

**Optimal Tuning Technique Linear Quadratic Regulator Based Speed Analysis
For DC Motor**Arun Kumar Singh Tomar¹, Narottam Dutt Upadhyay²Department of Electrical Engineering, RJIT, Gwalior¹Department of Electrical Engineering, MPCT, Gwalior²

Abstract — Linear Quadratic Regulator (LQR) algorithm is one of the controller methods to control a system. Dc motor speed is controlled by its driving voltage. The higher the voltage, the higher the motor speed. Linear quadratic regulator (LQR) provides an optimal control law for a linear system. It's a control strategy based on minimizing a quadratic performance index. In despite of the good results obtained from this method, the control design is not a straight forward task due to the trial and error method involved in the definition of weight matrices. In such cases, may be hard tuning the controller parameters in order to obtain the optimal behavior of the system. In this work, it proposes a states feedback technique in which there are no trial and error processes involved and the control design is carried out to fulfill specifications, for minimize overshoot and minimize settling and rising times. The proposed technique is based on the use a genetic algorithms. The controller is modeled in MATLAB environment, the simulation results show that the proposed controller gives better performance and less settling time when compared with the traditional PID controller.

Keywords- DC Motor, Optimal Speed Control, Linear Quadratic Regulator (LQR), PID controllers, MIMO System.

I. INTRODUCTION

Electrical drives involving various types of DC motors turn the wheel of industry. The main reason for their popularity is the ability to control their torque and flux easily and independently. Therefore, DC motors are comprehensively used in various industrial applications such as electrical equipment, computer peripherals, robotic manipulators, actuators, steel rolling mills, electrical vehicles, and home appliances. Its applications spread from low horse power to the multi-mega watt due to its wide power, torque, speed ranges, high efficiency, fast response, and simple and continuous control characteristics [1-4]. Controlling the speed of a DC motor is a pivotal issue. The speed of DC motor can be changed by controlling the armature and field voltages. Over the past decades, many techniques have been developed for the DC motor control. Some of these methods were based on classical and also intelligent approaches [5-10]. For DC motors, factors such as unknown load characteristic and parameter variation influence seriously the controlling effect of speed controller. The most commonly used controller for the speed control of DC motors is conventional PID controller. The reason is that the conventional PID controller is easy to implement either by hardware or by software. No deep mathematical theory is necessary to understand how the conventional PID controller works, so everybody is able to imagine what is happening inside the controller during the control process. Traditional PID controllers have very simple control structure and inexpensive cost. In spite of the major features of the fixed PID controller, it has some disadvantages such as the high starting overshoot in speed, the sensitivity to controller gains and the sluggish response due to sudden change in load torque disturbance. Therefore, a great deal of attention has been focused on adaptive or self-tuning of conventional PID controller gains. Tuning PID controller parameters is very difficult, poor robustness; therefore, it's difficult to achieve the optimal state under field conditions in the actual production. In order to overcome some problems that faced by conventional PID controller and achieve accurate control performance of speed control of a DC motor, the other type of control methods can be developed such as linear quadratic regulator. Linear quadratic regulator design technique is well known in modern optimal control theory and has been widely used in many applications. It has a very nice robustness property. This attractive property appeals to the practicing engineers. The linear quadratic regulator technique seeks to find the optimal controller that minimizes a given cost function (performance index). This cost function is parameterized by two matrices, Q and R, that weight the state vector and the system input respectively. In this paper, to achieve accurate control performance of speed control of DC motor, optimal linear quadratic regulator technique is presented. The remainder of the paper is organized as follows: at first the dynamic model of DC motor is briefly reviewed for the purpose of speed control. The next section the basic concept and design of linear quadratic regulator controller is briefly reviewed. Then the simulation results are presented. Finally, the last section states the main conclusion.

II. PLANT MODEL

Direct current motors are widely used for various industrial and domestic applications. There are two main method of controlling a DC motor. The first one is called field control, and has a constant voltage to set up the armature current, while a variable voltage applied to the stator induces a variable magnetic flux. The second named armature control consists of maintaining the stator magnetic flux constant, and varying the armature current. In this paper, the separately

excited DC motor model is chosen due to its good electrical and mechanical performances more than other DC motor models. The circuit of the separately excited DC motor is shown in figure 1. Objective is to control the speed of the separately excited DC motor by armature voltage control [1-4].

