Scientific Journal of Impact Factor (SJIF): 5.71

e-ISSN (O): 2348-4470
p-ISSN (P): 2348-6406

International Journal of Advance Engineering and Research Development

Volume 5, Issue 02, February -2018

Frontier of soft semi #gα-closed sets in soft bi-topological space

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Abstract —*In this paper , we introduce the concepts of frontier of soft semi #gα-closed sets in soft bi-topological spaces, which is denoted by* $(\mathbf{1},\mathbf{2})^*$*soft semi #gα-fr(F,A), where (F,A) is any soft set of (X,E).*

Keywords-$(\mathbf{1},\mathbf{2})^*$*soft semi #gα-open set;* $(\mathbf{1},\mathbf{2})^*$*soft semi #gα-closed set;* $(\mathbf{1},\mathbf{2})^*$*soft semi #gα-interior;* $(\mathbf{1},\mathbf{2})^*$*soft semi #gα-closure;* $(\mathbf{1},\mathbf{2})^*$*soft semi #gα-frontier.*

I. INTRODUCTION

In 1963, the concepts of bi-topological spaces was originally initiated by J.C.Kelly[3]. The theory of generalized closed sets in topological spaces which was found by Levine [8] in 1970. The concepts of generalized and semi generalized closed sets was introduced and studied by Lellis [7] in classical topology. He defined a bi-topological space (X,τ_1,τ_2) to be a set X with two topologies τ_1 *and* τ_2 on X and initiated the systematic study of bi-topological spaces. The soft theory is rapidly processing in different field of mathematics. It was first proposed by Russian researcher Molodtsov [9] in 1999. Muhammad Shabir and Manazza Naz [10] introduced soft topological spaces in 2011. It was defined over an initial universe with a fixed set of parameters. N.Cagman and S.Karatas[2] introduced topology on a set called "soft topology" and initiated the theory of soft topological spaces in 2013. In this paper we defined and examined the basic properties of $(1,2)^*$ soft semi #gα-frontier in soft bi-topological spaces and study their properties.

II. PRELIMINARIES

In this section we have presented some of the basic definitions and results of soft set, soft topological space, bi-topological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, P(X) is the power set of X, and A⊆ X.

- *2. Definition 2.1.*

Let $\tilde{\tau}$ *be the collection of soft sets over X, then* $\tilde{\tau}$ *is called a soft topology on X if* $\tilde{\tau}$ *satisfies the following axioms:*

(i)$\emptyset$, $\tilde{X}$ belongs to $\tilde{\tau}$.
(ii)The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
(iii)The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet (X, $\tilde{\tau}$,E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X, $\tilde{\tau}$,E) as X throughout the work.

- Definition 2.2.

A set X together with two different topologies is called bi-topological space. It is denoted by (X, τ_1,τ_2).

- Definition 2.3.

A soft set (F,A) of a soft topological space (X, $\tilde{\tau}$,E) is called

(i) soft $\alpha -$ closed [4] if $\tilde{s}cl(\tilde{s}\,int\ (\tilde{s}cl$(F,A))) $\tilde{\subseteq}$ (F,A). The complement of soft α-closed set is called soft α-open.

(ii) soft semi – closed [2] if $\tilde{s}int(\tilde{s}cl$(F,A)) $\tilde{\subseteq}$ (F,A). The complement of soft semi – closed set is called soft semi-open.

(iii) soft g-closed [5] if $\tilde{s}cl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in (X , $\tilde{\tau}$, E). The complement of soft g-closed set is called soft g-open.

(iv) soft $g^{\#}$ α- closed [6] if $\tilde{s}\,\alpha cl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft g-open in (X , $\tilde{\tau}$, E). The complement of soft $g^{\#}$ α -closed set is called soft $g^{\#}$ α -open.

(v) soft #gα- closed [8] if $\tilde{s}\,\alpha cl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft $g^{\#}$α- open in (X , $\tilde{\tau}$, E). The complement of soft #gα -closed set is called soft #gα -open.

