

**UPON THE METHOD FOR THE OPTIMIZATION OF THE SPHERE-
CYLINDER GRP VESSEL DESIGN UNDER INTERNAL PRESSURE**

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Abstract- This paper presents the optimization of the design with the method proposed by Dora Florea for the sphere-cylinder shell GRP under the pressure through the condition to achieve a constant circumferential stress in the shell equal with the maximum circumferential stress which exist in the shell with the constant thickness. The results obtained by author concerning at the stress state and the optimized design which obtained quickly, are better than results obtained by J.Leach [4], which studied 13 designs for the sphere-cylinder shell and then decide an acceptable solution for the design. The variation of the circumferential stress for the optimized shell through the method Dora Florea with the option proposed, is $\Delta\sigma_\theta = 0.76$ MPa comparing with the variation of the circumferential stress $\Delta\sigma_\theta = 3.65$ MPa obtained by J.Leach for a design selected by him.

Keyword- design optimization, stress, NASTRAN V.4.0, GRP

I. INTRODUCTION

The literature regarding to the design for the sphere-cylinder shell GRP subject to the internal pressure is not too large. In 1956 Fessler and Rose [1] suggested that the reduction of the thickness in the sphere is possible until 50%. An analytic determination of the stress in the sphere-cylinder shell was attempted by Rose [3] and note that a precision of the solution may be obtained through using the computer, which allows a precise specification of the design.

J.Leach and S.W.Saden [4] investigated the design of the joint for the sphere-cylinder GRP, the results being presented through the definition of the concentration factors independent of the elasticity modulus Young E and the coefficient Poisson μ . The condition used for the choice of the optimum design of the studied shell was that the ratio between the maximum stress in the sphere (circumferential stress) and the circumferential stress in the cylinder what represent the concentration factor Sc_f , to be near the unity. The analysis of the stress state it made with the BOSOR4 programme which resolves the problem of the axisymmetric thin shell for a material in the elastic linear field used the difference finite method. For the compute of the concentration coefficient Sc_f it observed that the stress became constant far away the joint sphere-cylinder. The peak of the stress it yield on the circumferential direction on the external surface of the cylinder. The maximum of the stress in the sphere it yield on the circumferential direction on the joint between cylinder and sphere. After the study of 13 designs J.Leach consider that the design from fig.1, case 12 is an acceptable solution from point of view of stress distribution.

In fig.2 it presents the variation of the circumferential and meridional stress in the shell studied of the constant thickness $h=10$ mm and in the fig.3 it shows the variation of the circumferential and meridional stress for the shell selected that being optimized.

In this paper it presents an original method proposed by Dora Florea for the design optimization imposing the condition that on the whole surface of the shell to acquire an uniform circumferential stress equal with the maximum circumferential stress which exist in the shell in the situation of the shell sphere-cylinder has the constant thickness.

II. THEORETICAL CONSIDERATION

The analysis of the stress state in the sphere-cylinder shell for the shell with the constant thickness $h=10$ mm was shown in the paper [6].

In fig.4 it presents the graphics of the stresses for the shell cylinder-sphere obtained with the analysis stress through the programme NASTRAN V.4.0 [7] by method of finite element: radial stress σ_r , circumferential stress σ_θ , axial stress σ_x and meridian stress σ_ϕ , shear stress τ_{xy} , VonMises stress σ_{vm} for a case of the sphere-cylinder shell with the constant thickness. The circumferential stress σ_θ has the values between 10.41 MPa and 20.51 MPa, the difference between the maximum and minimum value of the circumferential stress σ_θ being $\Delta\sigma_\theta = 10.1$ MPa and the Von Mises stress σ_{vm} has the values between 9.354 MPa and 17.82 MPa, the difference between the maximum and minimum value of the Von Mises stress σ_{vm} being $\Delta\sigma_{vm} = 8.466$ MPa.

