim(0, Q_k) \\ V_k &\sim(0, R_k) \end{aligned}$$

Apply a Taylor series expansion of the state equation around $x_{k-1} = \hat{x}_{k-1}^+$ & $w_{k-1} = 0$ to obtain the following:

$$\begin{aligned} x_{k-1} &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) + \frac{df_{k-1}}{dx} |_{\hat{x}_{k-1}^+} (x_{k-1} - \hat{x}_{k-1}^+) + \frac{df_{k-1}}{dw} |_{\hat{x}_{k-1}^+} (w_{k-1}) \\ &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) + F_{k-1}(x_{k-1} - \hat{x}_{k-1}^+) + L_{k-1}w_{k-1} \\ &= F_{k-1}x_{k-1} + [f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) - F_{k-1}\hat{x}_{k-1}^+] + L_{k-1}w_{k-1} \\ &= F_{k-1}x_{k-1} + \tilde{u}_{k-1} + w_{k-1} \dots\dots\dots (1) \end{aligned}$$

F_{k-1} and L_{k-1} are defined by the above equation. The known signal \tilde{u}_k and the noise signal \tilde{w}_k are defined as follows :

$$\begin{aligned} \tilde{u}_k &= f_k(\hat{x}_k^+, u_k, 0) - F_k \hat{x}_k^- \\ \tilde{w}_k &\sim(0, L_k Q_k L_k^T) \end{aligned}$$

Now linearize the measurement equation around $x_k = \hat{x}_k^+$ and $v_k=0$ to obtain

$$\begin{aligned} y_k &= h_k(\hat{x}_k^-, 0) + \frac{dh_k}{dx} |_{\hat{x}_k^-} (x_k - \hat{x}_k^-) + \frac{dh_k}{dv} |_{\hat{x}_k^-} v_k \\ &= h_k(\hat{x}_k^-, 0) + H_k (x_k - \hat{x}_k^-) + M_k v_k \\ &= H_k x_k + [h_k(\hat{x}_k^-, 0) - H_k \hat{x}_k^-] + M_k v_k \\ &= H_k x_k + Z_k + \tilde{v}_k \dots \dots \dots (2) \end{aligned}$$

H_k and M_k are defined by the above equation. The known signal Z_k and the noise signal \tilde{v}_k are defined as :

$$\begin{aligned} Z_k &= h_k(\hat{x}_k^-, 0) - H_k \hat{x}_k^- \\ \tilde{v}_k &\sim (0, M_k R_k M_k^T) \end{aligned}$$

We have a linear state space system in equation (1) and a linear measurement in equation (2). That means we can use the standard Kalman filter equations to estimate the state. This results in the following equations for the discrete time extended Kalman filter.

$$\begin{aligned} P_k^- &= F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T \\ K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1} \\ \tilde{x}_k^- &= f_{k-1}(\tilde{x}_{k-1}^-, u_{k-1}, 0) \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^- - Z_k) \\ &= \hat{x}_k^- + K_k [(y_k - h_k(\hat{x}_k^-, 0))] \end{aligned}$$

2.2 Algorithm for EKF:

The discrete time EKF can be summarized by an algorithms follows :

1. The system and measurement equations are given as follows :

$$\begin{aligned} x_k &= f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \\ y_k &= h_k(x_k, v_k) \\ W_k &\sim (0, Q_k) \\ V_k &\sim (0, R_k) \end{aligned}$$

2. Initialize the filter as follows:

$$\begin{aligned} \hat{x}_0^+ &= E(x_0) \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned}$$

3. For $k=1,2,3,\dots$ perform the following

- a. Compute the following partial derivative matrices:

$$\begin{aligned} F_{k-1} &= \frac{df_{k-1}}{dx} |_{\hat{x}_{k-1}^+} \\ L_{k-1} &= \frac{df_{k-1}}{dw} |_{\hat{x}_{k-1}^+} \end{aligned}$$

- b. Perform the time update of the state estimate and estimation error covariance as follows :

$$\begin{aligned} P_k^- &= F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T \\ \tilde{x}_k^- &= f_{k-1}(\tilde{x}_{k-1}^-, u_{k-1}, 0) \end{aligned}$$

- c. Compute the following partial derivative matrices :

$$\begin{aligned} H_k &= \frac{dh_k}{dx} |_{\hat{x}_k^-} \\ M_k &= \frac{dh_k}{dv} |_{\hat{x}_k^-} \end{aligned}$$

