

## Impact assessment of speed breaker on traffic flow behaviour at urban mid-block section using cellular automata

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**ABSTRACT** - Cellular automata (CA) have been used by many researchers for homogeneous traffic flow modelling. In country like India, non lane based traffic prevails; hence, designing control systems for such situations is a challenging task. Traffic simulation helps the analyst to model the behaviour of such complex systems. To represent multiple vehicle types, a multi cell representation was adopted. The position and speed of vehicles are assumed to be discrete in developed model. The speed of each vehicle changes in accordance with its interactions with other vehicles and is governed by some pre-assigned (stochastic) rules. In this paper, mid block section two-lane heterogeneous traffic flow model(MBTFM) using CA is presented. A conventional CA model is modified to handle heterogeneous traffic. While addressing heterogeneity, essentials features of the CA like uniform cell size, simple rules, and the computational efficiency achieved in homogeneous models are retained to a great extent by modifying cell size and developing certain rules which control vehicles as well as the road characteristics. The cell size is reduced to accommodate more ranges of speed, acceleration, size of vehicles and deceleration as well as to represent aggregate behaviour of vehicles. Further, to address the issue of non lane based movement, lateral movement rules were proposed.

The speed breakers are typically used to reduce the speed of vehicles near certain Institutions such as schools and hospitals. New rules are developed to analyse the impact of speed breaking on traffic characteristics. Calibration and validation of the model is also carried out at macroscopic level using real-world data. The model is verified for the microscopic characteristics with a conventional car-following simulation model. The model performed reasonably well in predicting the speed at bump and overall delay. The results show that developed model can be used as an effective tool for heterogeneous traffic flow modelling considering the computational benefit and simplicity in developing CA models.

**Key words:** Cellular automata, Traffic flow modelling, Simulation, Macroscopic, Microscopic, Homogeneous and Heterogeneous traffic flow.

### I. INTRODUCTION

Traffic flow modelling describes the dynamics of different type of vehicles and their interactions using mathematical relations to derive traffic flow characteristics. Micro simulation approach which incorporates the car-following theories is commonly adapted to model traffic. They require considerable computational power and many input parameters and are known to be computationally inefficient for large scale applications. Recently, Cellular automata (CA) have been successfully used for homogeneous traffic flow simulation. Cellular automata models have been widely used as discrete models of physical systems. They offer some distinct advantages for practical simulation of traffic flow and the most significant among them is computational efficiency [1]. In addition CA model are simple enough to allow analytical treatment [2] and complex enough to simulate and recover most relevant aspect of traffic [3].

Many researchers have used a CA model for the simulation of traffic flow. They have developed a Boolean model representing individual vehicles by 1-bit variables [4]. Nagel and Schreckenberg have developed a CA model, commonly known as NaSch model, for freeway. Since the introduction of this model, many researchers ([5]; [6]; [7]; [8]; [9]) have developed traffic flow models using CA by modifying or introducing new rules for better representation of certain phenomena like congested flow, meta stability, hysteresis, and incidents. Many of these models include rules for multi-lane and lane changing operations [10]. Most of the microscopic models developed so far are based on homogeneous traffic flow [11]. Some researchers have made efforts to incorporate heterogeneity at aggregate level in the recent past [12]. However, this approach has limited scope for micro-level details. Some researchers, on the other hand, tried to describe mixed traffic by converting the vehicles into a standard vehicle by using passengercar equivalent (PCE) factor [11] and then the homogeneous models are applied to derive traffic flow characteristics. The main drawback of this approach is its difficulty to incorporate characteristics like vehicle size, acceleration, deceleration, speed, etc. Many researchers have attempted to model heterogeneity using micro simulation, but these models need considerable computational power as individual vehicles are modelled with their micro-characteristics ([13]; [14]; and [15]). Most of them are based on grid-based approach where the road stretch is divided into a grid of cells and a vehicle is assumed to occupy multiple cells governed by its size. These models are not computational efficient as they are developed based on the principle of car following theory. In this context, CA based model developed by [1] deserves much attention. This model is a two-lane traffic flow model with two different type of vehicles, characterized by two different values of the maximum allowable speed for each type of vehicle ( $v_{max}^k$ ). However, in this model cell size is taken as 0.9 x 0.9 meter, in spite of taking two different vehicles having a different static and dynamic property.