It remarks that the stress which have the great values is the circumferential stress σ_θ in the cylinder $\sigma_{\theta max} = 20.51$ MPa.

Because maximum stress which presents in the sphere-cylinder shell under the pressure is the circumferential stress and this presents a jump of stress with the doubling of value in the joint, Dora Florea proposed in this paper a method for the optimization the sphere-cylinder GRP vessel design used the circumferential stress σ_θ imposed to obtain an uniform value for the circumferential stress in the whole shell.

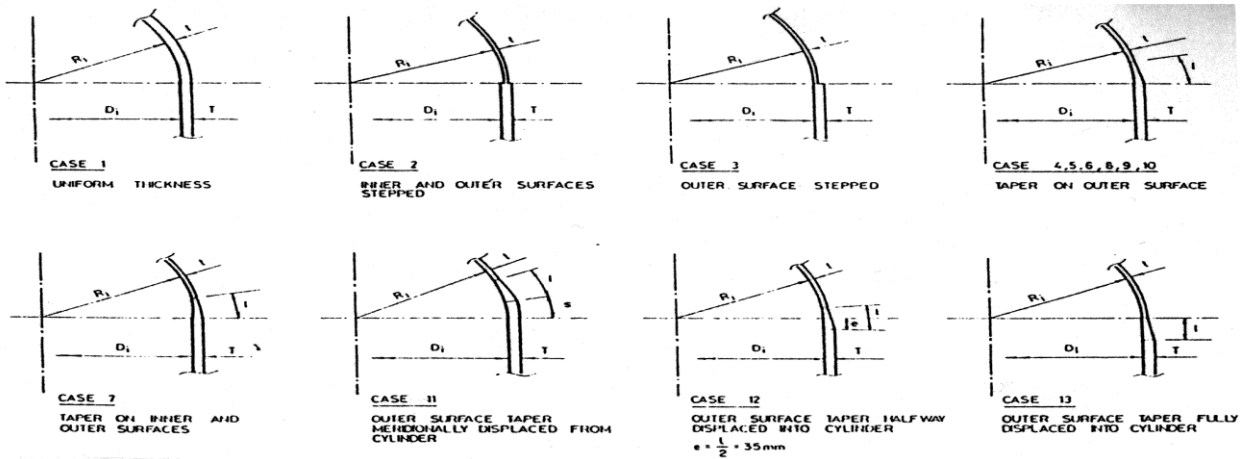


Fig.1 The study of design by J.Leach [4]

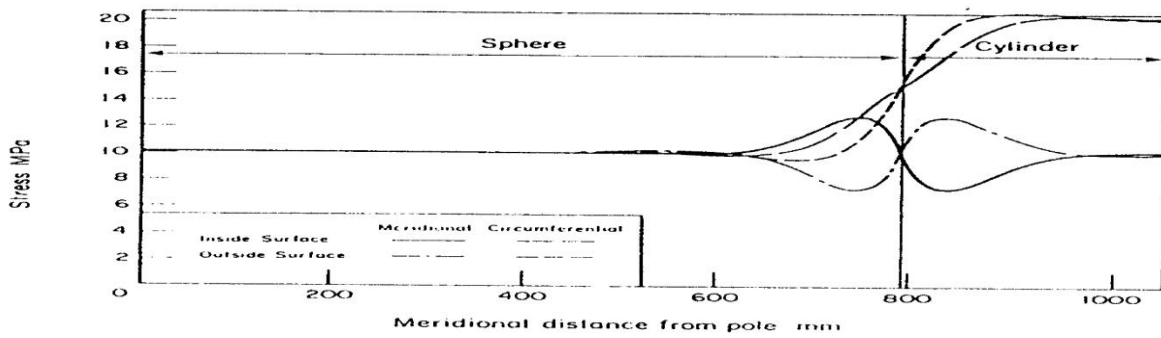


Fig.2 The graphic of the circumferential and meridional stress for thickness $h=10$ mm studied by J.Leach [4]