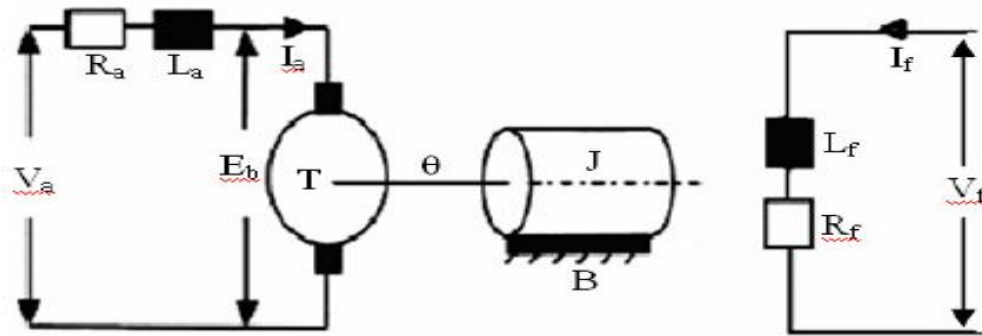


Figure 1. DC motor Model

Assuming constant field excitation the armature circuit electrical equation is written as:

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E_b \quad (1)$$

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + K_b \omega \quad (2)$$

Where

V_a = Input terminal voltage (armature voltage) in volt,

E_b = Back emf in volt,

R_a = Armature resistance in ohm,

L_a = Armature inductance in H.

K_b = Back emf constant in Vs/rad,

ω = Angular speed in rad/s,

i_a = Armature current in A.

The dynamics of the mechanical system is given by the following torque balance equation

$$T = K_T i_a = J \frac{d\omega}{dt} + B\omega \quad (3)$$

Where J = moment of inertia of the motor in kgm^2/s^2 ,

T = motor torque in Nm,

B = viscous friction coefficient in Nms,

K_T = torque factor constant in Nm/A.

Equation (1), equation (2) and (3)

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a} \omega + \frac{V_a}{L_a} \quad (4)$$

$$\frac{d\omega}{dt} = \frac{K_T}{J} i_a - \frac{B}{J} \omega \quad (5)$$

To design a desired controller using the LQR technique, the system should be expressed in the state space form. In the state space model of a separately excited DC motor, the equation (4) and equation (5) can be expressed by choosing the angular speed (ω) and armature current (i_a) as state variables and the armature voltage (V_a) as an input.

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_T}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} V_a \quad (6)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} \quad (7)$$

III. LINEAR QUADRATIC REGULATOR

Linear Quadratic Regulator (LQR) means the settings of a (regulating) controller governing either a machine or process are found by using a mathematical algorithm that minimizes a cost function with weighting factors supplied by a human (engineer). The "cost" (function) is often defined as a sum of the deviations of key measurements from their desired values. In effect this algorithm therefore finds those controller settings that minimize the undesired deviations, like deviations from desired altitude or process temperature. Often the magnitude of the control action itself is included in this sum as to keep the energy expended by the control action itself limited.