(vi) soft semi #gα-closed [9] if $\tilde{s}\,scl$(F,A) $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft #gα- open in (X , $\tilde{\tau}$, E). The complement of soft semi #gα -closed set is called soft semi #gα -open.

(vii) The union of all soft semi #gα open sets [10] each contained in a set (F,A) of (X , $\tilde{\tau}$, E) is called soft semi #gα interior of (F,A) which is denoted by $\tilde{s}$ semi #gα-int(F,A).

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(viii) The intersection of all soft semi #gα- closed sets [10], each containing a set (F,A) of (X , $\tilde{\tau}$, E) is called soft semi #gα-closure of (F,A), which is denoted by $\tilde{s}$ semi #gα-closure of (F,A).

- Definition 2.4.

Let X be a non-empty soft set on the universe X , $\tilde{\tau}_1$, $\tilde{\tau}_2$ are different soft topologies on $\tilde{X}$. Then ($\tilde{X}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$) is called a soft bi-topological space.

- Definition 2.5.

Let F_A ∈ S(U). Power soft set of F_A is defined by , $\tilde{P}$ (F_A) = {F_{Ai} $\tilde{\subseteq}$ F_A: i ∈ I}

And its cardinality is defined by $|\tilde{P}\ (F_A)|$ = 2 $\sum_{x\in E}|f_A(x)|$, where $|f_A(X)|$ is cardinality of $f_A(X)$.

- Example 2.6.

Let U = $\{u_1\ ,u_2\ ,u_3\ \}$, E = $\{x_1\ ,x_2\ \}$ and F_E = X = {(x_1 ,{ u_1 ,u_2 ,u_3}), (x_2 ,{ u_1 ,u_2 ,u_3})}. And let ($\tilde{X}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$) be a soft bi-topological space , where $\tilde{\tau}_1$ = { ∅, F_{A_2} , F_{A_3} , F_{A_5} , X } , $\tilde{\tau}_2$ = { ∅, F_{A_2} , F_{A_8} , $F_{A_{14}}$, X }, then $\widetilde{\tau_{1,2}}$ soft open sets are { ∅, F_{A_2} F_{A_3} , F_{A_5} , F_{A_8} , $F_{A_{14}}$, $F_{A_{17}}$, $F_{A_{32}}$, X } and $\widetilde{\tau_{1,2}}$ soft closed sets are {∅, F_{A_4} F_{A_6} , F_{A_7} , $F_{A_{12}}$, $F_{A_{31}}$, $F_{A_{44}}$, $F_{A_{46}}$,X }. Then ,