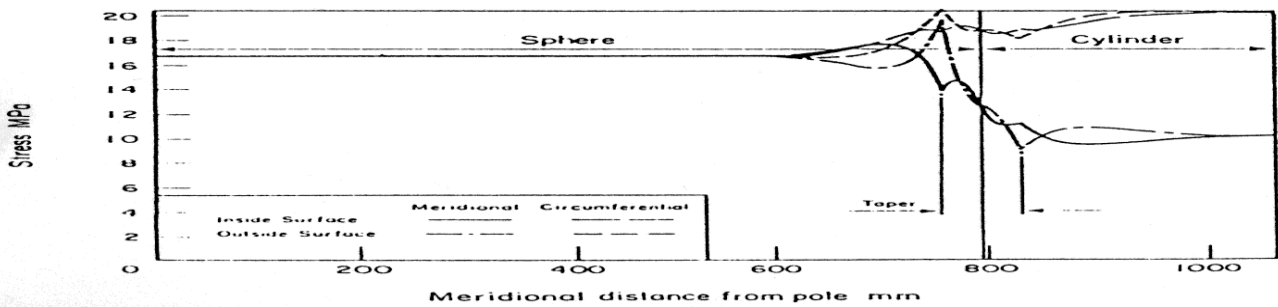


Fig.3 The graphic of the circumferential and meridional stress for the optimized shell by J.Leach [4]

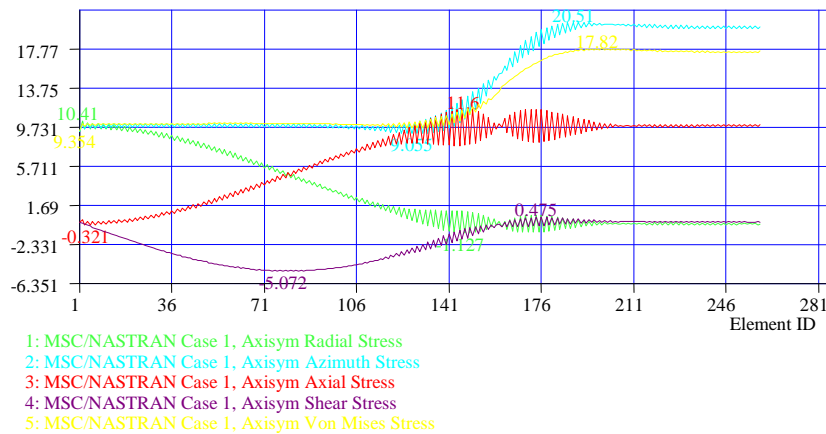


Fig.4 The graphics of the stress for the shell with the constant thickness $h=10$ mm obtain by NASTRAN V.4.0

For the circumferential stress allowed $\sigma_{\theta ad}$ it selected the maximum circumferential stress $\sigma_{\theta max}$ which exist in the cylinder

for the sphere-cylinder shell with the constant thickness with the purpose to achieve an uniform state of the circumferential stress in the whole shell and that is the condition by the determination an optim design with the minimum utilization of the material.

The mathematical model of the optimization problem is:

$$\Psi(u) = V(u) \quad (1)$$

where V is the volume of the shell under the behaviour of the restriction:

$$b_i(u) = \frac{\max[|\sigma_1|, |\sigma_2|, |\sigma_3|]}{\sigma_{ad}} - 1 = 0 \quad (2)$$

with $i=1..n$ and where

$$u = [u_1 \dots u_n]^t \quad (3)$$

U represents the vector of coordinates for the exterior design acquired.