## **II. WORKING PRINCIPLE OF CELLULAR AUTOMATA**

In cellular automata, space, time, and state variables are discrete which make them ideally suited for high-performance computer simulations. However, CA modelling differs in several respects from continuum models, which are usually based on differential equations and often cannot be treated analytically. One has to solve the latter numerically and therefore the equations have to be discretized. In general, only space and time variables become discrete whereas the state variable (e.g. the density or velocity) is still continuous. CA is discrete in space and time variables and this discreteness is already taken into account in the definition of the model and its dynamics. This helps in obtaining the desired behavior in a much simpler way. The numerical solution of (discretized) differential equations is only accurate in the limit  $\Delta x, \Delta t \rightarrow 0$ . This is different in the CA where  $\Delta x$  and  $\Delta t$  are finite and accurate results can be obtained since the rules (dynamics) are designed such that the discreteness is an important part of the model. A cellular automaton is a discrete dynamical system. Each point in a regular spatial lattice, called a cell, can have any one of a finite number of states. All cells in CA are identical and have discrete state. The states in the cells of the lattice are updated according to local rules. That is, the state of the cell at a given time depends only on its own state one time step previously, and the states of its nearby neighbors at the previous time step. All cells in the lattice are updated synchronously. The state of the lattice advances in discrete time steps. The rule is some well-defined function, which if given an initial configuration will always evolve the system in a certain way based on this configuration. CA which strictly comply with the definition given above, are referred to as deterministic CA. However, it may be very convenient for some applications to have a certain degree of randomness in the rule. It may be desirable, for instance, that a rule selects one outcome among the several possible states, with a probability  $p$ . CA whose updating rule is thus driven by external probabilities are called probabilistic CA. The updating of a given cell requires one to know only the state of the cells and its vicinity. The spatial region in which a cell needs to search for this purpose is called the neighbour hood. For two-dimensional CA, two neighbor hoods are often considered. The key idea of neighbour hood is that when updation occurs, the cells within the block evolve only according to the state of that block and also depend on what is in the adjacent blocks. In practice when simulating a given cellular automata rule, it is not possible to deal with an infinite lattice. The system must be finite and have boundaries. A site belonging to the lattice boundary doesn't have the same neighborhood as other internal sites. In order to define the behavior of these sites, a different evolution rule can be considered, which sees the appropriate neighborhood.

### **2.1 Lane changing in CA models**

Lane changing behaviour could be implemented easily in CA models and a number of models were developed in the past. For instance, [16] have developed multi-lane traffic model considering heuristic rules of human behaviour. However, on validation some of these rules resulted in unrealistic behaviour. [17] examined a two-lane model with completely deterministic rules. These rules, however, gave a situation where several cars just oscillated between the links instead of moving ahead. To overcome this drawback randomness was introduced by [18]. The above approach further extended by introducing asymmetric models with different lane changing rules for different lanes and type of vehicle [1]. New CA rules were developed for multi-lane traffic and calibrated with the data from German highways [13]. They introduced a two-lane CA model for homogeneous traffic flow where the vehicles move in opposite directions; passing being allowed in one or both lanes [19]. They did extensive work to summarize different approaches to lane changing and proposed a general scheme with realistic rules for lane changing [20]. Most of the above models were developed based on homogeneous traffic conditions. Lane changing in heterogeneous traffic is more complex due to large number of factors involved, for instance, size and dynamic characteristics of each type of vehicles like acceleration, deceleration, and maximum speed. Hence, these models require some modifications while implementing in mixed traffic conditions.