F_{A_1} = ∅
$F_{A_2} = \{(x_1,\{u_1\})\}$
$F_{A_3} = \{(x_1,\{u_2\})\}$
$F_{A_4} = \{(x_1,\{u_3\})\}$
$F_{A_5} = \{(x_1,\{u_{1,}u_2\})\}$
$F_{A_6} = \{(x_1,\{u_{2,}u_3\})\}$
$F_{A_7} = \{(x_1,\{u_{3,}u_1\})\}$
$F_{A_8} = \{(x_2,\{u_1\})\}$
$F_{A_9} = \{(x_2,\{u_2\})\}$
$F_{A_{10}} = \{(x_2,\{u_3\})\}$
$F_{A_{11}} = \{(x_2,\{u_{1,}u_2\})\}$
$F_{A_{12}} = \{(x_2,\{u_{2,}u_3\})\}$
$F_{A_{13}} = \{(x_1,\{u_{3,}u_1\})\}$
$F_{A_{14}} = \{(x_1\ ,\{u_1\}\ ,(x_2\ ,\{u_1\})\}$
$F_{A_{15}} = \{(x_1\ ,\{u_1\}\ ,(x_2\ ,\{u_2\})\}$
$F_{A_{16}} = \{(x_1\ ,\{u_1\}\ ,(x_2\ ,\{u_1,u_2\ \})\}$
$F_{A_{17}} = \{(x_1\ ,\{u_2\}\ ,(x_2\ ,\{u_1\})\}$
$F_{A_{18}} = \{(x_1\ ,\{u_2\}\ ,(x_2\ ,\{u_2\})\}$
$F_{A_{19}} = \{(x_1\ ,\{u_2\}\ ,(x_2\ ,\{u_1,u_2\})\}$
$F_{A_{20}} = \{(x_1\ ,\{u_3\}\ ,(x_2\ ,\{u_1\})\}$
$F_{A_{21}} = \{(x_1\ ,\{u_3\}\ ,(x_2\ ,\{u_2\})\}$
$F_{A_{22}} = \{(x_1\ ,\{u_3\}\ ,(x_2\ ,\{u_{1,}u_2\})\}$
$F_{A_{23}} = \{(x_1\ ,\{u_3\}\ ,(x_2\ ,\{u_{3,}u_1\})\}$
$F_{A_{24}} = \{(x_1\ ,\{u_1\}\ ,(x_2\ ,\{u_3\})\}$
$F_{A_{25}} = \{(x_1\ ,\{u_1\}\ ,(x_2\ ,\{u_{2,}u_3\})\}$
$F_{A_{26}} = \{(x_1\ ,\{u_2\}\ ,(x_2\ ,\{u_{3,}u_1\})\}$
$F_{A_{27}} = \{(x_1\ ,\{u_2\}\ ,(x_2\ ,\{u_3\})\}$
$F_{A_{28}} = \{(x_1\ ,\{u_2\}\ ,(x_2\ ,\{u_{2,}u_3\})\}$
$F_{A_{29}} = \{(x_1\ ,\{u_1\}\ ,(x_2\ ,\{u_3,u_1\})\}$
$F_{A_{30}} = \{(x_1\ ,\{u_3\}\ ,(x_2\ ,u_3\})\}$
$F_{A_{31}} = \{(x_1\ ,\{u_3\}\ ,(x_2\ ,\{u_{2,}u_3\})\}$
$F_{A_{32}} = \{(x_1\ ,\{u_{1,}u_2\}\ ,(x_2\ ,\{u_1\})\}$
$F_{A_{33}} = \{(x_1\ ,\{u_{1,}u_2\}\ ,(x_2\ ,\{u_2\})\}$
$F_{A_{34}} = \{(x_1\ ,\{u_{1,}u_2\}\ ,(x_2\ ,\{u_1,u_2\})\}$
$F_{A_{35}} = \{(x_1\ ,\{u_{2,}u_3\}\ ,(x_2\ ,\{u_1\})\}$
$F_{A_{36}} = \{(x_1\ ,\{u_{2,}u_3\}\ ,(x_2\ ,\{u_2\})\}$
$F_{A_{37}} = \{(x_1\ ,\{u_{2,}u_3\}\ ,(x_2\ ,\{u_1,u_2\})\}$
$F_{A_{38}} = \{(x_1\ ,\{u_{3,}u_1\}\ ,(x_2\ ,\{u_1\})\}$
$F_{A_{39}} = \{(x_1\ ,\{u_{3,}u_1\}\ ,(x_2\ ,\{u_2\})\}$
$F_{A_{40}} = \{(x_1\ ,\{u_{3,}u_1\}\ ,(x_2\ ,\{u_1,u_2\})\}$
$F_{A_{41}} = \{(x_1\ ,\{u_{1,}u_2\}\ ,(x_2\ ,\{u_3\})\}$