In relation (2) σ_1 and σ_2 are the principal stresses [8] expressed by relations:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\frac{[\sigma_x - \sigma_y]^2}{2^2} + \tau_{xy}^2} \quad (4)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\frac{[\sigma_x - \sigma_y]^2}{2^2} + \tau_{xy}^2} \quad (5)$$

In the relation (4) and (5) σ_x and σ_y are σ_θ (the circumferential stress) and σ_ϕ (the meridional stress) for the sphere , respective σ_θ (the circumferential stress) and σ_x (the axial stress) for the cylinder. For the case in study because the load of pressure imposed is axisymmetrical the tangential stresses τ_{xy} are nulls.

In relation (2) σ_{ad} is the maxim circumferential stress which exist in the shell with constant thichness for the method proposed by Dora Florea in this paper:

$$\sigma_{ad} = \sigma_{\theta max} \quad (6)$$

In the relation (4) and (5), the circumferential stress σ_θ for the sphere has the expression:

$$\sigma_\theta = \frac{pr}{4h} e^{-\lambda\psi} \left(\cos \lambda\psi + \frac{3\nu}{\sqrt{3(1-\nu^2)}} \sin \lambda\psi \right) + \frac{pr}{2h} \quad (7)$$

and the meridian stress σ_ϕ for the sphere has the expression :

$$\sigma_\phi = \frac{3pr}{4h\sqrt{3(1-\nu^2)}} e^{-\lambda\psi} \sin \lambda\psi + \frac{pr}{2h} \quad (8)$$

in which $\psi = \alpha - \phi$ represent the angles of reference for the calculation of the stresses in the sphere: $\alpha = 90^\circ$, $\phi \in [90^\circ..0^\circ]$ and the expression for λ is:

$$\lambda^4 = 3(1-\nu)(r/h)^2 \quad (9)$$

In the relation (4) and (5), the circumferential stress for the cylinder σ_θ has the expression:

$$\sigma_{\theta} = \frac{3\nu p \zeta_{\beta x}}{4\beta^2 h^2} - \frac{pr}{4h} \theta_{\beta x} + \frac{pr}{h} \quad (10)$$

and the axial stress for the cylinder σ_x has the expression:

$$\sigma_x = \frac{3p \zeta_{\beta x}}{4\beta^2 h^2} + \frac{pr}{2h} \quad (11)$$

in witch:

$$\theta_{\beta x} = e^{-\beta x} \cos \beta x \quad (12)$$

$$\zeta_{\beta x} = e^{-\beta x} \sin \beta x$$

and

$$\beta = \sqrt{3(1-\nu^2)/(r^2 h^2)} \quad (13)$$

In the relations (7) at (13) r is the internal radius of the sphere , h is the thickness of the shell , l is the length of the cylinder , E is the Young modul of elasticity , ν is the Poisson coefficient , p is the internal constant axysymmetrical and uniform pressure .

Because for the shell optimization it used the condition to obtain the maxim value of the circumferential stress σ_{θ} in the whole shell without the jump of stress ,the condition for the optimized shell imposed for the method proposed by Dora Florea in this paper is:

$$|\sigma_{\theta}(u_i) - \sigma_{\theta ad}| < Eps \quad (14)$$

where u_i represents the coordinates for the exterior design acquired and $i=1..n$ and $\sigma_{\theta ad}$ is the maxim value of the circumferential stress $\sigma_{\theta max}$ which it exist in the cylinder for the sphere-cylinder shell with the constant thickness.

For solution the optimization model proposed by Dora Florea it used the programme OPTIMISES [5] wrote by Dora Florea which determined the optimization shell through calculation of the coordinates u_i for the exterior optimized design. The theoretical aspects with regard at the OPTIMISES programme used for the optimization in the elastic linear field for a design of the thin shell sphere-cylinder under the internal pressure was show in the paper [5] .It make the observation that the programme OPTIMISES has the option to determine a design which respect the condition to obtain an uniforme circumferential stress in the shell. The determination of the stress state in the sphere-cylinder shell for a concret case it achieved with PCETJ1 programme[6] wrote by Dora Florea.