- d. Perform the measurement up date of the state estimate and estimation error covariance as follows :

$$\begin{aligned} K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^- - Z_k) \\ &= \hat{x}_k^- + K_k [(y_k - h_k(\hat{x}_k^-, 0))] \\ P_k^+ &= (I - K_k H_k) P_k^- \end{aligned}$$

III. MODELING OF THE SYSTEM:

Consider an approximately periodic, non -sinusoidal signal, in the additive white Gauss in noise e . A non-Sinusoidal signal may be considered to consist of an infinite number of sinusoidal components. Two Sets of parameters

can characterize the signal: the fundamental frequency and the amplitude of each harmonic component. The signal is not exactly periodic since frequencies, amplitudes and phases change slowly over time. A Fourier series representation of this signal can be written as

$$y(t) = \sum_{k=1}^{\infty} r_k \sin(kW_f t + \varphi_k)$$

Here we use a discrete time domain (i.e $t=0,1,2,\dots$) rather than a continuous domain. As our signal $y(t)$ is not exactly periodic, but parameters frequency W_f , amplitudes r_k , and phases Φ_k , are slowly time varying. So we can states them as follows:

$$\begin{aligned} w_f &= W_f(t) \\ r &= r_k(t) \\ \varphi_k &= \varphi_k(t) \end{aligned}$$

We assume for model that the signal $y(t)$ is corrupted by noise. So measurements are given as follows :

$$Z(t) = y(t) + v(t)$$

Now the task is to estimate the values $r_1(t), \dots, r_m(t), w_{f1}(t), \dots, w_{fm}(t)$. From the measurements, where m denotes the number of the significant harmonics. Parameters are only estimated up to m th harmonics. The higher harmonics are assumed to be negligible. A total of $2m$ parameters are must be estimated.

We are estimating amplitudes as well as the frequency of harmonic components presents in the signal up to m^{th} order. This requires establishing the estimator that uses both that the energy in the fundamental and in the higher order harmonics for estimating frequency of the signal. Information's about the frequency contained in any of the harmonic component depends on the energy of that harmonic. So if the particular harmonic component is strong, then estimator of a frequency component must give more weight for the information available in the strong harmonic component and less weight to forthe information available in the weak harmonic components.

The estimator determines the frequency of harmonic component by first estimating the harmonic amplitude of that harmonic component. Knowledge of the frequency of harmonic component, the model can assists for the calculation of amplitudes harmonic components.

State space representation of a signal can represented as follows:

$$\begin{aligned} x(t+1) &= \varphi x(t) + w(t) \\ Z(t) &= h(x(t) + v(t)) \\ &= y(t) + v(t) \end{aligned}$$

Where,

$$x(t) = [r_1(t), r_2(t) \dots \dots r_m(t), w_{f1}(t), w_{f2}(t), \dots \dots w_{fm}(t)]^T$$

And,

$$\varphi = \begin{bmatrix} I_m & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_m \end{bmatrix}$$

Where, I_m is a m^{th} order identity matrix.

$$h(x(t)) = \sum_{k=1}^{\infty} r_k \sin(kW_f t + \varphi_k)$$

And $w(t)$ is white Gaussian noise, with a zero mean and a variance represented as Follows:

$$E[w(t)w(t)^T] = Q$$

The observation noise $v(t)$ is also a white Gaussian noise with zero mean and has a variance represented as follows:

$$E[v(t)v(t)^T] = R$$

Which is uncorrelated with $w(t)$ can be represented as follows:

$$E[w(t)v(t)] = 0$$

we have noise variance Q matrix which is diagonal. From the equation of $x(t+1)$ in the state space representation, we can conclude that the amplitudes of harmonic components are evolving randomly over a time. Also the same argument is true for the fundamental frequency of the signal. The rate of the random walk can be determined by diagonal Q matrix. A zero Q matrix will correspond to the constant amplitude, frequency and the phase. An Extended Kalman filter will be applied for estimating $x(t/t)$ or $x(t/t-1)$ of $x(t)$ from the measurement $z(t)$.Here $x(t/t)$ denotes estimation of $x(t)$ with given measurements at time and $x(t/t-1)$ denotes estimation of with given measurements at time $t-1$.