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$F_{A_{42}} = \{(x_1\ ,\{u_1, u_2\}\ ,(x_2\ ,\{u_2\ ,u_3\})\}$
$F_{A_{43}} = \{(x_1\ ,\{u_2, u_3\}\ ,(x_2\ ,\{u_3\})\}$
$F_{A_{44}} = \{(x_1\ ,\{u_2, u_3\}\ ,(x_2\ ,\{u_2, u_3\})\}$
$F_{A_{45}} = \{(x_1\ ,\{u_3, u_1\}\ ,(x_2\ ,\{u_3\})\}$
$F_{A_{46}} = \{(x_1\ ,\{u_3, u_1\}\ ,(x_2\ ,\{u_2, u_3\}$
$F_{A_{47}} = \{(x_1\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{48}} = \{(x_1\ ,\{u_1, u_2, u_3\}, (x_2\ ,\{u_1\})\}$
$F_{A_{49}} = \{(x_1\ ,\{u_1, u_2, u_3\}, (x_2\ ,\{u_2\})\}$
$F_{A_{50}} = \{(x_1\ ,\{u_1, u_2, u_3\}, (x_2\ ,\{u_1, u_2\})\}$
$F_{A_{51}} = \{(x_1\ ,\{u_1, u_2, u_3\}, (x_2\ ,\{u_3\})\}$
$F_{A_{52}} = \{(x_1\ ,\{u_1, u_2, u_3\}, (x_2\ ,\{u_2, u_3\})\}$
$F_{A_{53}} = \{(x_1\ ,\{u_1, u_2, u_3\}, (x_2\ ,\{u_3, u_1\})\}$
$F_{A_{54}} = \{(x_1\ ,\{u_1\}, (x_2\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{55}} = \{(x_1\ ,\{u_2\}, (x_2\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{56}} = \{(x_1\ ,\{u_1, u_2\}, (x_2\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{57}} = \{(x_1\ ,\{u_3\}, (x_2\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{58}} = \{(x_1\ ,\{u_2, u_3\}, (x_2\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{59}} = \{(x_1\ ,u_1\ ,u_3\}, (x_2\ ,\{u_1, u_2, u_3\})\}$
$F_{A_{60}} = \{(x_1\ \{u_1, u_3\}, (x_2, \{u_1, u_3\})\}$
$F_{A_{61}} = \{(x_1\ \{u_1, u_2\}, (x_2, \{u_1, u_3\})\}$
$F_{A_{62}} = \{(x_1\ \{u_2, u_3\}, (x_2, \{u_1, u_3\})\}$
$F_{A_{63}} = \{\ (x_2, \{u_1, u_2, u_3\})\}$
$F_{A_{64}} = \{(x_1\ ,\{u_1\ ,u_2\ ,u_3\}, (x_2\ ,\{u_1, u_2, u_3\})\}$ = X. Are all soft subsets of F_E. So $\left|\tilde{P}\ (F_E)\ \right| = 2^6 = 64$

- Definition 2.7.

  A soft set (F,A) of a soft bi-topological space $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E) is called $(1,2)^*$ soft semi #gα-closed if $\tilde{s}$ scl (F,A) $\tilde{\subseteq}$ (U,E) , whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is $(1,2)^*$ soft #gα-open in $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E). The complement of $(1,2)^*$ soft semi #gα-closed set is called $(1,2)^*$ soft semi #gα-open.
- Theorem 2.8.

  If (F,A) and $\widetilde{\tau_{1,2}}$(G,B) are soft subset of (X,E), then
  (i) (F,A) is $(1,2)^*$ soft semi #gα-open iff $(1,2)^*$ soft semi #gα-int(F,A) $\cong$ (F,A).
  (ii) $(1,2)^*$ soft semi #gα-int (F,A) is $(1,2)^*$ soft semi #gα − open.
  (iii) (F,A) is $(1,2)^*$ soft semi #gα-closed iff $(1,2)^*$ soft semi #gα-cl(F,A) $\cong$ (F,A).
  (iv) $(1,2)^*$ soft semi #gα-cl (F,A) is $(1,2)^*$ soft semi #gα-closed.
  (v) $(1,2)^*$ soft semi #gα-cl ((X,E)\(F,A)) $\cong$ (X,E)\ $(1,2)^*$ soft semi #gα-int(F,A).
  (vi) $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)) $\cong$ (X,E)\ $(1,2)^*$ soft semi #gα-cl(F,A).
  (vii) If (F,A) is $(1,2)^*$ soft semi #gα-open in $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E) and $\widetilde{\tau_{1,2}}$(G,B) is $(1,2)^*$ soft semi #gα-open in $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E), then, (F,A)$\tilde{\cap}$ $\widetilde{\tau_{1,2}}$(G,B) is $(1,2)^*$ soft semi #gα-open in $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E)
  (viii) A point x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-cl (F,A) iff every $(1,2)^*$ soft semi #gα-open set in (X,E) containing x intersects (F,A).
  (ix) Arbitary intersection of $(1,2)^*$ soft semi #gα-closed sets in $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E) is also $(1,2)^*$ soft semi #gα-closed in $(\tilde{X}, \tilde{\tau_1}, \tilde{\tau_2}$,E).
- Definition 2.9.

