

**LORENTZIAN ESTIMATION FOR RICIAN DE-NOISING IN MRI IMAGES**

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Abstract - The accuracy of neuro imaging techniques like MRI exclusively depends in the visual analysis of an image. Noise in the MRI is modeled as Rician distribution introduces a signal dependent bias thereby reducing the image contrast. In this paper an efficient algorithm is proposed for Rician de-noising using Adaptive Lorentzian estimation from the most similar neighborhood pixel extracted using Rank Ordered Absolute Difference Statistic(ROAD) statistic after noise estimation. The performance of the algorithm is evaluated quantitatively using PSNR and by visual analysis. The results show the efficiency of the algorithm for noise reduction and radiological analysis.

Keywords- ROAD, Rician noise, Lorentzian estimator.

I.INTRODUCTION

Magnetic resonance imaging (MRI) is a powerful diagnostic technique used in radiology to visualize detailed internal structures of the human body. The magnitude images are formed by calculating the magnitude, pixel by pixel, from the real and the imaginary images. This form is a nonlinear mapping and therefore the noise distribution is no longer Gaussian. It is assumed that the noise in each signal to have a Gaussian distribution with zero mean and each channel to be contaminated with white noise.

NOISE in magnitude Magnetic Resonance (MR) images is usually modeled by means of a Rician distribution, due to the existence of zero-mean uncorrelated Gaussian noise with equal variance in both the real and imaginary parts of the complex k-space data [1], [2]. This noise may affect the performance of different post processing techniques applied to MR data, such as segmentation, registration or tensor estimation in diffusion tensor MRI (DT-MRI) [3].

In this paper, an adaptation of the Median Absolute Deviation (MAD) estimator in the wavelet domain is used for Rician noise estimation. This robust and efficient estimator has been proposed by Donoho [4] for Gaussian noise and since has been widely used in image processing.

Robust estimation is presented to remove rice noise effectively. The proposed algorithm uses simple fixed length window of size 3 x 3 for noise detection based on Rank Ordered Absolute difference (ROAD) The Robust estimation holds well in retaining the local features and edges in the image and to deal with intensity discontinuities.

II.RELATED WORK

Nowak and Robert[5] [16] proposed a simple wavelet based de-noising technique which could not differentiate the small structures from noise. Spatial and temporal filtering proposed in [6][1] eliminated high frequency but introduced blurring effect of fine details in image. Anisotropic Diffusion methods [7][9] removes noise using gradient information. In [8][13] discusses a noise adaptive non-linear filter which requires prior noise information during image acquisition for de-noising. Wavelet Shrinkage technique [9][20] for denoising uses a threshold based shrinking technique. The disadvantage is the requisite of variance of wavelet coefficients depending on Gaussian distribution. In Transform domain filters[10][25] the noise is separated and smoothed based on prior information.

III.RANK ORDERED ABSOLUTE DIFFERENCE STATISTIC (ROAD)

The ROAD statistic [11] is a measure of how close a pixel value is to its n most similar neighbors. This technique is based on the fact that,

- 1) Unwanted impulses will vary greatly in intensity from most or all of their neighboring pixels.
- 2) Pixels composing the actual image should have at least half of their neighboring pixels of similar intensity. This is true even pixels on an edge where there is a sharp transition between the colors.

A. Pixel Neighborhood

Let $x=(x_1, x_2)$ be the location of the pixel under consideration and let $P(N)$ be a set of points in a $(2N+1) \times (2N+1)$ neighborhood centered around x for some positive integer N given by

$$P(N)=\{x_{i,j}, -N \leq i,j \leq N\} \quad (2)$$

In this discussion we consider $N=1$.

Let P_x^0 represents the set of points in a 3 x 3 neighborhood of x . For each $y \in P_x^0$ let $d_{x,y}$ denote the absolute difference between the pixels x and y given by,

$$d_{x,y}=|u_x-u_y| \quad (3)$$

$d_{x,y}$ values are sorted in increasing order to define $ROAD_m(x)$ given by

$$ROAD_m(x) = \sum_{i=m1}^{m2} r_i(x) \quad (4)$$

The value of $m1$ and $m2$ depend on the statistic of the noise free neighborhood pixels.

IV. ROBUST STATISTICS

The field of robust statistics is concerned with estimation problems in which the data contains outliers. Robust estimation algorithms can be classified into three large types of estimators: M-estimator, L-estimator, and R-estimator. An M-estimator is a maximum likelihood-type estimator, and it is obtained by solving a minimization problem..