$$\begin{aligned} \hat{x}_{t/t} &= \hat{x}_{t/t-1} + G(t)[Z(t) - h(\hat{x}_{t/t-1})] \\ \hat{x}(t+1/t) &= \varphi \hat{x}(t/t) \\ G(t) &= P(t)H^T(t)(H(t)P(t)H^T(t) + R)^{-1} \\ P(t+1) &= \varphi[P(t) - G(t)H(t)P(t)]\varphi^T + Q \end{aligned}$$

Where H(t) is the jacobian of h(t). That is :

$$\begin{aligned} H(t) &= \frac{dh(\hat{x}(t/t-1))}{d\hat{x}(t/t-1)} \\ H(t) &= [\sin(w_f t + \varphi_k) \dots \sin(w_f t + \varphi_k) \dots \hat{r}_1 \cos(w_f t + \varphi_k) \dots \hat{r}_k \cos(w_f t + \varphi_k) \dots] \end{aligned}$$

And the initial values are

$$\begin{aligned} \hat{x}(0) &= E[x(0)] = x'(0) \\ P(0) &= E[(x(0) - x'(0))(x(0) - x'(0))^T] \end{aligned}$$

IV. PROPOSED SYSTEM:

In the simulation we let

$$z(t) = r_1 \sin(2\pi 0.04t + 0.01) + r_2 \sin(4\pi 0.04t + 0.01) + r_3 \sin(6\pi 0.04t + 0.01) + r_4 \sin(8\pi 0.04t + 0.01) + v(t) \dots (3)$$

where v(t) is a zero mean, white Gaussian noise process with a variance 0.1. The fundamental frequency of the signal $w_f = 2\pi 0.04$ and the amplitudes of the signal r_k are constant over a time. we use amplitude as written below.

