

**Stability condition for PSV: cyclostationary detection**

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**Abstract**—the Uses spectral correlation of cyclostationary signals to reduce effect of noise & interference on desire signal. PSV filter exploits the cyclic frequencies of signal the more different cyclic frequencies the signals have, the more effective the filter will be. The stability criteria for two-dimensional (2-D) periodically shift variant (PSV) filters, represented in state space by the first model of FornasiniMarchesini (FM) with periodic coefficients is interesting challenge. GivoneRoesser Model is an extension for 2-D representation an algorithm with using linear matrix inequality is proposed to determine the stability of a given 2-D PSV system by sufficient conditions derived for stability analysis.

**Keywords**— PSV, FM, GR,2-D

**I. INTRODUCTION**

Two dimensional (2-D) PSV filters have applications in processing digital video with cyclo-stationary noise, and 2-D multirate filter banks. PSV filters are also important for designing 2-D filters with power-of-two coefficients. PSV stands for Periodically Shift Variant system (filter). As its name suggests, it is in between shift invariant & shift variant system. There are a number of ways to relax the time-invariance implicit in the assumption of stationarity - our research centers on the assumption of *cyclostationary* fields to describe a pulse train. Cyclostationary fields are fields whose field statistics are shift invariant with respect to a specific time shift. Hence, for a general 2D transfer function or transfer matrix it is desirable to obtain a state-space realization with as low order as possible. Note that by “a general system” we mean that there is not any restriction on the coefficients in the transfer function or transfer matrix of the system.

**II. SYSTEM MODELING**

It is considerable that not like 1D one dimensional case, it is tough to get minimum state space realization in 2D state system with some exception along special categories. ‘A general system’ implies no condition on boundaries on coefficient in transfer matrix of the system.

Difference equation and state space representation is

$$y(i, j) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{mn}(i, j) y(i-m, j-n) + \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} b_{mn}(i, j) y(i-m, j-n)$$

Where  $(m, n) (0, 0)$  for  $a_{mn}$ . The coefficient are periodically shift variant with period P and Q.

$$a_{mn}(i, j) = a_{mn}(i+P, j+P) = a_{mn}(i+P, j) = a_{mn}(i, j+Q)$$

$$b_{mn}(i, j) = b_{mn}(i+P, j+P) = b_{mn}(i+P, j) = b_{mn}(i, j+Q)$$

With help of Fornasini-Marchesini 1 model have been searched with attention for stability of 2D discrete system. The main research challenge is FM1 model of 2D system sufficient condition for asymptotic stability.

Linear matrix inequalities is form of problem of system regarding system and control [19]. To solve the problem interior point methods have been applied. High dimensional matrixes can be sufficient for these algorithms like problem with large number of LMIs. This paper discuss to solve the LMI feasible issue which is used for sub gradient simplex based cutting plane method. By this method we can have feasible solution with repetitively cutting off the not feasible part of given polyhedron

Main step for cutting plan methods is calculation of query point. Finding query point in three level process efficiently is done with sub gradient simplex based cutting plan. To obtain query point easily half way along sub gradient.

A sphere inscribed in a corner or the Chebyshev center is calculated based on simplex tableaus to ensure the query points are deep inside. Redundant constraints can also be pruned based on simplex tableaus. Linear matrix inequality (LMI) techniques are powerful design tools in system and control areas. In linear system design and robust control analysis, a large number of design specifications and constraints can be formulated as LMIs, i.e., linear combinations of decision variables with constant matrix coefficients. A typical application is the system stability [20] problem to search for a common Lyapunov function over the intersection of a set of LMIs. For example, a robust control system is stable if a set of Lyapunov inequalities resulting from different assumptions are satisfied. For a switched system described by a family of linear time invariant Subsystems to be stable, these subsystems are desirable to share a common Lyapunov function.

### III. LYAPUNOV STABILITY

Lyapunov stability is named after Aleksandr Lyapunov, a Russian mathematician who published his book *The General Problem of Stability of Motion* in 1892. Lyapunov was the first to consider the modifications necessary in nonlinear systems to the linear theory of stability based on linearizing near a point of equilibrium. His work, initially published in Russian and then translated to French, received little attention for many years. Interest in it started suddenly during the Cold War (1953-1962) period when the so-called “Second Method of Lyapunov” was found to be applicable to the stability of aerospace guidance systems which typically contain strong nonlinearities not treatable by other methods. A large number of publications appeared then and since in the control and systems literature. In control engineering, a state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. “State space” refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space.