  (i) For any soft subset (F,A) of $\widetilde{\tau_{1,2}}$(X,E), its $(1,2)^*$soft #gα-border is defined by,
  $(1,2)^*$ soft bd (F,A) $\cong$ (F,A) \ $(1,2)^*$ soft int(F,A).

  (ii) For any soft subset (F,A) of $\widetilde{\tau_{1,2}}$(X,E),its $(1,2)^*$soft #gα-frontier is defined by,
  $(1,2)^*$ soft fr (F,A) $\cong$ $(1,2)^*$ soft cl(F,A) \ $(1,2)^*$ soft int(F,A).

III. FRONTIER OF SOFT SEMI #gα -CLOSED SETS IN SOFT BI-TOPOLOGICAL SPACES

In this section, we introduce and study the concepts of frontier of soft semi #gα-closed sets in soft bi-topological spaces.

3. Soft semi #gα-frontier of a set in soft bi-topological spaces:
3.1.1. Definition:
For any soft subset (F,A) of $\widetilde{\tau_{1,2}}$ (X,E), its $(1,2)^*$soft semi #gα-frontier is defined by ,
$(1,2)^*$ soft semi #gα -fr (F,A) $\cong$ $(1,2)^*$ soft semi #gα- cl(F,A) \ $(1,2)^*$ soft semi #gα- int(F,A).

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3.1.2. Theorem.

In a soft bi-topological space (X, $\widetilde{\tau_{1,2}}$,E), for any soft subset (F,A) of $\tilde{\tau}_{1,2}$ (X, E), the following statements hold.

(i)$(1,2)^*$ soft semi #gα-fr (∅) ≅ $(1,2)^*$ soft semi #gα-fr (X,E) ≅ $\widetilde{\emptyset}$.

(ii) $(1,2)^*$ soft semi #gα-cl (F,A) ≅ $(1,2)^*$ soft semi #gα- int (F,A) $\widetilde{\cap}$ $(1,2)^*$ soft semi #gα-fr (F,A).

(iii)$(1,2)^*$ soft semi #gα- int (F,A) $\widetilde{\cap}$ $(1,2)^*$ soft semi #gα-fr (F,A) ≅ $\widetilde{\emptyset}$.

(iv) $(1,2)^*$ soft semi #gα-bd (F,A) $\widetilde{\subseteq}$ $(1,2)^*$ soft semi #gα-fr (F,A) $\widetilde{\subseteq}$ $(1,2)^*$ soft semi #gα- cl (F,A) .

(v) If (F,A) is $(1,2)^*$ soft semi #gα-closed, then (F,A) ≅ $(1,2)^*$soft semi #gα-int (F,A) $\widetilde{\cup}$ $(1,2)^*$ soft semi #gα-fr(F,A).

Proof:

(i) Let us take $(1,2)^*$ soft semi #gα-fr(∅) is in $(1,2)^*$ soft semi #gα-fr (X,E) and is an empty set.

(ii) Since intersection of $(1,2)^*$ soft semi #gα-int (F,A) and $(1,2)^*$ soft semi #gα-fr(F,A) should be in $(1,2)^*$ soft semi #gα-cl (F,A) .

(iii) Since intersection of $(1,2)^*$ soft semi #gα-int (F,A) and $(1,2)^*$ soft semi #gα-fr (F,A) is always empty.

