Scientific Journal of Impact Factor (SJIF): 5.71

e-ISSN (O): 2348-4470
p-ISSN (P): 2348-6406

International Journal of Advance Engineering and Research Development

Volume 5, Issue 02, February -2018

"Amalgamation of Two methods of Partial Differential Equations"

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ABSTRACT :- *In partial differential complex domain, by comparing two equations or transform one equation in to another is done by considering the coupling of two equations (1) and (2). The for the most part imperative statement is that the combination equation has considerably lots of variables and so the sense of the solution is not so inconsequential. The result is applied to the problem of analytic continuation of the solution. In this paper, I will in attendance a novel draw near to the study of nonlinear partial differential equations in the composite domain. Since the explore is still in the initial stage, as a replica study I will converse about only the subsequent two partial*

Differential equations in the first stage, as a reproduction swot up I will confer lone the following two partial differential equations.

$$\frac{\partial y}{\partial x} = P\left(x,t,y,\frac{\partial y}{\partial t}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

(Where (x, t) ∈ C2 are variables and y = y(x, t) is the unknown function)

$$\frac{\partial \theta}{\partial x} = Q\left(x,t,y,\frac{\partial \theta}{\partial t}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

(where (x, t) ∈ C2 are variables and θ= θ (x, t) is the unknown function).

INTRODUCTION

In partial differential complex domain we can say that the two equations (1) and (2) are equivalent when we can transform (1) into (2) (or (2) into (1)) One way to treat this sentence is to consider the coupling of (1) and (2), and to solve their coupling equation. The coupling of two partial differential equations (1) and (2) means that, we consider the following partial differential equation with infinitely many variables (x, t, y0, y1, . . .)

$$\frac{\partial \alpha}{\partial x} + \sum_{n\geq 0} A^n\, [P](x,t,y_0,y_1,\ldots\ldots\ldots,y_{m+1})\ \frac{\partial \alpha}{\partial ym} = Q(x,t,\alpha,A[\alpha])\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(where α = α (x, t, y0, y1, . . .) is the unknown function), or the following partial differential equation with infinitely many variables (x, t,y0, y1, . . .)

$$\frac{\partial \beta}{\partial x} + \sum_{n\geq 0} An\ [Q](x,t,\theta\,0,\theta\,1,\ldots\ldots\ldots\ldots,\theta\,m{+}1)\ \frac{\partial \beta}{\partial \theta m} = Q(x,t,\beta,A[\beta])\ldots\ldots\ldots(4)$$

(where = β (x, t, θ 0, θ 1, . . .) is the unknown function). In the equation (3) (resp. (4)), the notation D means the following vector field with infinite many variables (x, t, θ 0, θ 1, . . .)

In the previous segment, I determination provide a use to the difficulty of analytic continuation of the solution. The consequence is just the similar as in Kobayashi and Lope-Tahara .This prove the competence of our pioneering come near in my paper. In the coffer of normal differential equations, the possibility of the coupling of two differential equations was explained in next section of G´erard-Tahara.

Coupling of Two Ordinary Differential Equations

Previous to the conversation in partial differential equations, let us provide a short review on the coupling of two ordinary differential equations in previous section of G´erard-Tahara.

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Initially, let us think about the subsequent two ordinary differential equations:

$$\frac{dy}{dx} = P(x, y) \ldots\ldots.(5)$$

$$\frac{d\theta}{dx} = q(x, \theta) \ldots\ldots\ldots\ldots(6)$$

where $p(x, y)$ (resp. $q(x, \theta)$) is a holomorphic meaning defined in a neighborhood of the source of $Ct \times Cu$ (resp. $Ct \times Cw$).