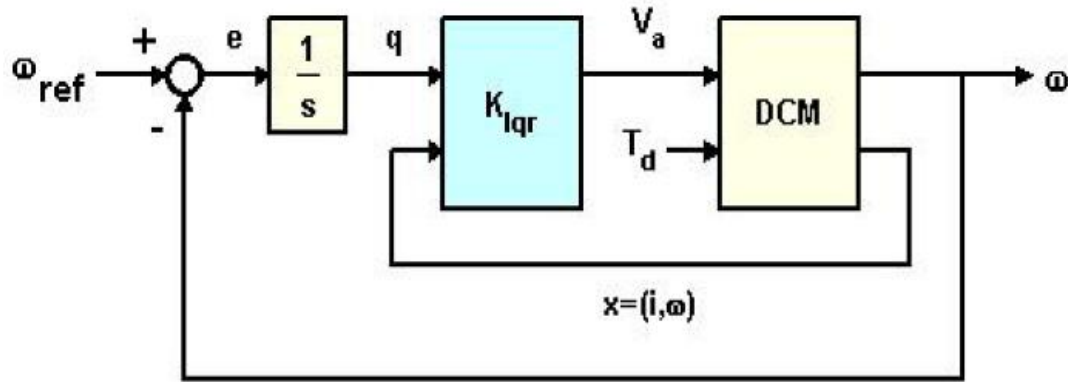


Figure 2. LQR Model

The LQR algorithm takes care of the tedious work done by the control systems engineer in optimizing the controller. However, the engineer still needs to specify the weighting factors and compare the results with the specified design goals. Often this means that controller synthesis will still be an iterative process where the engineer judges the produced "optimal" controllers through simulation and then adjusts the weighting factors to get a controller more in line with the specified design goals. The LQR algorithm is, at its core, just an automated way of finding an appropriate state-feedback controller. And as such it is not uncommon to find that control engineers prefer alternative methods like full state feedback (also known as pole placement) to find a controller over the use of the LQR algorithm. With these the engineer has a much clearer linkage between adjusted parameters and the resulting changes in controller behavior. Difficulty in finding the right weighting factors limits the application of the LQR based controller synthesis [6]. Linear Quadratic Regulator (LQR) is the most common approach to modern control design, and one of the controller that are commonly used by users to control the system besides Proportional Integral Derivatives (PID) because its stability.

Suppose that the space model is

$$\begin{aligned} \dot{x} &= A_x + B_u \\ y &= C_x + D_u \end{aligned}$$

Suppose that we have sensors to measure the entire state and that we use a controller (regulator)

$$u = -Kx$$

that seeks to drive the state to zero. You could use pole placement via Ackerman's formula. Here, we use the LQR methodology to specify the gain K. For this, let

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

Where $Q = Q \geq 0$ and $R = R > 0$.

The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function and we seek to find the gain vector K to minimize this "cost function".

IV. SIMULATION RESULTS

In order to verify the validity of the linear quadratic regulator controller, simulation tests are carried out using MATLAB software. The performance of linear quadratic regulator controller has been investigated and compared with the conventional PID controller. Figure 3 and figure-4 shows the step responses of PID Controller and LQR controller respectively. According to the simulation results, linear quadratic regulator method give the better performance compared to traditional PID controller.

