

Channel Estimation for MIMO based-Polar Codes

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Abstract— The popular scheme for estimating the wireless channel is through the transmission of pilot or training symbols. pilot symbols are a predetermined fixed set of symbols which are transmitted over the wireless channel. This set of symbols is known to the wireless receiver as it is programmed beforehand. The receiver observes the output corresponding to the transmitted pilot symbols and with knowledge of the transmitted pilot symbols, proceeds to estimate the unknown fading channel coefficient.

Keywords—multiple input multiple output (MIMO); Channel State Information(CSI); Channel Capacity,Polar Codes.

I.Introduction

Channel Estimation in Wireless Systems consider the wireless channel model given as $y(k) = hx(k) + n(k)$ where h is the flat-fading channel coefficient. The estimate $\hat{x}(k)$ of the symbol $x(k)$ can then be recovered from $y(k)$ simply as $\hat{x}(k) = y(k)$. This termed the zero-forcing receiver in wireless system. It can be seen now that in order to detect the transmitted symbol $x(k)$ at the receiver, one needs to know the channel coefficient h. The process of computing channel coefficient h at the receiver is termed channel estimation and is an important procedure in every wireless communication system. The popular scheme for estimating the wireless channel is through the transmission of pilot or training symbols. pilot symbols are a predetermined fixed set of symbols which are transmitted over the wireless channel. This set of symbols is known to the wireless receiver as it is programmed beforehand. The receiver observes the output corresponding to the transmitted pilot symbols and with knowledge of the transmitted pilot symbols, proceeds to estimate the unknown fading channel coefficient. This procedure for pilot-based channel estimation is described below.

Consider the transmission of L(p) pilot symbols $x^{(p)}(1), x^{(p)}(2), x^{(p)}(3), \dots, x^{(p)}(L^{(p)})$ for the purpose of channel estimation. Let the corresponding received outputs be $y^{(p)}(1), y^{(p)}(2), y^{(p)}(3), \dots, y^{(p)}(L^{(p)})$, i.e. each $y^{(p)}(k)$, is the output corresponding to the transmitted pilot symbol $x^{(p)}(k)$. The model for these received pilot symbol is given as $y^{(p)}(k) =$

To simplify the derivation below, let us assume for a moment that all the quantities $y^{(p)}(k), x^{(p)}(k), n(k)$ and the channel coefficient h are real. Due to the presence of noise $n(k)$ in the above system, it is clear that for any k. Thus, one has to determine an estimate of h from the noisy observation samples $y(k)$. Intuitively then, a reasonable estimate of h can be derived as a minimizer of the cost function

$$\hat{h} = \arg \min_h \left\{ \left(y^{(p)}(1) - hx^{(p)}(1) \right)^2 + \left(y^{(p)}(2) - hx^{(p)}(2) \right)^2 + \dots + \left(y^{(p)}(L^{(p)}) - hx^{(p)}(L^{(p)}) \right)^2 \right\}$$

$$= \sum_{k=1}^{L^{(p)}} \left(y^{(p)}(k) - hx^{(p)}(k) \right)^2 = \xi(h)$$

The above minimization aims to find the best estimate of h which corresponds to the lowest observation error $\xi(h)$ and is, hence, termed the least-squares estimate. Naturally, the convenient way to minimize the error function $\xi(h)$ above is differentiate it and set it equal to zero. This procedure yields

$$\frac{d\xi(h)}{dh} = \sum_{k=1}^{L^{(p)}} 2 \left(y^{(p)}(k) - hx^{(p)}(k) \right) x^{(p)}(k)$$

$$0 = \sum_{k=1}^{L^{(p)}} x^{(p)}(k) \left(y^{(p)}(k) - \hat{h}x^{(p)}(k) \right)$$

$$\text{therefore } \hat{h} = \frac{\sum_{k=1}^{L^{(p)}} y^{(p)}(k)x^{(p)}(k)}{\sum_{k=1}^{L^{(p)}} \left(x^{(p)}(k) \right)^2}$$

Thus, one can compute the channel estimate of the fading channel coefficient h. Let us now derive a more elegant matrix-based framework to derive the result above. The vector model for the pilot-symbol transmission reception is given as

$$\begin{bmatrix} y^{(p)}(1) \\ y^{(p)}(2) \\ \vdots \\ y^{(p)}(L^{(p)}) \end{bmatrix} = h \begin{bmatrix} x^{(p)}(1) \\ x^{(p)}(2) \\ \vdots \\ x^{(p)}(L^{(p)}) \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2) \\ \vdots \\ n(L^{(p)}) \end{bmatrix}$$

Hence, the vector model for the above system can be comprehensively given as

$$y^{(p)} = hx^{(p)} + n$$

where $y^{(p)}$, $x^{(p)}$, n are $L^{(p)}$ dimensional vectors.