The mathematical condition used by OPTIMISES programme which realize the thickness computing $h_{i,i=1..n}$ of the sphere in n points with the step δh desired is (15) where the circumferential stress σ_{θ} for the sphere in the exterior shell ,has the expression from (7):

$$|\sigma_{\theta sphere} - \sigma_{\theta max}| < Eps \quad (15)$$

Through the solution of the relation (15) it obtain $n1$ values for the thickness $h_{i,i=1..n1}(\psi)$ for the spherical shell , the value $n1$ beeing determined by plan $\Delta\phi$ allow in interval $[0^0,90^0]$ and than it compute u_i represents the coordinates for the exterior design acquired.

For the cylindrical shell the mathematical condition is (16) where the circumferential stress σ_{θ} for the cylinder in the exterior shell ,has the expression from (10):

$$|\sigma_{\theta cylinder} - \sigma_{\theta max}| < Eps \quad (16)$$

Through the solution of the relation (16) it obtain $n2$ values for the thickness $h_{i,i=1..n2}(x)$ for the cylindrical shell , the value $n2$ is determine of the pas Δx allow in interval $[0,1]$ where l is the length of cylinder and than it compute u_i represents the coordinates for the exterior design acquired.

For the obtain the solution of the transcendental neliniar ecuations (15) with the circumferential stress $\sigma_{\theta sphere}$ for the sphere and (16) with the circumferential stress $\sigma_{\theta cylinder}$ for the cylinder , Dora Florea used in the programme OPTIMISES the bisect method .

III.SHOWING OF THE RESULTS ACQUIRED WITH THE OPTION OF THE OPTIMISES PROGRAMME

The programme OPTIMISES wrote in the Turbo Pascal V7.0 for a PC computer by Dora Florea was run with the option for the optimization by circumferential stress , for a concrete case of the thin shell with 1) the geometrical constants : the internal radius of the sphere $r=500$ mm , the thickness of the shell $h=10$ mm , the length of the cylinder $l=500$ mm and with 2) the material characteristics: the Young modul of elasticity $E=7E+3$, the Poisson coefficient $\mu =0.34$ and with 3)

the function parameters :the internal constant axysymmetrical and uniform pressure $p=0.4$ MPa.

The optimized design it obtain for the optimization condition to achieve the maximum circumferential stress $\sigma_{\theta\max}=20.51$ MPa and uniform in the whole shell without the jump of stress.

Through the runing the programme OPTIMISES ,it obtain the thickness $h_{i,i=1\dots n}$ of the shell in n points for the optimized design and the coordinates u_i for the exterior contour of the shell.

The programme OPTIMISES determine so :

1) the graphic of the thickness for the optimized shell by method proposed by Dora Florea in this paper , $h=f(x)$ in function of x presents in fig.5

2) the graphic of the circumferential stress $\sigma_{\theta}=g(x)$ in function of the x for the optimized design showing in fig.6 where itremark the approximative linear variation of the circumferential stress σ_{θ} with the variation of the values between 19.74 MPa and 20.51MPa ,the difference between the maxim and minim value of the circumferential stress σ_{θ} being $\Delta\sigma_{\theta}=0.77$ MPa.

3) the graphic of the equivalent Von Mises stress $\sigma_{vm}=q(x)$ in function of the x for the optimization design determined with the variation of the values between 17.10 MPa and 21.07 MPa it show in fig.7 ,the difference between the maxim and minim value of the Von Mises stress σ_{vm} being $\Delta\sigma_{vm}=3.97$ MPa.

4) the design of the optimized design acquired with the programme OPTIMISES and the detail of the design it show in fig.8a and fig.8b.

The programme OPTIMISES [5] allow the optimization profil of the sphere-cylinder shell for any value of geometrical constants , the material characteristics and the function parameters.