(iv) Border and frontier of (F,A) is always contained in $(1,2)^*$ soft semi #gα-cl (F,A).

(v) is follows from (ii) and Theorem 2.8 (ii).

3.1.3. Theorem.

For any soft subset (F,A) of $\widetilde{\tau_{1,2}}$(X,E) in a soft bi-topological space (X, $\widetilde{\tau_{1,2}}$,E), the following hold.

(i) $(1,2)^*$ soft semi #gα-fr (F,A)≅ $(1,2)^*$ soft semi #gα-cl (F,A) $\widetilde{\cap}$ $(1,2)^*$ soft semi #gα-cl ((X,E)\ (F,A)).

(ii) A point x $\widetilde{\in}$ $(1,2)^*$ soft semi #gα-fr (F,A), if and only if $(1,2)^*$ soft semi #gα-open set containing x intersects both (F,A) and its complement $\widetilde{\tau_{1,2}}$(X,E) \ (F,A).

(iii) $(1,2)^*$ soft semi #gα-cl (($(1,2)^*$ soft semi #gα-fr(F,A)) ≅ $(1,2)^*$ soft semi #gα-fr(F,A), that is $(1,2)^*$ soft semi #gα-fr(F,A), is $(1,2)^*$ soft semi #gα-closed.

(iv) $(1,2)^*$ soft semi #gα-fr (F,A) ≅ $(1,2)^*$ soft semi #gα-fr ((X,E) \ (F,A)).

(v) (F,A) is $(1,2)^*$ soft semi #gα-closed iff $(1,2)^*$ soft semi #gα-fr (F,A) ≅ $(1,2)^*$ soft semi #gα-bd (F,A), that is, (F,A) is $(1,2)^*$ soft semi #gα-closed iff (F,A) contains its $(1,2)^*$ soft semi #gα-frontier .

Proof:

From (i), we can prove (iii) by applying the results of Theorem2.8(iii) and (ix). Proof of (iv) is similar.

(v): If (F,A) is $(1,2)^*$ soft semi #gα-closed, then (F,A) ≅ $(1,2)^*$ soft semi #gα-cl (F,A). Hence by definition 3.1.1., $(1,2)^*$ soft semi #gα-fr (F,A) ≅ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) ≅ $(1,2)^*$ soft semi #gα-bd (F,A).

Conversly, suppose that $(1,2)^*$ soft semi #gα-fr (F,A) ≅ $(1,2)^*$ soft semi #gα-bd(F,A), using Definition 3.1.1. and definition of soft border, we get $(1,2)^*$ soft semi #gα-cl (F,A) ≅ (F,A).

3.1.4. Theorem.

For any soft subset (F,A) of $\widetilde{\tau_{1,2}}$(X,E) in a soft bi-topological space (X, $\widetilde{\tau_{1,2}}$,E), the following hold.

(i) (F,A) is $(1,2)^*$ soft semi #gα-regular iff $(1,2)^*$ soft semi #gα-fr (F,A) ≅ $\widetilde{\emptyset}$.

(ii)$(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-int(F,A)) $\widetilde{\subseteq}$ $(1,2)^*$ soft semi #gα-fr(F,A). .

(iii)$(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-cl (F,A)) $\widetilde{\subseteq}$ $(1,2)^*$ soft semi #gα-fr(F,A).

(iv)$(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-fr (F,A)) $\widetilde{\subseteq}$ $(1,2)^*$ soft semi #gα-fr (F,A)).

(v) $\widetilde{\tau_{1,2}}$(X,E) ≅ $(1,2)^*$ soft semi #gα-int (F,A) $\widetilde{\cup}$ $(1,2)^*$ soft semi #gα-int ((X,E) \ (F,A))$\widetilde{\cup}$ $(1,2)^*$ soft semi #gα − fr(F,A).

(vi) $(1,2)^*$ soft semi #gα-int(F,A) ≅ (F, A)\ $(1,2)^*$ soft semi #gα-fr (F,A).