The least-squares estimate of the channel coefficient h given as

$$\begin{aligned} \hat{h} &= \arg \min_h \|y^{(p)} - hx^{(p)}\|^2 \\ &= \arg \min_h \{(y^{(p)} - hx^{(p)})^T (y^{(p)} - hx^{(p)})\} \\ &= \arg \min_h \{(y^{(p)})^T y^{(p)} - 2((x^{(p)})^T y^{(p)})h - ((x^{(p)})^T x^{(p)})h^2\} = \{\xi h\} \end{aligned}$$

As illustrated previously, to minimize the observation error, one can now differentiate the above cost function $\xi(h)$ and set it equal to zero,

compute the estimate \hat{h} as

$$\begin{aligned} \frac{d\xi(h)}{dh} &= -2((x^{(p)})^T y^{(p)}) + 2((x^{(p)})^T x^{(p)})\hat{h} \\ &= -2((x^{(p)})^T y^{(p)}) + 2((x^{(p)})^T x^{(p)})\hat{h} \\ \hat{h} &= \frac{(x^{(p)})^T y^{(p)}}{(x^{(p)})^T x^{(p)}} \\ \hat{h} &= \frac{\sum_{k=1}^L y^{(p)}(k)x^{(p)}(k)}{\sum_{k=1}^L (x^{(p)}(k))^2} \end{aligned}$$

which is identical to the expression derived above in equation. Further, one can now easily derive the expression for the channel estimation for complex numbers h , by simply replacing the transpose operator above with the Hermitian operator. hence, the general expression for the channel estimate when the various quantities are complex numbers is given as

$$\hat{h} = \frac{(x^{(p)})^H y^{(p)}}{(x^{(p)})^H x^{(p)}} = \frac{(x^{(p)})^H y^{(p)}}{\|x^{(p)}\|^2}$$

II.MIMO CHANNEL MODEL

The basic form of antenna technology is Single Input Single Output (SISO) having one antenna at the transmitter and one at the receiver and has a simple configuration with many advantages. The performance of this system is poor owing to multipath fading and interference, and based on the channel capacity theorem, channel bandwidth is allocated to the system. Blockier et al (2002) have discussed the delay spread channels having more advantages and compared to flat fading channel in terms of capacity. The channel performance is determined in terms of bit error rate, mean square error and symbol error rate in this single input single output orthogonal frequency division multiplexing system.

III.CHANNEL ESTIMATION

MIMO system has multiple antennas at the transmitting and receiving ends that provide high link reliability and data rate. In wireless communication system, the received signal is degraded owing to the multipath fading and the characteristics of the channel and so, the channel should be estimated to recover the original signal in the receiver. Ye et al(2002) have presented the signal detection with enhanced channel estimation. Channel estimation plays a vital role in MIMO-OFDM system as it is used to estimate the channel coefficient corresponding to all transmit and receive antenna pairs on all subcarrier positions. It is selected based on time variation of the channel, Implementation and computational complexity of the system. Berna & Reyat (2005) discussed Linear Mean Square (LMS) and Recursive Mean Square (RLS) algorithms are used to estimate the channel using three transmit antennas. Here, Least Square (LS) program is proposed to estimate the MIMO-OFDM systems.

IV. Least Square Channel Estimation

Least Square (LS) technique is used to minimize the square distance between the original signal and the received signal. The channel is estimated without any knowledge on the statistics of the channel. It estimates the least value of the square of the error and requires matrix inversion and pilot symbols and it has low computational complexity, but high mean square error. The channel coefficients are calculated by using this equation

$$H_{LS} = X^{-1}Y$$

H_{LS} - Channel Matrix

X - Input Matrix Y - Output Matrix

For 2x2 MIMO-OFDM system

The received signals can be given as

Channel coefficients of 2x2 MIMO-OFDM systems

Table.1 Channel Coefficients for 2x2 MIMO-Polar Codes

H11	H12	H21	H22
0.0964	0.0754	0.0586	0.0699
0.1160	0.0770	0.0622	0.0516
0.0717	0.0904	0.0583	0.0123
0.0777	0.0890	0.0667	0.0554
0.1179	0.0572	0.0812	0.0464

The channel coefficients of 2x2 MIMO-OFDM systems for only five subcarriers are shown in Table Least square technique estimates the channel coefficients which are pre-owned to calculate the bit error rate.