IV.CONCLUSION

It appreciate that the obtained results by author corresponds with the imposed condition to obtain a design for which the circumferential maxim stress $\sigma_{\theta\max}=20.55$ MPa to become constant in the whole shell without the jump of the [4] circumferential stress. The difference between the maxim circumferential stress $\sigma_{\theta\max}=20.51$ MPa and the minim circumferential stress $\sigma_{\theta\min}=19.74$ MPa for the optimized shell are very small $\Delta\sigma_{\theta}=0.77$ MPa. For the model selected byJ.Leach presents in fig.1,case 12 the variation of the circumferential stress between the maximum circumferential stress

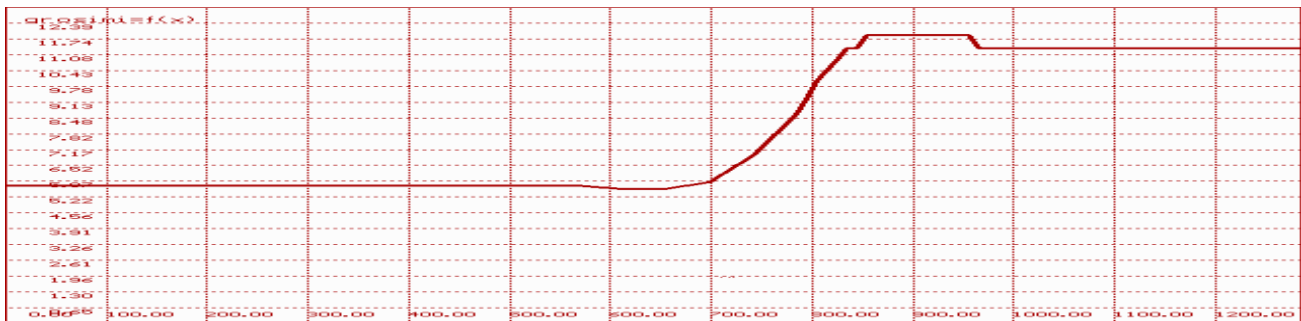


Fig.5 The graphic of the thickness $h=f(x)$

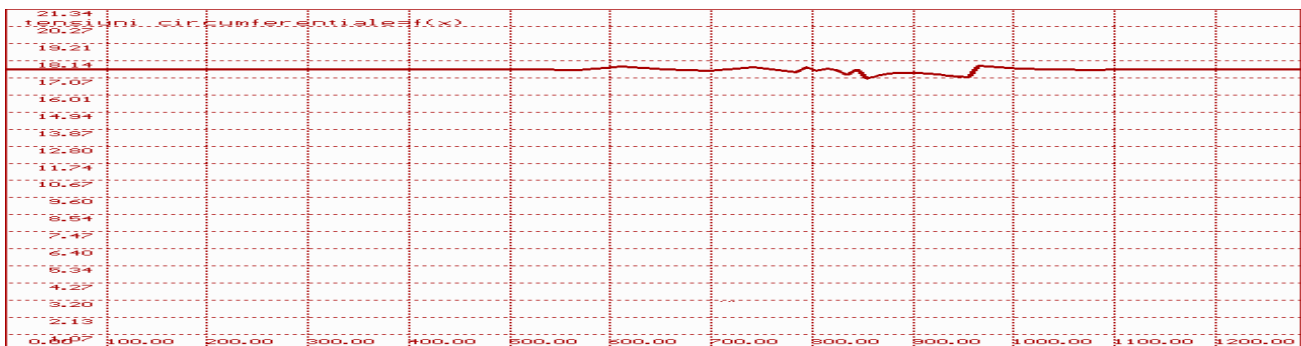


Fig.6 The graphic of the circumferential stress $\sigma_{\theta}=g(x)$

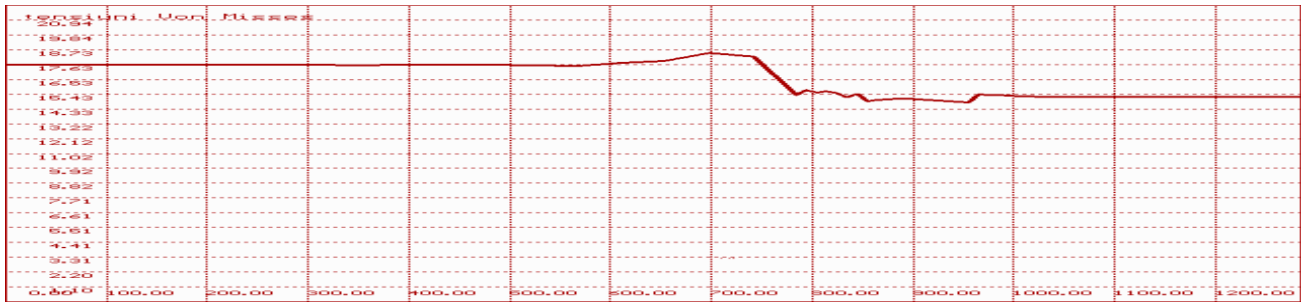


Fig.7 The graphic of the equivalente Von Mises stress σ_{vm} for the optimized design $\sigma_{vm}=q(x)$