(vii) If (F,A) is $(1,2)^*$ soft semi #gα-open, then (F,A) $\widetilde{\cap}$ $(1,2)^*$ soft semi #gα-fr (F,A) ≅ $\widetilde{\emptyset}$, that is, $(1,2)^*$ soft semi #gα-fr (F,A) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$(X,E) \ (F,A).

Proof:

From Theorem 2.8(i) and (iii) and Definition 3.1.1 , (i) can be proved. Since $(1,2)^*$ soft semi #gα-int (F,A) is $(1,2)^*$ soft semi #gα-open. Similarly $(1,2)^*$ soft semi #gα-cl (F,A) is $(1,2)^*$ soft semi #gα-closed, (i) and (ii) can proved. Since $(1,2)^*$ soft semi #gα-fr (F,A) is $(1,2)^*$ soft semi #gα-closed, invoking the above theorem (iv), (v) can be proved.

To prove (iv), since (X,E) ≅ $(1,2)^*$ soft semi #gα-cl (F,A) $\widetilde{\cup}$ ((X,E) \ $(1,2)^*$ soft semi #gα-cl (F,A)), but from the above theorem (ii) $(1,2)^*$ soft semi #gα-cl (F,A) ≅ $(1,2)^*$ soft semi #gα-int (F,A) $\widetilde{\cup}$ $(1,2)^*$ soft semi #gα-fr(F,A). Also $\widetilde{\tau_{1,2}}$(X,E) \ $(1,2)^*$ soft semi #gα-cl (F,A) ≅ $(1,2)^*$ soft semi #gα-int((X,E) \ (F,A)). Hence $\widetilde{\tau_{1,2}}$ (X,E) ≅ $(1,2)^*$ soft semi #gα-int (F,A) $\widetilde{\cup}$ $(1,2)^*$ soft semi #gα- fr (F,A) $\widetilde{\cup}$ $(1,2)^*$ soft semi #gα-int ((X,E) \ (F,A)). Thus (v) is proved.

Proof of (vi) follows from the Theorem 2.8 (v). If (F,A) is $(1,2)^*$ soft semi #gα-open, (F,A) ≅ $(1,2)^*$ soft semi #gα-int (F,A). Hence (vii) follows from (iii).

3.1.5. Theorem.

If (F,A) is any $(1,2)^*$ soft subset of (X,E), then $(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-fr (F,A)) ≅ $(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-fr (F,A)).

Proof:

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If $(1,2)^*$ soft semi #gα-fr (F,A) is $(1,2)^*$ soft semi #gα-closed. Then (F,A) is $(1,2)^*$ soft semi #gα-closed in (X,E), then (X,E) \ (F,A) is $(1,2)^*$ soft semi #gα-closed and hence we have $(1,2)^*$ soft semi #gα-int ((X,E) \ (F,A)) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-int(X,E) \ $(1,2)^*$ soft semi #gα-fr (X,E) \ (F,A) ≅ $(1,2)^*$ soft semi #gα-cl ((X,E) \ (F,A)).

In both the cases using Theorem 3.1.4 (vii), we get $(1,2)^*$ soft semi #gα-fr (($(1,2)^*$ soft semi #gα-fr(F,A)) ≅ $(1,2)^*$ soft semi #gα-fr (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-fr ((X,E) \ (F,A)) ≅ $(1,2)^*$ soft semi #gα-fr (F,A).

3.1.6. Theorem.

If (F,A) and $\widetilde{\tau_{1,2}}$(G,B) are $(1,2)^*$ soft subsets of $\widetilde{\tau_{1,2}}$(X,E) such that (F,A) $\tilde{\cap}$ $\widetilde{\tau_{1,2}}$(G,B) ≅ $\widetilde{\emptyset}$, where (F,A) is $(1,2)^*$ soft semi #gα-open in (X,E) , then (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-cl(G,B) ≅ $\widetilde{\emptyset}$.