V.Bit Error Rate

In the communication system, information is lost over a communication channel owing to the external noise, distortion, bit synchronization and interference. Bit error rate is formalize as the number of bits lost branched by the total number of transmitted bits in the system.

Bit Error Rate = Number of bits lost / Total number of bits sent to reduce the bit error rate in this system, proper modulation schemes are selected. As bit error rate and data rate are inversely proportional to each other so that bit error rate should be minimized to increase the data rate.

MIMO- OFDM setup with 64 Quadrature Amplitude Modulation (QAM) is plotted in terms of bit error rate with regard to signal to noise ratio. From this figure it is clear that bit error rate reduces in the order of 1x1 (SISO), 2x1 (MISO), 1x2 (SIMO) and 2x2 (MIMO) OFDM systems as the signal to noise ratio is increased. From these simulation results, it is form that in MIMO-OFDM system, the bit error rate keeps on reducing better than other OFDM systems.

VI. Mean Square Error

Mean square error is one of the significant parameters to estimate the difference between the theoretical values of an estimator and true value of the quantity being estimated. It calculates the average of the squares of the error. In Figure the performance of SISO, SIMO, MISO and MIMO- OFDM systems with 64

Quadrature Amplitude Modulation (QAM) is plotted for 64 subcarriers in terms of mean square error with respect to signal to noise ratio. From this figure it is inferred that mean square error reduces in the order of 1x1 (SISO), 2x1 (MISO), 1x2 (SIMO) and 2x2 (MIMO) OFDM

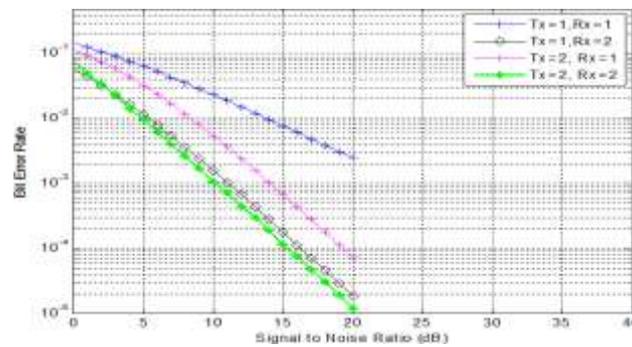
systems as the signal to noise ratio is increased, it is found that in MIMO-OFDM system, the mean square error keeps on reducing better than other OFDM systems

Symbol Error Rate

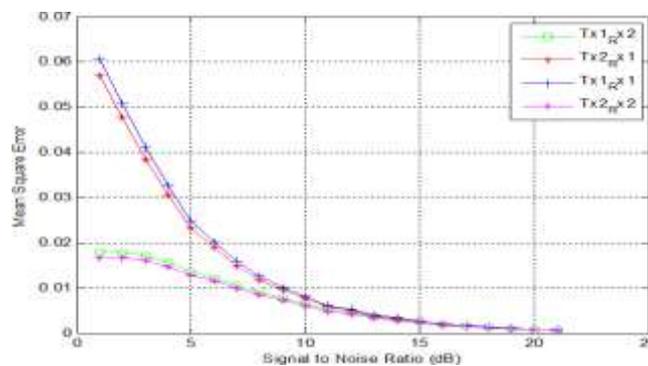
It is defined as the number of symbol changes in transmission channel per second. It is measured for 1x1, 1x2, 2x1 and 2x2 MIMO-OFDM systems. Figure 3.7 shows the graph of symbol error rate with signal to noise ratio. From this graph, it is clear that the symbol error rate is reduced as signal to noise ratio increases for all the four types of OFDM systems namely Single Input Single Output (SISO), Single Input Multiple Output (SIMO), Multiple Input Single Output (MISO) and Multiple Input Multiple Output (MIMO).

Conclusion

LS channel estimator is used to calculate the channel coefficients. The four different OFDM systems are analyzed and simulated. The results consists of four parameters namely bit error rate, mean square error, symbol error rate and capacity of the channel for MIMO-OFDM systems. The bit error rate values are minimized in 2x2 MIMO-OFDM systems compared to other 1x1, 1x2, 2x1 OFDM systems. Similarly channel capacity is maximized in 2x2 MIMO-Polar code systems, compared to the 1x1 SISO-OFDM systems. The number of antenna such as 3x3 and 4x4 are increased, the error rate will be minimized. Due to computational complexity and cost of added antenna, maximum 2x2 MIMO channels are taken and analyzed in this work.



The signal to noise ratio for varied transmitting and receiving antennas shown in fig up to maximum two transmit and receive antennas with respect to BER and MER.



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