and the minimum circumferential stress computed with the BOSOR [4] programme by J.Leach (fig.3) through the finit difference method is $\Delta\sigma_0=(20.55-16.9)$ MPa =3.65 MPa. The conclusion is that through the method proposed by Dora Florea in this paper used the OPTIMISES [5] programme, with the option for the optimization by circumferential stress, it compute an optimized design for which the variation of the circumferential stress $\Delta\sigma_0=0.76$ MPa is very small in comparison with the result acquired by J.Leach [4].

It made the observation that J.Leach[4] studied 13 designs of the sphere-cylinder shell and than proposed a design fig.1,case 12. The optimized design obtain with the method proposed by Dora Florea in this paper ,it acquire very quick after the running the OPTIMISES [5] programme and the results from point of view of state stress are better than obtain by J.Leach[4].

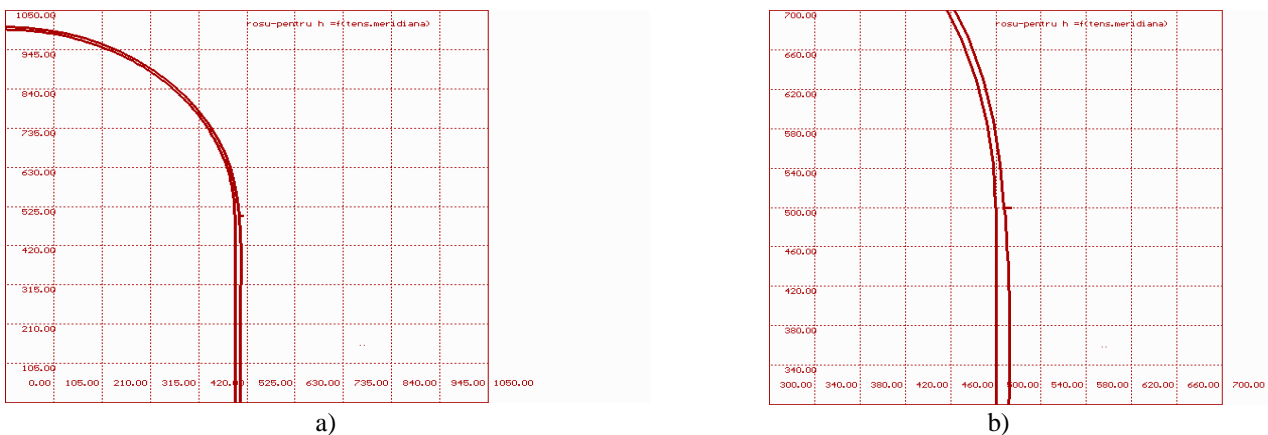


Fig.8 The design of the optimized design with OPTIMISES programme a) section profil b)detail of the joint section profil

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