## **III. METHODOLOGY**

The objective of the present study is to develop a heterogeneous traffic flow model based on the principles of cellular automata. While addressing heterogeneity, the essential features of the CA like uniform cell size, simple speed and position updating rules, and the computational efficiency of the homogeneous models are retained to a great extent. Accordingly, the basic CA model developed by [21] has been modified to take into account the heterogeneity of traffic. Here, the vehicles are differentiated by their dynamic characteristics like maximum speed, acceleration, deceleration, and lane changing behaviour. The cell size is also reduced considering the presence of the smaller vehicle and acceleration point of view. In addition, an effort has been made to develop lane changing rules for heterogeneous traffic conditions. For this, the rules developed by [20] are modified to handle dynamic vehicular characteristics. The lane changing rule is developed based on the psychological behaviour of the driver on the road. Main components of the models are input, initialization, and application of CA rules, vehicle generation, and lane changing.

### **3.1 Input and initialization**

The proposed model requires information like cell size, to represent common vehicle, types of vehicle, maximum speed, acceleration and deceleration of each type of vehicle, initial lane density (cells occupied with vehicles), mean arrival rate of each lane, classified volume of each lane, noise probability ( $p$ ), and lane changing probability ( $pl$ ) of each lane. In addition, incident details like location and duration of any incident are required. Once the input is given, each lane of the

road is divided into equal-sized cells and each cell either hold a vehicle or remains empty. The vehicles are generated to occupy the cells as per the given initial density and then the clock is set to zero. The vehicle type and the speeds are generated randomly, guided by the vehicle composition and the speed distributions.

### 3.2 Application of CA rules

The vehicle movement rules are similar to the single lane rules of the NaSch model. The cell size and time-steps are crucial in CA model as they affect all dynamic properties of the vehicles. Traditionally, CA models have taken a cell size of 7.5 meters because of uniform size of vehicles. However, in the present study, the cell size is reduced to 0.9 meters considering heterogeneous nature of vehicles. It may be noted that this cell size is decided based on type of vehicle. It is decided in such a way that it represents the actual size of vehicles and the total width of the road as close as possible. The physical representation of the vehicle should be kept slightly more than the actual size of vehicle to provide some clearance. The cell length also depends upon the dynamic characteristics of the vehicular movement, as in the cellular automata, distance-headway and speed is considered in terms of number of cells. If cell size is taken small, it can represent the physical features of vehicles more accurately but at the same time it will reserve the considerable memory for computation. The time-step should be closer to the reaction time and the flow characteristics should not differ much during this time. Since these characteristics are similar to homogeneous traffic, the present study also uses a time-step of one second similar to most of the CA models. If a vehicle moves with the speed of 42 cells/time-step (136 kmph) then the minimum possible distance headway is 37.8 meters. Whereas in NaSch model if the vehicle moves with a speed of 5 cells/time-step (135 kmph) then the minimum possible distances headway is 37.5 meters. Therefore, a cell size of 0.9 meter gives considerably closer dynamics to that of NaSch model and at the same time it offers more speed options. In the case of CA modelling, speed and position of the vehicles are updated by following certain rules. These rules are applied to each vehicle at each time-step and for each lane. The notations are explained first. Let,  $X_n$  denotes the position of  $n^{\text{th}}$  vehicle (follower) represented by the cell number it occupy. Similarly,  $X_{n+1}$  denotes the position of  $n+1^{\text{th}}$  vehicle (leader). Let,  $V_n$  denote the speed of the  $n^{\text{th}}$  vehicle in cells/time-step and  $gap_n^f$  represents gap between the  $n^{\text{th}}$  vehicle and the  $n+1^{\text{th}}$  vehicle in front, expressed in number of cells. Here, the speed is taken as number of cells per time-step since in CA the variables are discrete. Similarly, the front gap of a vehicle is expressed in number of cells. The  $gap_n^f$  is given by  $X_{n+1} - X_n$ . This means that even when vehicles are present in adjacent cells, the gap ( $gap_n^f$ ) is one. At each time-step  $t \rightarrow t + \Delta t$ , where  $\Delta t$  is selected as 1 second, the arrangement of all the vehicles in the cells is updated simultaneously according to the following rules:

**Rule 1 (Acceleration):** If speed of the vehicle ( $V_n$ ) at any time-step is lower than  $V_{max}^k$ , then the driver's tendency is to reach the maximum speed at every time-step as per the vehicle's acceleration ( $a_k$  which is speed increase in number of cells per time step). If the speed of the vehicle is  $v_n$ , the maximum speed of this vehicle-type is  $V_{max}^k$  and the acceleration is  $a_k$  then the acceleration rules is given as;

$V_n \rightarrow \min(V_n + a_k \Delta t, V_{max}^k)$ . This reflects the general tendency of the drivers to drive as fast as possible, if allowed to do so, without crossing the maximum speed limit.