### IV. GIVONE-ROESSER MODEL

The 2-D state-space theory was introduced by Roesser. Since then, several other works have appeared and so far the use of 2-D systems do not cease to increase. Based on these works, several properties concerning 2-D systems such as controllability, observability and realization have been investigated. This paper concentrates on stabilization of 2-D systems. In fact, the stability of 2-D systems using the 2-D Lyapunov equation has already been studied in, while the state and output feedback stabilization problem is treated in, by solving a set of 2-D polynomial equations. Further, most of the available works in the literature of 2-D systems consider only state-feedback stabilization, or dynamic output-feedback control. However, state-feedback controllers require the measurement of every state, some of which may be difficult to measure. On the other hand, dynamic output-feedback controllers (which include systems with state observers) result in high order controllers which may not be practical in real applications. Instead, the static output-feedback controllers are less expensive to implement and more reliable so they will be studied in this paper. In 2-D systems area, static output-feedback stabilization problem is not fully

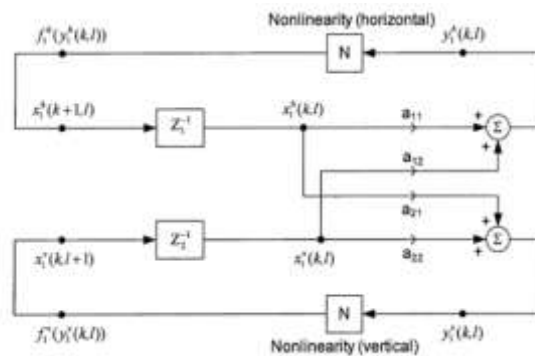


Fig 1

FM-2 system

The main issues in the design of any control system are stability analysis and stabilization. With the introduction of state-space models of 2-D discrete systems, various Lyapunov equations have emerged as powerful tools for the stability analysis and stabilization of 2-D discrete systems. Lyapunov based sufficient conditions for the stability of 2-D discrete systems have been studied. When the dynamics of practical systems are represented using state-space models, errors are inevitable as the actual system parameters would be different than the estimated system parameters, i.e., the model parameters. The cause of errors are the approximations made during the process modeling, differences in presumed and actual process operating points, change in operating conditions, system aging etc. Control designs based on these models, therefore, may not perform adequately when applied to the actual industrial process and may lead to instability and poor performances. This has motivated the study of robust control for the uncertain 2-D discrete systems. The aim of robust control is to stabilize the system under all admissible parameter uncertainties arising due to the errors around the nominal system. Many significant results on the solvability of robust control problem for the uncertain 2-D discrete systems have been proposed in.

Consider the 2-d periodically shift variant system (zero I / p) described by FM-2 model:

$$x(k+1, l+1) = (A_1 + OA_1)x(k, l+1) + (A_2 + OA_2)x(k+1, l)$$

$$\begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} - A^T(P_1 + P_2)A > 0$$

Checking stability of various derived theorem taking example of LSIV 2D attasi model filter

A linear 2-D PSV system represented by the first model of FM-1 is given as

$$X(h+1,k+1) = A_1(h,k)X(h,k+1) + A_2(h,k)X(h+1,k+1) + A_0(h,k) X(h,k) + B(h,k)U(h,k)$$

$$Y(h, k) = C(h, k) X(h, k) \dots$$

Where a state vector  $x(h,k) \in \mathbb{R}^{L \times 1}$  an input  $u(h, k)$ , and an output  $y(h, k)$  are scalar, state matrices and vectors  $A_0(h, k)$ ,  $A_1(h, k)$ ,  $A_2(h, k) \in \mathbb{R}^{L \times L}$ ,  $B(h, k) \in \mathbb{R}^{L \times 1}$  and  $C(h, k) \in \mathbb{R}^{L \times 1}$  are periodically shift variant with period  $(P, Q)$  where  $P$  and  $Q$  are positive integers, not both zero, i.e.  $A_0(h, k) = A_0(h+P, k) = A_0(h, k+Q)$ .

Similarly for FM2 models with coefficient of only  $A_1$  and  $A_2$  and their example with matrix form also with stability criteria is discuss further

LSIV 2-D Attasi's model filter using circulant matrices as following

$$A_1 = \begin{bmatrix} 0.5 & -0.5 & 0.125 & -0.125 \\ -0.125 & 0.5 & -0.5 & 0.125 \\ 0.125 & -0.125 & 0.5 & -0.5 \\ -0.5 & 0.125 & -0.125 & 0.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.5 & 0 & -0.015 & 0.25 \\ 0.25 & 0.5 & 0 & -0.015 \\ -0.015 & 0.25 & 0.5 & 0 \\ 0 & -0.015 & 0.25 & 0.5 \end{bmatrix}$$

$$B = [1 \ 0.39 \ -1 \ 0.45]$$

$$C = [1 \ -1 \ -1 \ 1]$$

Where  $A_0 = A_1 A_2$ .

Our derived purposed theorem 1

$$\begin{bmatrix} -P & PA & 0 & PH \\ A^T P & -Q & \epsilon E^T & 0 \\ 0 & \epsilon E & -\epsilon I & 0 \\ H^T P & 0 & 0 & -\epsilon I \end{bmatrix} < 0$$

$$\text{Where } A = [A_1 \ A_2]$$

Detail show above is related to FM1 and now consider the

For above matrices  $A_0 A_1 A_2$  has been satisfies using Matlab LMI toolbox hence the system is stable is proved, as we considered stable systems the condition is satisfying both side so we can say theorem is worked properly.