Proof:

If possible, let x $\tilde{\in}$ (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-cl (G,B). Then (F,A) is a $(1,2)^*$ soft semi #gα-open set containing x and also x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-cl (G,B). By Theorem 2.8 (viii), (F,A) $\tilde{\cap}$ $\widetilde{\tau_{1,2}}$(G,B) ≅ $\widetilde{\emptyset}$, which is contradiction . Thus (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-cl (G,B) ≅ $\widetilde{\emptyset}$.

3.1.7. Theorem.

If (F,A) and $\widetilde{\tau_{1,2}}$(G,B) are $(1,2)^*$ soft subsets of $\widetilde{\tau_{1,2}}$(X,E) such that (F,A) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}$(G,B) and $\widetilde{\tau_{1,2}}$ (G,B) is $(1,2)^*$ soft semi #gα-closed in (X,E) , then $(1,2)^*$ soft semi #gα-fr (F,A) $\tilde{\subseteq}$ (G,B).

Proof:

$(1,2)^*$ soft semi #gα-fr (F,A) ≅ $(1,2)^*$ soft semi #gα-cl (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα- cl (G,B) \ $(1,2)^*$ soft semi #gα-int (F,A) ≅ $\widetilde{\tau_{1,2}}$ (G,B) \ soft semi #gα- int (F,A) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}$(G,B).

3.1.8. Theorem.

If (F,A) and $\widetilde{\tau_{1,2}}$(G,B) are $(1,2)^*$ soft subsets of $\widetilde{\tau_{1,2}}$(X,E) such that (F,A) $\tilde{\cap}$ $\widetilde{\tau_{1,2}}$(G,B) ≅ $\widetilde{\emptyset}$, where (F,A) is $(1,2)^*$ soft semi #gα-open in (X,E) , then (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-fr (G,B) ≅ $\widetilde{\emptyset}$.

Proof:

Since $(1,2)^*$ soft semi #gα-fr (G,B) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-cl (G,B). If possible, let x $\tilde{\in}$ (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-int (G,B). Then (F,A) is a $(1,2)^*$ soft semi #gα-closed set containing x and also x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-int (G,B). We know that a point x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-cl (F,A) iff every $(1,2)^*$ soft semi #gα-open set in (X,E) containing x intersects both (F,A) and its complement $\widetilde{\tau_{1,2}}$(X,E) \ (F,A) ≅ $\widetilde{\emptyset}$, which is contradiction . Thus (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-int(G,B) ≅ $\widetilde{\emptyset}$.

3.1.9. Example.

Let U = $\{u_1 , u_2 , u_3\}$, E = $\{x_1 , x_2\}$ and F_E = X = $\{(x_1 ,\{u_1 , u_2 , u_3\}), (x_2 ,\{u_1 , u_2 , u_3\})\}$. And let $(\tilde{X} , \tilde{\tau}_1 , \tilde{\tau}_2)$ be a soft bi-topological space , where $\tilde{\tau}_1$ = { $\emptyset$, F_{A_2} , F_{A_3} , F_{A_5} , X } , $\tilde{\tau}_2$ = { $\emptyset$, F_{A_2} , F_{A_8} , $F_{A_{14}}$, X }, then $\widetilde{\tau_{1,2}}$ soft open sets are { $\emptyset$, F_{A_2} F_{A_3} , F_{A_5} , F_{A_8} , $F_{A_{14}}$, $F_{A_{17}}$, $F_{A_{32}}$, X } and $\widetilde{\tau_{1,2}}$ soft closed sets are {$\emptyset$, F_{A_4} F_{A_6} , F_{A_7} , $F_{A_{12}}$, $F_{A_{31}}$, $F_{A_{44}}$,
$F_{A_{46}}$, X } Then ,

Frontier of soft semi #gα-closed set is: [$\{x_1 , (u_2)\}$]

IV. CONCLUSION

In this paper , Frontier of soft semi gα-closed sets in soft bi-topological spaces introduced and studied with already existing sets in soft bi-topological spaces. The scope for further research can be focused on the applications of soft bi-topological spaces.

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