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**Rule 2 (Deceleration):** The vehicle reduces its speed if the front gap (spacing) is not sufficient to drive at current speed.

If the deceleration of the vehicle is  $d_k$ , then the deceleration rule is given as  $V_n \rightarrow \min(V_n, \frac{gap_n^f}{\Delta t} - d_k \Delta t)$ . ( $d_k$  is speed decreased in number of cells per time step)

**Rule 3 (Randomization):** As mentioned earlier, in CA models, stochastic driver behaviour is incorporated by introducing noise probability  $p$  and is applied using the randomization rule given by:  $V_n \rightarrow \max(V_n - 1, 0)$  with probability  $p$ . This step takes into account the different behavioural patterns of the individual drivers, especially non deterministic acceleration and overreaction while slowing down; this is crucial important for the spontaneous formation of traffic jams.

**Rule 4 (Position updation):** Application of the above rules will result in a new speed. By using this speed and the position in the current time-step, the position at the next time-step is updated. The rule is given as:  $X_n \rightarrow X_n + V_n \Delta t$ .

**Rule 5 (Speed breaker):**

**Case – I**

Vehicle is in motion and driver sees the speed breaker and if there is no vehicle at front between current vehicle and bump line. if  $\frac{X_n}{\Delta t} < V_n$  then,  $V_n \rightarrow \frac{X_n}{\Delta t} - d_{bump}^k \Delta t$  else  $V_n \rightarrow V_n - d_{bump}^k \Delta t$

**Case – II**

Vehicle is in motion and driver sees the speed breaker and if there is vehicle at front between current vehicle and bump line. if  $\frac{gap_n^f}{\Delta t} \leq V_n$  then,  $V_n \rightarrow \frac{gap_n^f}{\Delta t} - d_{bump}^k \Delta t$  else  $V_n \rightarrow V_n - d_{bump}^k \Delta t$ .

Where  $d_{bump}^k$  = Deceleration of vehicle type  $k$  in cell per time step at bump

$$= \frac{V_n^2 - V_{bump}^k}{2X_n}$$

and  $V_{bump}^k$  = Speed of vehicle type  $k$  in cell per time step at bump

### 3.3 Vehicle generation

After updating all vehicles in the system, new vehicles are to be generated in each lane. The vehicles are generated based on Poisson's distribution with a given mean arrival rate; the vehicle type (k) being governed by a given classified volume; and an initial speed is generated randomly between as per the observed distribution for the vehicle. Each vehicle generated is kept into a stack and it enters into the systems as soon as the first cell is found empty. The number of vehicles at each time-step in the stack indicates the queue at entry point. The individual delay at entry in queue is also recorded. The vehicles that are not in the queue are directly entered into the system with generated speed; otherwise they are entered with zero speed. The lane distribution (usage of lane and speed restriction, if any) and arrival pattern for each lane are input for the vehicle generation.

### 3.4 Lane-changing criteria

Lane-changing criteria are proposed based on two-lane model developed by [20]. This rule is modified for heterogeneous traffic by specifying probability of lane changing and considering the maximum speed for each vehicle type. The lane changing rule for a vehicle consists of the fulfilment of the following five criteria:

1.  $\frac{gap_c^f}{\Delta t} - 1 < V_{max}^{lc}$
2.  $\frac{gap_t^f}{\Delta t} - 1 > V_{max}^k$
3.  $X_n^t = 0$  (empty)
4.  $\frac{gap_T^b}{\Delta t} - 1 > V_{max}^{lt}$
5.  $p_l \geq rand()$

where,  $gap_c^f$  is the front gap in the current lane,  $gap_t^f$  front gap in the target lane,  $gap_T^b$  is the back gap in the target lane,  $v_{max}^{lc}$  is the maximum speed allowed on the current lane,  $v_{max}^{lt}$  is the maximum speed allowed in target lane,  $x_n^t$  indicate whether target cell n is empty or not, rand() is a function which generates a random number between 0 and 1, and  $p_l$  is the lane changing probability.