At this time, let us talk about the likeness of two differential equations. Let $p(x, z)$ and $q(x, z)$ be different functions of (x, z) in a area of $(0, 0)$ $C_t \times C_z$ as before, and let us think about the subsequent two equations:

$$\frac{\partial y}{\partial x} = P(x, y), y(x) \to 0 \ldots\ldots\ldots\ldots.(7)$$

$$\frac{\partial \theta}{\partial x} = q(x, \theta), \theta(x) \to 0 \ldots\ldots\ldots\ldots(8)$$

Characterize by *Sa* (resp. *Sb*) the set of all the holo-morphic solutions of [7] (resp. [8]) in a appropriate neighbourhood of x = 0. Let α (t, u) be a holo-morphic explanation of (2.1) fulfilling α (0, 0) = 0: if $u(t) \in Sa$ we have $\alpha\ (t, u(t)) \longrightarrow \alpha\ (0, 0) = 0$ (as $t \longrightarrow 0$).
The combination of (7) and (8) means that we regard as the subsequent partial differential equation [9] or [10]:

$$\frac{\partial \alpha}{\partial x} + p(x, y)\frac{\partial \alpha}{\partial y} = q(x, \alpha) \ldots\ldots\ldots\ldots.(9)$$

(Where (t, u) are variables and $\alpha = \alpha\ (t, u)$ is the indefinite function)

$$\frac{\partial \beta}{(\partial x} + q(x, \theta)\frac{\partial \beta}{\partial \theta} = p(x, \beta) \ldots\ldots\ldots\ldots\ldots(10)$$

(Where (t, θ) are variables and $\beta = \beta(t, \theta)$ is the unidentified function).

We call (9) or (10) the coupling equation of (5) and (6) . Let α (x, y) be a explanation of (9) and presume that the relation $\theta = \alpha$ (x, y) is equal to y =(t, θ) for some role β (t, θ); then β (t, θ) is a explanation of (10). Let ψ(t, θ) be a clarification of (10) and suppose that the relation y =β (t, θ) is equivalent to $\theta = \beta$ (x, y) for some meaning β (x, y); then β (x, y) is a Solution of (9).

CONCLUSION

We will prove only the part (1). Since $\theta = \alpha$ (x, y) is equivalent to y = β (t, θ), we get u $\equiv\beta$ (x, β (x,y)). By beginning this with admiration to x and y

we get

$$\begin{aligned}
&(\frac{\partial \beta}{\partial x} + q(x, \theta)\frac{\partial \beta}{\partial \theta}) \\
&= \frac{\partial \beta}{\partial x}\big(x, \alpha(x, y)\big) + q\big(x, \alpha(x, y)\big)\frac{\partial \beta}{\partial \theta}\big(x, \alpha(x, y)\big) \\
&= -\frac{\partial \beta}{\partial \alpha}\big(x, \alpha(x, y)\big)\frac{\partial \alpha}{\partial x}(x, y) + q\big(x, \alpha(x, y)\big)\frac{\partial \beta}{\partial \theta}\big(x, \alpha(x, y)\big) \\
&= \frac{\partial \beta}{\partial \alpha}\big(x, \alpha(x, y)\big)(-\frac{\partial \alpha}{\partial x}(x, y) + q\big(x, \alpha(x, y)\big))
\end{aligned}$$

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$$=\frac{\partial\beta}{\partial\alpha}\left(x,\alpha(x,y)\right)p(x,y)\frac{\partial\alpha}{\partial y}(x,y) = p(x,y)\ldots\ldots\ldots\ldots\ldots(11)$$

Coupling of Two Partial Differential Equations

Now, let us oversimplify the theory in this section to partial differential equations. In this segment we will give only a formal theory.

Let us regard as the subsequent two nonlinear partial differential equations:

$$\frac{\partial y}{\partial x} = P\left(x,t,y,\frac{\partial y}{\partial t}\right)\ldots\ldots\ldots\ldots.(12)$$

(where $(t, x) \in C2$ are variables and $u = u(t, x)$ is the unknown function)

$$\frac{\partial\theta}{\partial x} = Q\left(x,t,\theta,\frac{\partial\theta}{\partial t}\right)\ldots\ldots\ldots\ldots\ldots(13)$$

where (t, x) ∈ C2 are variables and θ = θ (t, x) is the unknown function).
For simplicity we assume that P(x, t, y0, y1) (resp. Q(x, t, θ0, θ1)) is a holo-morphic function defined in a neighbourhood of the origin of Ct × Cx × Cu0× Cu1 (resp. Ct × Cx × Cw0× Cw1). We can regard D as a vector field with infinitely many variables (x, u0, u1, . . .)

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