$$r_k = r_{k-1}/2 \quad k=2,3,4$$

V. RESULT AND DICUSSION:

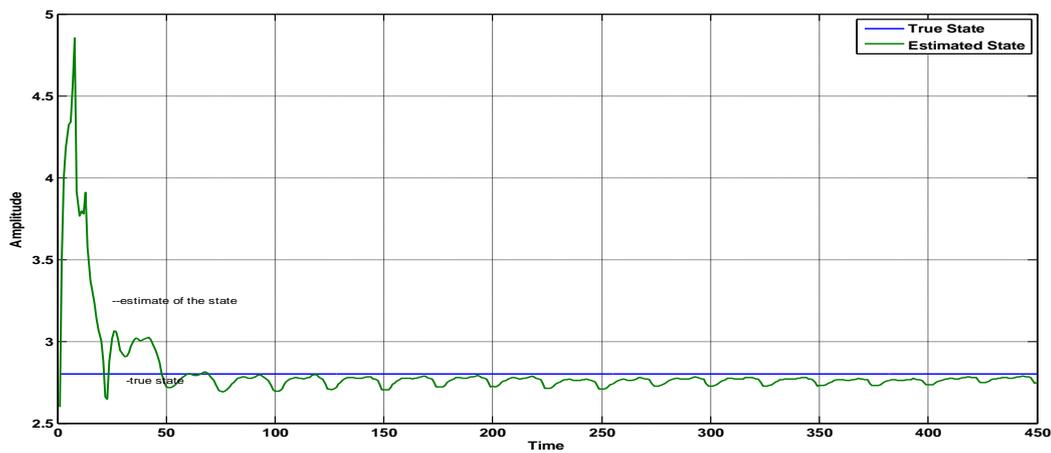


Figure.1 True and estimate value of the amplitude of first harmonic

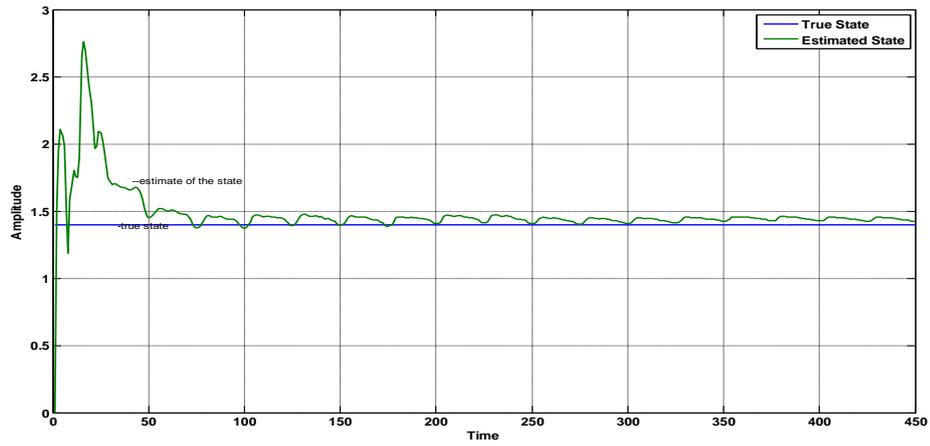


Figure.2 True and estimate value of the amplitude of second harmonic

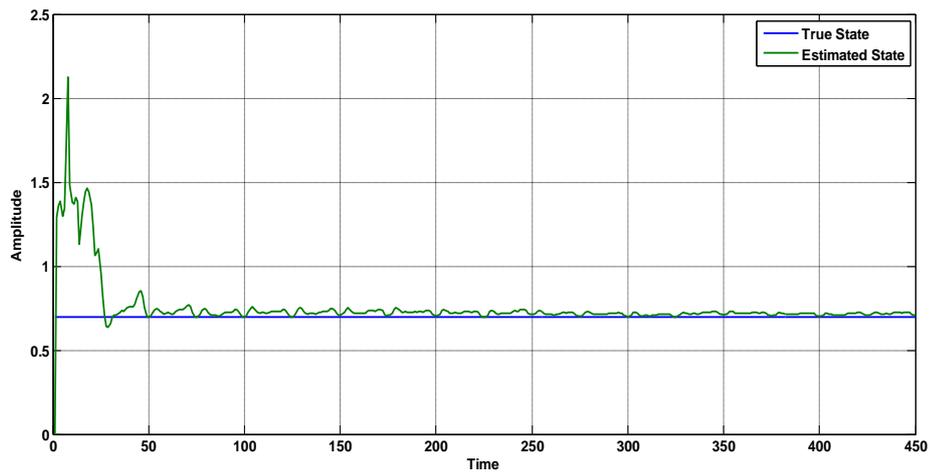


Figure.3 True and estimate value of the amplitude of third harmonic

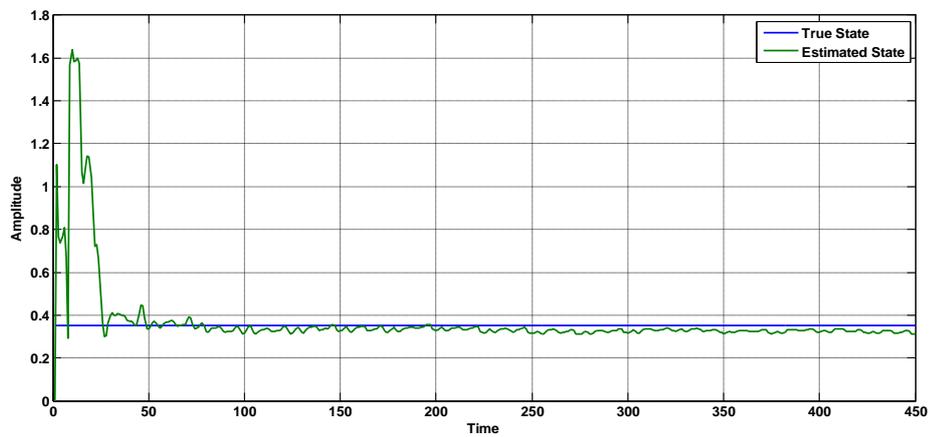


Figure.4 True and estimate value of the amplitude of fourth harmonic

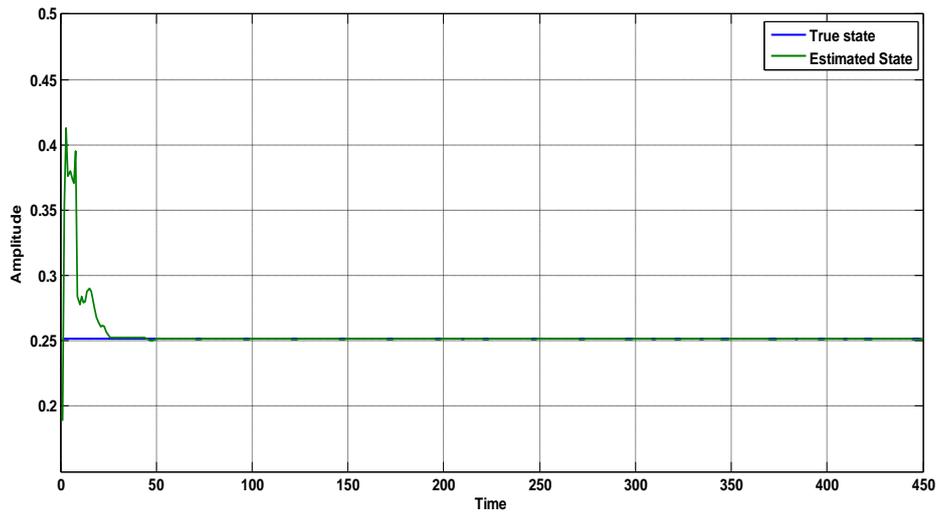


Figure.5 True and estimate value of the amplitude of fundamental frequency

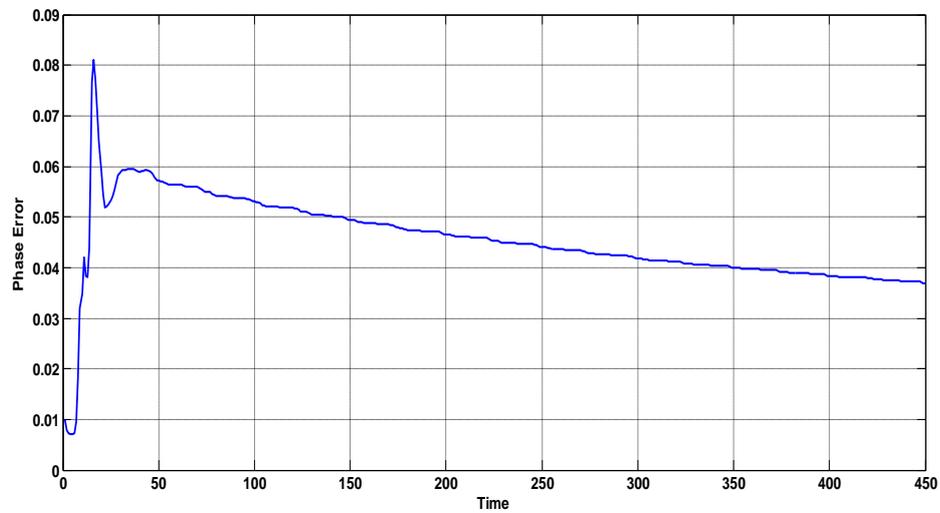


Figure.6 Phase error of first harmonic

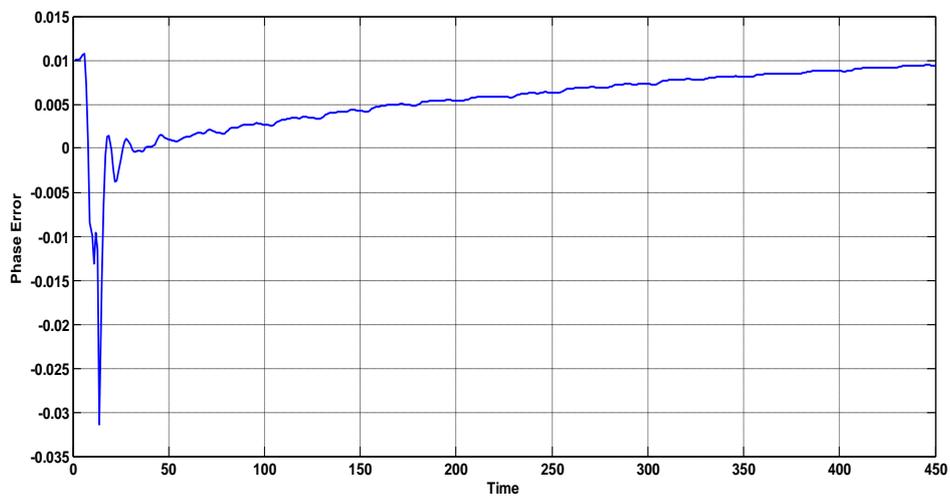


Figure.7 Phase error of second harmonic

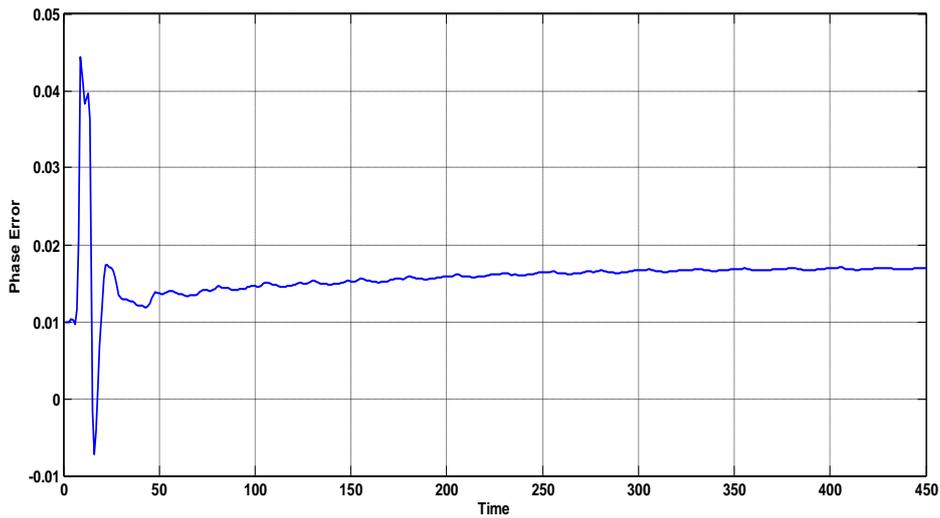