Criteria 1 ensures that when the front gap available in the current lane is less than the gap required to maintain the desired speed, then the lane changing could take place. Criteria 2 ensure that the vehicle has sufficient front gap in the target lane to move with desired speed. Criteria 1 and 2 are known as the trigger criteria or incentive criteria which imply that a vehicle move with desired speed. The criteria 3 looks if target cell is empty or not. Criteria 4 ensure that the speed of the following vehicle in the target lane is not affected. Criteria 3 and 4 are known as the safety criteria, which ensure that lane changing is safe by avoiding collision with the following vehicle in the target lane. The criteria 5 take care of the randomness in the behaviour of drivers in lane changing by introducing a lane changing probability ( $p_l$ ). If all the above criteria are satisfied for a vehicle then that vehicle is eligible for lane changing.

## IV. MODEL CALIBRATION AND VALIDATION

Calibration and validation are very important tasks in evaluating any traffic simulation model which ensures that the model reflects real system. Calibration and validations ensures the ability of the model to reproduce the behaviour of real traffic. Most of the CA models are not validated at the microscopic level because of the coarseness of the model [21]. Moreover, it is difficult to get speed and position of individual vehicle at every second from real-traffic [22]. Further, these models are expected to capture the essential features of the traffic and not micro-behaviour [2]. The present model is also calibrated and validated with the help of macroscopic data obtained from real traffic. Since the cell size is reduced to 0.9 meters, vehicle-type specific maximum speeds are introduced, and modified lane changing rules are proposed. The model is calibrated with the field data collected at macroscopic level by observing the average speed on the selected urban mid-block.

**Table:1 Field observed data and MBTFM data**

Vehicle type	Field data			MBTFM model			RMSPE
	Composition Classified volume	Avg. Speed (km/hr)	Max.speed (km/hr)	Composition Classified volume	Avg. Speed (km/hr)	Max.speed c/ts (km/hr)	
							for avg. speed
2W	69.46%	35.28	74.37	68.09%	42.18	23 (74.52)	19.56%
3W	12.31%	24.32	37.83	13.86%	22.03	14 (45.36)	9.42%
4W	17.63%	33.27	74.37	14.96%	36.05	24 (77.76)	8.36%
LCV	0.50%	28.29	60	2.08%	26.43	18 (58.32)	6.57%
HCV	0.10%	24.23	45	1.01%	16.12	12 (38.88)	33.47%

The macroscopic validation is then carried out with the data collected on 7.2 m wide road for one hour. Total 4028 vehicles are generated during an hour of simulation. The observed headway distribution (cumulative probability) is used

to generate vehicles. The maximum observed speed of each type of vehicles in urban mid-block is given in Table 1 respectively. These speeds were converted to the nearest cells per time step and are shown in Table 1. The results of the simulation run show that the model is capable of generating the classified volume and maximum speed considerably close to the real system, and the RMS percentage error (RMSPE) for the average speed difference for each type of vehicles are shown in Table 1. The validation of MBTFM model is conducted at microscopic and macroscopic levels. In microscopic validation, vehicles headway generated from the MBTFM model are compared to the field observed data. In macroscopic validation, speed-flow-density relationships are compared.

## V. Vehicle Behaviour at speed breaker

A survey was conducted to measure variation in the speed due to speed breaker at different distance after and before the speed breaker. Speed survey was conducted at seven different sections defined on the bump and 60m, 40m, 20m before and after the speed breaker with radar gun. Comparison of observed speed and MBTFM speed are shown in following Tables 2, 3, and 4 for car, 3W, and 2W respectively. Deceleration before the speed breaker and acceleration after the speed breaker are shown in following Fig. 3, 4, and 5 for car, 3W, and 2W. Results shows that the MBTFM output closely match the field data for average speed of any type of vehicle near speed breaker. Various error tests like mean percentage error (MPE), root mean error (RMS), inequality co efficient (U), chi square were carried out. All the error test results are within the acceptable range. The delay study with bump and without bump also carried out and results are shown in Table 5.