Following 2-D discrete system represented by FM second model

$$x(i+1, j+1) = A_1 x(i, j+1) + A_2 x(i+1, j) + B_1 u(i, j+1) + B_2 u(i+1, j)$$

$$z(i, j) = Cx(i, j) + Du(i, j) \quad i \geq 0, j \geq 0$$

Following results are proposed by us for stability conditions of a 2-D PSV system represented by FM-2 model. Same has been verified by numerical example also. The first results can be stated as follows. A 2-D PSV System represented by FM-2 model is globally asymptotically stable provided there exist  $n^* \times n$  positive definite symmetric matrices  $P_1$  and  $P_2$  and positive scalars  $\epsilon$  such that, for  $A = [A_1 \ A_2]$ ,

$$H = [H_1 \ H_2], \quad E = E_1 \oplus E_2$$

$$\begin{bmatrix} -P & PA & 0 & PH \\ A^T P & -Q & \varepsilon E^T & 0 \\ 0 & \varepsilon E & -\varepsilon I & 0 \\ H^T P & 0 & 0 & -\varepsilon I \end{bmatrix} < 0$$

Where  $P = P_1 + P_2$   
 $Q = P_1 \oplus P_2$

The second result can be stated as follows. 2-D PSV System represented by FM-2 model is globally asymptotically stable provided there exist  $n \times n$  positive definite symmetric matrices  $P_1, P_2$  and positive scalars  $\varepsilon_1, \varepsilon_2$  such that

$$\begin{bmatrix} P_1 - \varepsilon_1 E_1^T E_1 & 0 & A_1^T P & PH_1 & PH_2 \\ 0 & P_2 - \varepsilon_2 E_2^T E_2 & A_2^T P & 0 & 0 \\ PA_1 & PA_2 & P & 0 & 0 \\ H_1^T P & 0 & 0 & \varepsilon_1 I & 0 \\ H_2^T P & 0 & 0 & 0 & \varepsilon_2 I \end{bmatrix} > 0$$

Where  $P = P_1 + P_2$

For above matrices  $A_1, A_2$  has been satisfies using mat lab LMI toolbox hence the system is stable is proved, as we considered stable systems the condition is satisfying both side so we can say theorem is worked properly.

Zero-input stability of a GR state-space model is studied so that only the periodic coefficient matrices  $A_1(i, j), A_2(i, j), A_3(i, j), A_4(i, j)$  need to be considered. A zero-input 2-D PSV GR model can be written as

$$\begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = \begin{bmatrix} A_1(i, j) & A_2(i, j) \\ A_3(i, j) & A_4(i, j) \end{bmatrix} \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix}$$

$$w(i, j) = \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix}$$

Then the above can be modified as

$$w(i, j) = A^{10}(i-1, j)w(i-1, j) + A^{01}(i, j-1)w(i, j-1)$$

$$A^{10}(i, j) = \begin{bmatrix} A_1(i, j) & A_2(i, j) \\ 0 & 0 \end{bmatrix}$$

$$A^{01}(i, j) = \begin{bmatrix} 0 & 0 \\ A_3(i, j) & A_4(i, j) \end{bmatrix}$$

2-D PSV System represented by GR model is globally asymptotically stable provided there exist  $n \times n$  positive definite symmetric matrices  $P_1, P_2$  and positive  $k_1, k_2, k_3, k_4$  such that  $S > 0$

$$S = \begin{bmatrix} p_1 - k_1 E_1^T E_1 - k_3 E_3^T E_3 & 0 & A_1^T P & A_1^T P_1 & 0 & 0 & 0 & 0 \\ 0 & p_2 - k_2 E_2^T E_2 - k_4 E_4^T E_4 & A_2^T P & A_2^T P_2 & 0 & 0 & 0 & 0 \\ P_1 A_1 & P_1 A_2 & P_1 & 0 & P_1 H_1 & P_1 H_2 & 0 & 0 \\ P_2 A_3 & P_2 A_4 & 0 & P_2 & 0 & 0 & P_2 H_3 & P_2 H_4 \\ 0 & 0 & H_1^T P_1 & 0 & k_1 I & 0 & 0 & 0 \\ 0 & 0 & H_2^T P_2 & 0 & 0 & k_2 I & 0 & 0 \\ 0 & 0 & 0 & H_3^T P_1 & 0 & 0 & k_3 I & 0 \\ 0 & 0 & 0 & H_4^T P_2 & 0 & 0 & 0 & k_4 I \end{bmatrix}$$

Using above attasi model circulant matrix and converting it in to GR model matrix we have checked stability criteria for both equations hence we can say derived criteria gives best results

Conclusion: After taking examples of attasi model filter with circular matrix as a PSV system coefficient for checking our derived criteria for stability, it is successfully gives stable result for all the theorems but for GR model first and GR model second theorem gives best result compare to FM1 and FM2 stability criteria

## V. CONCLUSION

A review on the stability of 2-D discrete systems described by FM second model has been presented in this paper. Example of cyclostationary signal is processing with PSV system and FM, GR models necessary and sufficient condition verified. Among the three model the FM2 and GR is most appropriate model for system stability and reliability of system. For the future scope the different type of system can be characterize using the FM1, FM2 and GR model for get the stability analysis.

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