Figure.8 Phase error of third harmonic

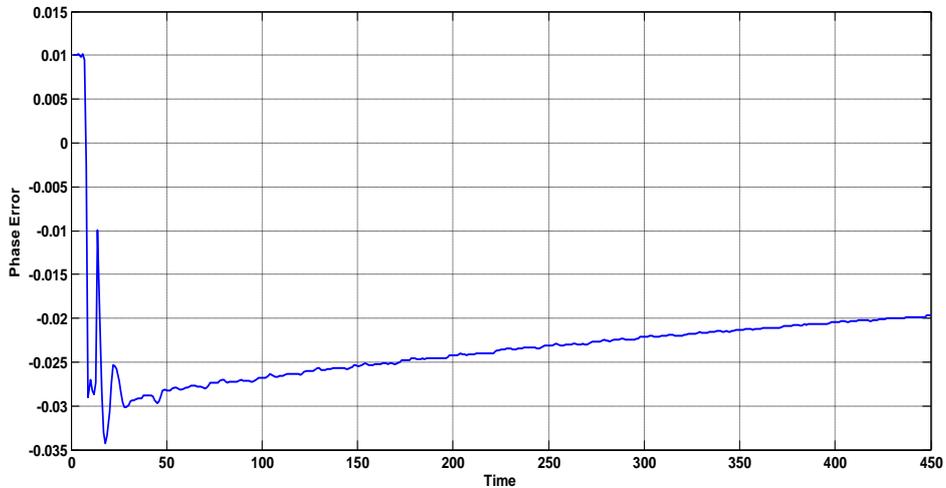


Figure.9 Phase error of fourth harmonic

Fig 6 shows the true and estimated value of the fundamental frequency from 450 samples. The estimated value of the frequency tracks the true value after $t=30$ and locks onto it very closely. Fig 1 shows the true and estimated values of the amplitude of the first harmonic component. Tracking is achieved by $t=50$. Fig 2 shows the true and estimated values of the amplitude of the second harmonic component. Tracking is achieved by $t=50$, although the initial estimate value $r_2(0)$ is zero.

VI. CONCLUSION:

In radar, nuclear magnetic resonance, power networks problem of estimating frequency and magnitude of non-sinusoidal signal with white noise has been extensively studied. This paper represents the estimation of phase and amplitude of harmonic components presents in distorted voltage. Extended Kalman filter accurately estimate the magnitudes and phase of harmonic components presents in distorted voltage waveforms with presence of noise.

In the simulation, an 18 dB signal-to-noise ratio was used. The performance of the filter becomes poor when lower signal-to-noise ratios were used. Extended Kalman filter utilize linearization for computing the state and error covariance matrices for resulting a more accurate estimation of the parameters of a non-sinusoidal signal. So if more number of harmonics components is considered for harmonic analysis, state and error covariance matrices become bulky and linearization process become complicated.

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