Table 2 Car speed at various locations after/before speed breaker					
Car speed at various location	Avg. Speed of vehicle (km/hr)		Error		
	Car (Field data)	Car (MBTFM)	MPE%	RMSE	U
at 60 m before Speed breaker	54.23	66.31	22.27%	12.08	0.100201
at 40 m before Speed breaker	44.28	53.00	19.70%	8.72	0.089665
at 20 m before Speed breaker	25.09	30.71	22.44%	5.63	0.100869
at Speed breaker	15.48	17.36	12.13%	1.88	0.057165
at 20 m after Speed breaker	25.28	32.26	27.60%	6.98	0.121263
at 40 m after Speed breaker	38.83	43.21	11.28%	4.38	0.053372
at 60 m after Speed breaker	45.85	53.46	16.60%	7.61	0.076629
<b>Average</b>			<b>18.86%</b>	<b>6.75</b>	<b>0.085595</b>
Chi-square test value: 9.58 (Critical chi-square value: 11.07 )					

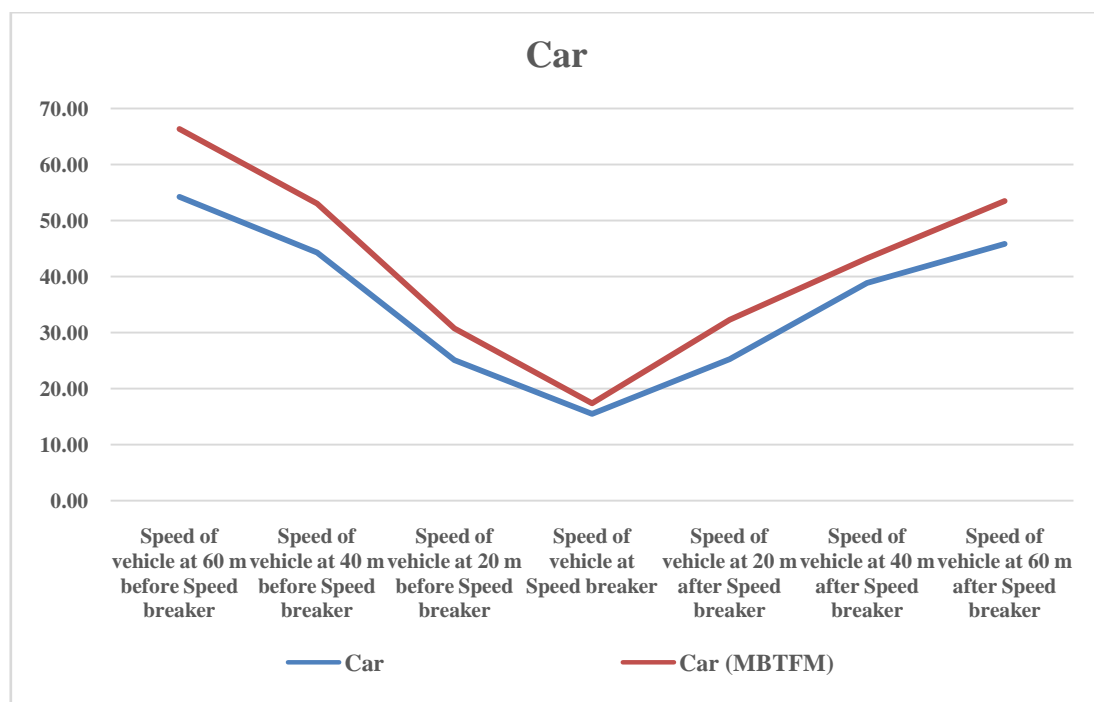


Fig. 3 Car speed at various locations after/before speed breaker

Table 3 2-Wheeler speed at various locations after/before speed breaker					
2-wheeler speed at various location	Avg. Speed of vehicle (km/hr)		Error		
	2Wheeler (Field data)	2 Wheeler (MBTFM)	MPE%	RMSE	U
at 60 m before Speed breaker	45.28	56.99	25.87%	11.71	0.11454
at 40 m before Speed breaker	35.87	40.3	12.36%	4.43	0.0582
at 20 m before Speed breaker	24.91	28.33	13.73%	3.42	0.06423