The M-estimators were initially proposed by Huber (1964)[12] as a generalization of the maximum likelihood estimator. The M estimator addresses the problem of finding best fit to the model $d = \{d_0, d_1, d_2, \dots, d_{s-1}\}$ to another model. $e = \{e_0, e_1, e_2, \dots, e_{s-1}\}$ in cases where the data differs statistically from the model assumptions. It finds the value that minimizes the size of the residual errors between d and e . This minimization can be written as using the equation.

$$\min \sum_{s \in S} \rho((e_s - d_s), \sigma) \quad (5)$$

where σ scale parameter that controls the outlier rejection is point, and ρ is M-estimator.

Reducing ρ will cause the estimator to reject more measurements as outliers. S is the set of all chosen values. d_s is the input model and e_s is the best fit model. To minimize above, it is necessary to solve the equation (6) & (7)

$$\sum \Psi((e_s - d_s), \sigma) = 0 \quad (6)$$

Where the influence function given by the equation (7),

$$\Psi(x, \sigma) = \frac{\partial \rho(x, \sigma)}{\partial x} \quad (7)$$

Generally, the robustness is measured using two set of parameters: influence function and breakdown point. The influence function gives the change in an estimate caused by insertion of outlying data as a function of the distance of the data from the (uncorrupted) estimate. Breakdown point is the largest percentage of outlier data points that will not cause a deviation in the solution.

To increase robustness, re-descending estimators are considered for which the influence of outliers tends to zero with increasing distance. Lorentzian estimator [13][14], an Influence function that tends to zero for increasing estimation distance and maximum breakdown value.

The Lorentzian estimator $\rho_{LOR(x)}$ is defined by the equation (8)

$$\rho_{LOR}(x) = \log\left(1 + \frac{x^2}{2\sigma^2}\right) \quad (8)$$

and it is described by the influence function $\psi_{LOR(x)}$ given by the equation (9)

$$\psi_{LOR}(x) = \rho'_{LOR}(x) - \frac{2x}{2\sigma^2 + x^2} \quad (9)$$

Where x is the Lorentzian estimation distance and σ is the breakdown point.

V PROPOSED ALGORITHM

This method uses $ROAD_m$ value for selecting neighborhood pixels and Robust Estimation on the selected pixels using Lorentzian estimator for noise estimation is illustrated in Fig. 1.

The technique has two phases,

- (A) Rice Noise Estimation
- (B) Neighborhood pixel Selection and
- (B) Estimation phase

(A). Rice Noise Estimation

Median Absolute Deviation (MAD) estimator in the wavelet domain is used for Rician noise as it is robust and efficient.

(B). Neighborhood pixel Selection

Neighborhood pixel Selection has the following steps,

1. The 3×3 neighborhood for each pixel is extracted.
2. The absolute differences of the centre and neighborhoods are sorted in ascending order.
3. Select the requisite values computed in the previous step to find the $ROAD_m$ value. $ROAD_m$ is based on the correlation of the pixel with its noise neighbors.

Let $AD(i)$ be the absolute difference matrix and n the count of pixels. The following test conditions are used to find the $ROAD_m$ value.

if estimated noise < 30
 select the first six pixels closely related to the centre pixel
 else
 select the first four pixels closely related to the centre pixel

(C). Estimation phase

Lorentzian estimator is used for estimation. The estimation phase has the following steps [15],

1. The selected neighborhood pixels are used for estimation.
2. Find x , the difference of each selected pixel with the median value and compute the function $f(x)$ given in the equation (10)

$$f(x) = 2x / (2\sigma + x) \quad (10)$$

Where σ is outlier rejection point, is given by the equation (11),

$$\sigma = \frac{\tau_s}{\sqrt{2}} \quad (11)$$

Where τ_s is the maximum expected outlier and is given by,

$$\tau_s = \zeta \sigma_N \quad (12)$$

Where σ_N is the local estimate of the image standard deviation and ζ is a smoothening factor. Here $\zeta = 0.3$ is taken for medium smoothening.

3. Pixel is estimated using the equations (13) and (14)

$$S_1 = \sum_{l \in L} \frac{\text{pixel}(l) * f(x)}{x} \quad (13)$$

$$S_2 = \sum_{l \in L} \frac{f(x)}{x} \quad (14)$$

Where L is number of selected pixels in the window.