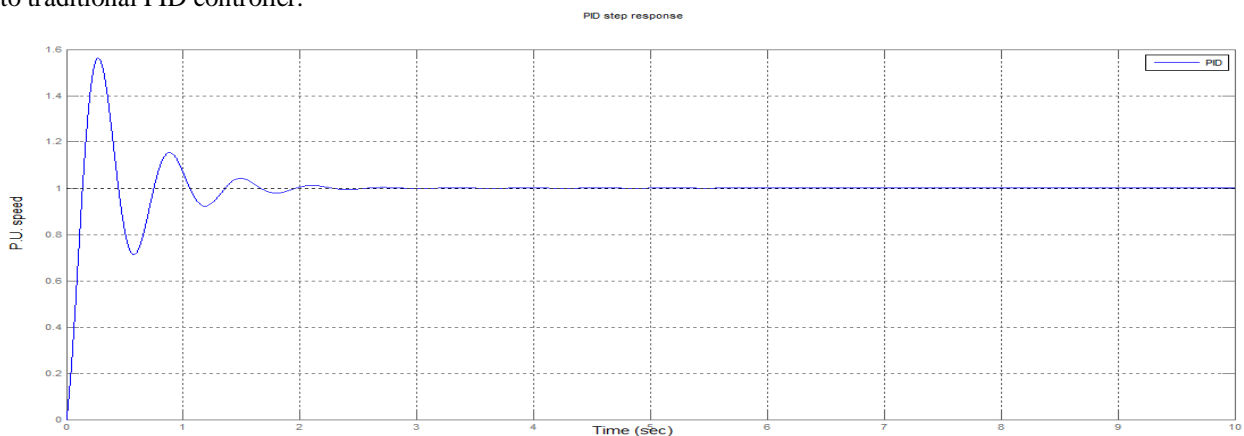


Figure-3 Step response of PID Controller

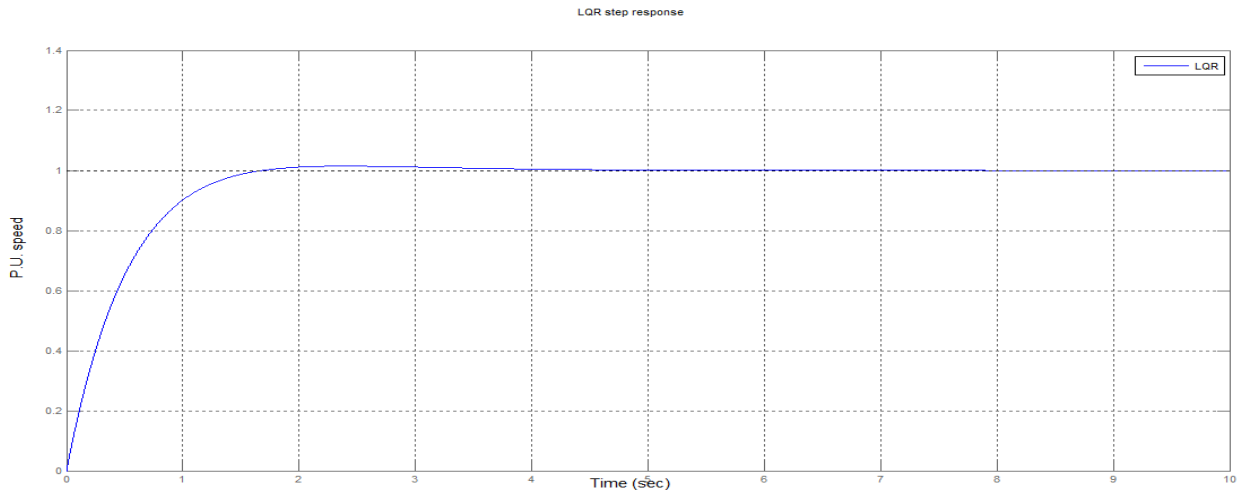


Figure-4 Step Response of LQR Controller

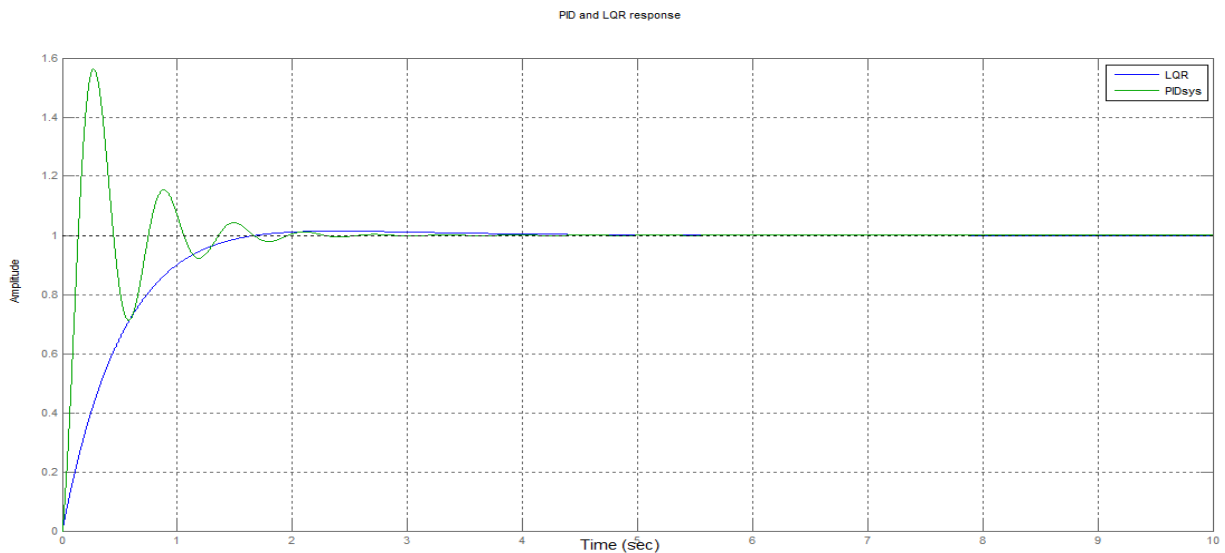


Figure-5 Step Response of PID and LQR Response

The time response parameters percent overshoot; settling time, rise time, and steady state error for LQR and PID controller are presented in Table 1.

Table-1 Performance of LQR and PID Controller

Time Response Specification	LQR	PID
Settling Time (T_s)	1.43	1.83
Peak Amplitude	1.01	1.56
Overshoot %	1.44	56.4

V. CONCLUSIONS

Optimal LQR controller and conventional PID controller have been considered in this paper for controlling the speed of DC motor. The performance of the two controllers is validated through simulations. Based on the comparative simulation results, one can conclude that the linear quadratic regulator controller realizes a good dynamic behavior of the DC motor with a rapid settling time, less peak amplitude and minimum overshoot compared to conventional PID controller under nominal condition. The comparison between the speed control of the separately excited DC motor by linear quadratic regulator technique and conventional PID controller shows clearly that the linear quadratic regulator technique gives better performances than conventional PID controller against parameter variations. Furthermore, the simulation results so obtained show that the conventional PID controller gives greatest value of percent overshoot and longer settling time.

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