at Speed breaker	15.99	15.61	-2.37%	0.38	0.01199
le at 20 m after Speed breaker	23.27	26.36	13.30%	3.09	0.0623
at 40 m after Speed breaker	32.87	34.67	5.49%	1.8	0.02671
at 60 m after Speed breaker	38.78	40.93	5.53%	2.15	0.0269
<b>Average</b>			<b>10.56%</b>	<b>3.86</b>	<b>0.0521</b>
Chi-square test value: 4.69 (Critical chi-square value: 11.07 )					

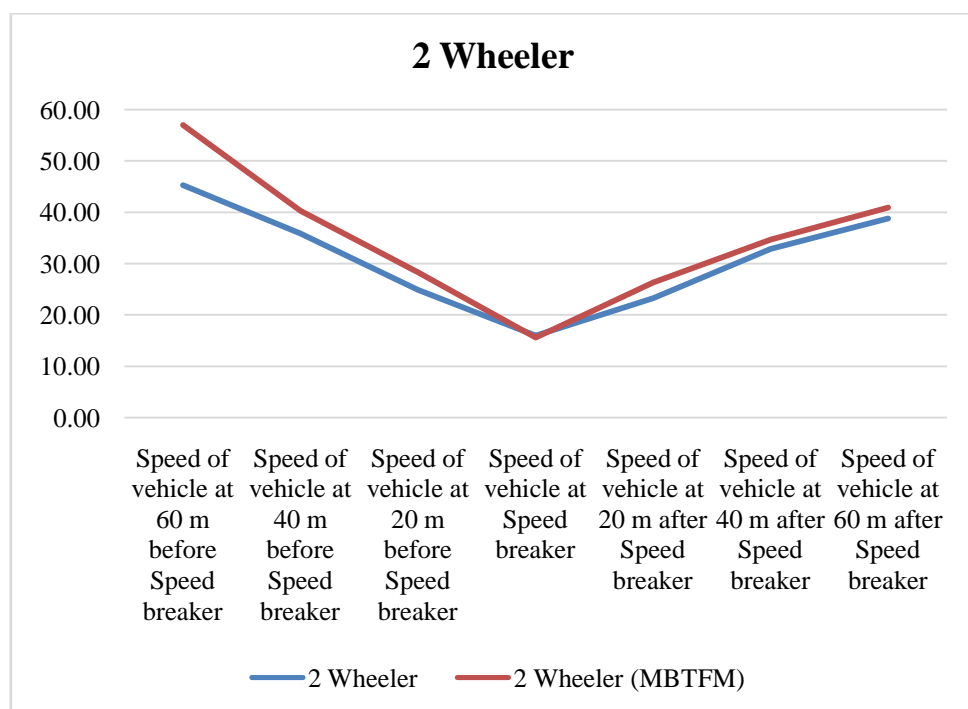


Fig.4 Wheeler speed at various locations after/before speed breaker

Table 4 3-Wheeler speed at various locations after/before speed breaker					
3-wheeler speed at various location	Avg. Speed of vehicle (km/hr)		Error		
	3Wheeler (Field data)	3Wheeler (MBTFM)	MPE%	RMSE	U
At 60 m before Speed breaker	37.39	42.12	12.65%	4.73	0.0594
At 40 m before Speed breaker	33.25	36.92	11.05%	3.67	0.0523
At 20 m before Speed breaker	23.47	28.62	21.94%	5.15	0.0988
At Speed breaker	14.69	15.05	2.48%	0.36	0.0122
At 20 m after Speed breaker	20.64	21.4	3.68%	0.76	0.018
At 40 m after Speed breaker	27.26	26.33	-3.43%	0.93	0.0174
At 60 m after Speed breaker	28.27	30.65	8.40%	2.38	0.0403
<b>Average</b>			<b>8.11%</b>	<b>2.57</b>	<b>0.0426</b>
Chi-square test value: 2.40 (Critical chi-square value: 11.07 )					