6. Ratio of S_1 and S_2 gives the estimated pixel value.

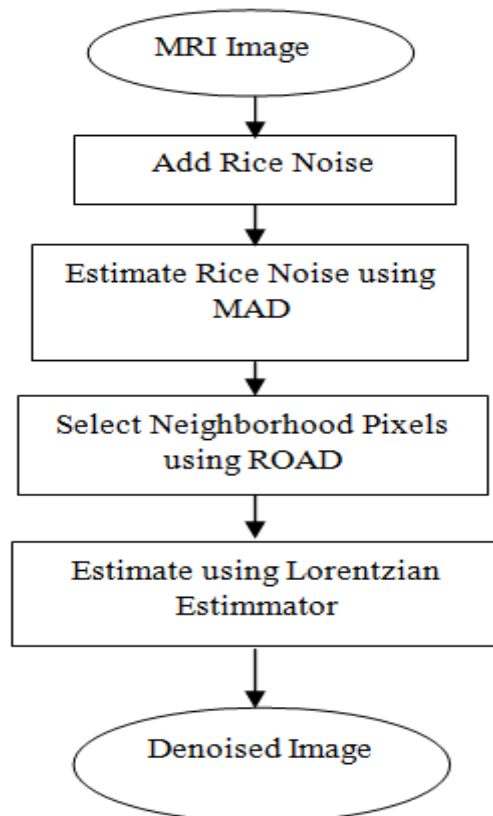


Figure 1. Proposed Method

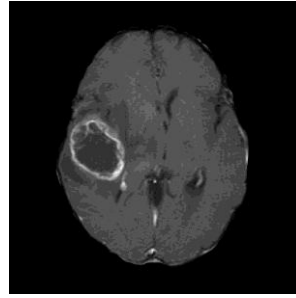


Figure 2. Axial T1C+MRI Image

Table 1. Performance of the algorithm with the images shown in Figure 2

Noise	Estimated Noise	PSNR	MSE
5	4.50	44.50	2.30
10	8.89	40.35	5.99
15	12.77	37.79	10.79
20	16.68	36.16	15.71
25	20.75	34.99	20.56
30	24.57	34.08	25.39

VI. PERFORMANCE EVALUTION

The performance of the proposed method is evaluated using two parameters, Mean Square Error (MSE) value and Peak Signal to Noise Ratio(PSNR).

A. Mean Square Error

Mean Square Error Value (MSE) is a risk function, corresponding to the expected value of the squared error loss or quadratic loss. MSE measures the average of the square of the error. The error is the amount by which the estimator differs from the quantity to be estimated. In this case The MSE is the cumulative squared error between the de-noised image and the original image. Mean Square Error (MSE) is computed using the equation (15).

$$MSE = \frac{1}{MN} \sum_{y=1}^M \sum_{x=1}^N [I(x, y) - I'(x, y)]^2 \quad (15)$$

Where $I(x,y)$ is the original image, $I'(x,y)$ is the reconstructed image and M,N are the dimensions of the images. A lower value for MSE means lesser error

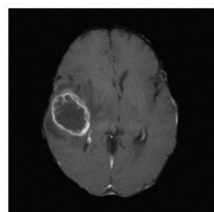
B. Peak Signal to Noise Ratio

The PSNR block computes the peak signal-to-noise ratio, in decibels, between two images. This ratio is often used as a quality measurement between the original and estimated image. The higher the PSNR, the better the quality of the estimated, or reconstructed image. PSNR in decibels (dB) is computed by using the equation (16)

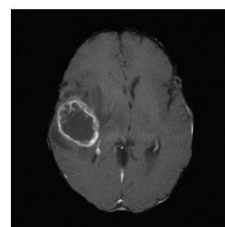
$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right) \quad (16)$$

Where MSE is given by equation (15)

Table I and Figure 3 shows that the method performs well than these existing methods and even at high noise densities the increase in MSE values is less. The visual results in Fig. 2 show that the method is good in retaining edges, avoids blurring and removing noise. So it is proved that the PA is good in preserving image details, high PSNR, low MSE values and performs effectively for rice noise removal.



(a) Rice noise 5%



(b)De-noised Image

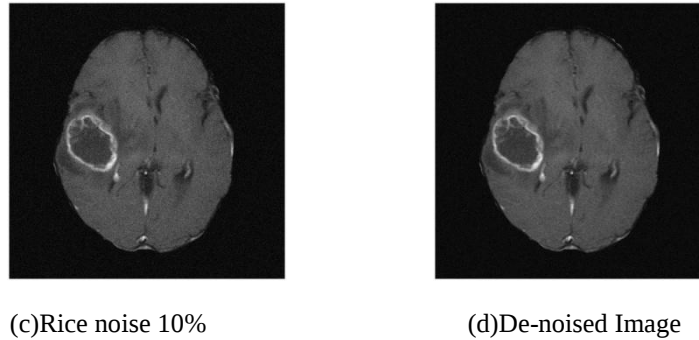


Figure 3. Visual Analysis

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