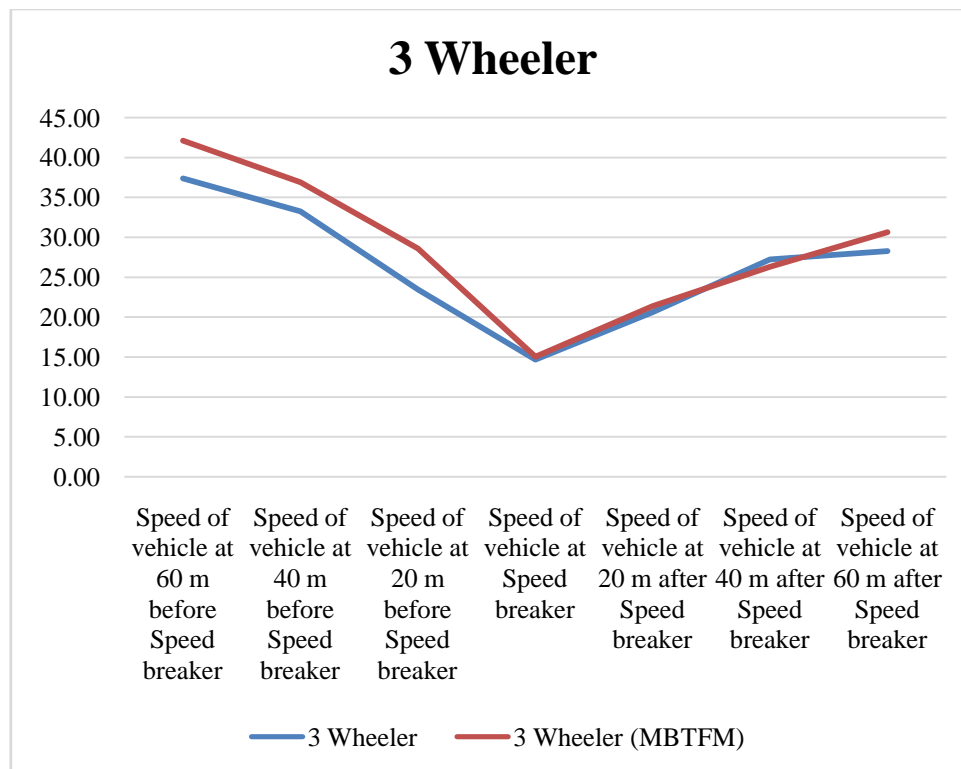


Fig. 53-Wheeler speed at various locations after/before speed breaker

## VI. CONCLUSION

Two-lane heterogeneous traffic flow modelling using cellular automata is attempted in this study. A conventional CA model is modified to handle heterogeneous traffic. While addressing heterogeneity, essentials features of the CA like uniform cell size, simple rules, and the computational efficiency achieved in the homogeneous models are retained to a great extend. The vehicular characteristics like acceleration, deceleration, maximum speed, and lane changing behaviour are also incorporated in the modelling. Further, the cell size is reduced to 0.9 meters to take care of wide variations in traffic. Lane changing rule is also improved by distinguishing each lane by its maximum speed and lane-changing probability. The model is calibrated and validated with the data collected from heterogeneous traffic at macroscopic level. It can be seen that the simulated speed-flow-density relationship derived from the MBTFM are well matched with the field data under various traffic flow levels. The model is applied to study the impact of speed breaker on traffic flow. The model performed reasonably well in predicting the speed at bump and overall delay. Thus, this model is a valuable tool in understanding heterogeneous traffic flow. These studies demonstrated the potential of CA models for large scale simulations making it suitable for real-time applications. However, before any such deployments, considerable testing with heterogeneous traffic having diverse vehicular composition and road condition is necessary.

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