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Predication of Population Growth with Rate of Migration using Cubic Spline Method

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Abstract - *This paper presents the method of prediction of population growth over a period of time at various locations of a striped region. It is considered that population growth follows of diffusion equation where the nature of migration rate of population growth and diffusion rate of molecules will be similar. The numerical approach has been adopted to find the solution of constructed Partial Differential Equation (PDE) of population growth equation by Cubic Spline – Explicit and Implicit methods with different initial and boundary conditions.*

Keywords- *Population Growth, PDE, Migration Rate, Cubic Spline, Explicit method, Implicit method.*

I. INTRODUCTION

The rapid growth of population has been being the biggest obstacle in the development of any country. At present UN-DESA, 2017 report says that world is overpopulated [1] and the growth in resources has not been keeping pace with the growth in population. As per the World Economic Forum's (WEF) annual Global Shapers surveys that, the top most concerning world issues regarding overpopulation, according to futuristic are Lack of economic opportunity and unemployment is 14.2%, Food and water security is 15.1%, Lack of education is 16.5%, Safety, security, and wellbeing are 18.1%, Poverty is 31.1%, Large scale conflict and wars are 38.5%, Climate change and destruction of natural resources are 45.2% [2]. So the increasing trend of population is a warning against development of a nation. Moreover, modernization in social, cultural and economic development also leads the increasing trend of population. Nowadays every individual is running towards the modernization and they are putting their best efforts to adopt it. As a result, the standards of living, the thinking pattern and there facilities utilization have been changed. Also the technological revolution is a byproduct of modernization [2]. Due to this, people can be easily connected with each other, deal with each other and update themselves. In short, it has provided a large platform to everyone for their own growth and development, which provokes people to get all the facilities in easy and quick manner. To fulfill their dream everyone likes to adapt the new changed living environment and therefore they are ready to move from their native towards developed area. Especially towards those areas, which are well developed but already populated like as urban areas. Thus, it is very much difficult for any country to manage such kind of drastic movement and as a result that particular region becomes overcrowded. It is a serious problem when population growth of any country goes beyond its optimum level, causes the scarcity of facilities viz. house, medical, hygiene, food and many more, that creates unemployment, poverty, pollution etc.[2] So overcrowding in one area or at a location or in a city is a critical issue for government. They should seriously take smart steps in the direction of managing this overpopulation, can also develop more developed area accordingly. So that, overpopulation should be migrate to the well-developed area where the people should get all basic facilities like resident, hospital, employment and transportation, etc. In short migration is the better solution to manage overpopulation.

People moves from one place to another with the intentions of settling, permanently in the new location is called "Migration" [6]. It is observed that this migration happens for the certain purpose or in certain time. It happens automatically and it is unpredictable. Hence if somehow we can fix the nature of migration then we will be able to manage and calculate the overcrowded population.

II. MIGRATION EQUATION OF POPULATION GROWTH

The nature of migration of population growth is similar to the diffusion of molecules in the space, in which the rates of diffusion of the flow of molecules are changed with respect to time and space [3]. Here Diffusion flux means how much molecules are diffused in particular area at particular time. Thus, it is a function of both time and space. There are two types of diffusion equations, steady state and non-steady state. Here we are considering non-steady state equation because of the diffusion flux is changed with respect to time [3]. For instant a dye is injected at the one end of the tube at same time molecules of dye start to move from higher concentration area to lower concentration area.

The same nature of Diffusion equation is followed by migration of population as well [6]. The overcrowding population at one location (particularly residential region), a population may enter in other location (residential area) and this movement is randomly. The migration of population is changed with respect to change in time and space according to its

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rate of migration. Hence we are adopting the same concept for migration equation as well. Here, population depends upon two independent variables time (t) and space (x). Also 'm' is rate of migration. It is the difference between the number of people coming into an area and the number of people leaving an area throughout the year. When the number of people coming into an area is larger than the number of people leaving an area called a positive migration rate. Migration rate will counts as per 1000 people [6]. Thus, the one dimensional partial differential equation model in the context of diffusion said as migration of population w.r.t time and space will be as follows [3].

$$\frac{\partial P}{\partial t} = m\frac{\partial^2 P}{\partial x^2}\ ; \quad m > 0 \tag{1}$$

The solution of equation (1) shows an amount of population (P) in time t at place x, will be uniquely determined by different analytical and numerical methods along with required initial and boundary conditions.

Here we are adopting numerical approach especially Cubic Spline Method [5] for the solution of this partial differential equation by considering two different types of initial and boundary conditions to observe the nature of growth of population in future.

Case-(a): $P(x,0) = C_0 + C_1e^{-\alpha x}$, $P(0,t) = C_0 + C_1$
Case-(b): $P(x,0) = C_0 + C_1e^{-\alpha x}$, $P(0,t) = C_0 + C_1e^{-\alpha t}$
Where, $P(x,t)$ indicates population in time t and at location x.

III. SPLINE APPROACH [4][5]

There were several numerical approximation methods in mathematics which have been proposed for the solution of partial differential equation like as Finite Difference Method (FDM), Finite Element Method (FEM), and Splines etc. Here we were considering Cubic Spline approach for the solution of one dimensional diffusion equation. Spline is nothing but it is piecewise connecting polynomials and the connecting polynomials could be of any degree. So we have different types of spline polynomials viz., linear spline, quadratic spline, cubic spline, quantic spline, etc. Out of these, the cubic spline of degree three has been found the most popular approximation method. Higher order polynomials are more oscillated as compare to the lower order. Also the linear equation is not applicable at everywhere. So with this limitation of the higher order and linear we will consider the fourth order (third degree) polynomial for cubic spline. It is also observed that fourth order or third degree polynomial is more acceptable as an approximate function than the actual graph. Here we follow cubic spline explicit and implicit method.

3.1. Explicit Method [4][5]

The numerical solution to the partial differential equation form in equation (1) is to find the growth of population w.r.t space and time of one space variable through explicit spline collocation method describe as below.
Consider equation (1) with the initial and boundary conditions taken in case-(a) and (b). We divide the region x into, n-equal subintervals of width h. Let us represent the points of subintervals by $x_0, x_1, x_2, \dots, x_n$. Here, we have the results in time $j\Delta t$ (where j=0,1,…) at the mesh points $x_0, x_1, x_2, \dots, x_n$. $P_{i,j}$ is represented as a value of P at the i^{th} mesh point in time $j\Delta t$.

For estimating the function P by a cubic spline method let us consider $S(x)$ is a polynomial of degree k over each subinterval $[x_i, x_{i+1}]$. For computing the polynomial of $S(x_i)$ first we calculate $S''(x_i)$ by solving set of simultaneous equations for, i =0,1, 2, 3… n-1. We should note that values of P at $x = x_0$ and $x = x_n$ are known by initial conditions. Now discretizing left side of equation (1) by forward difference formula and replacing right side by the second derivatives $S''(x_i)$ at j^{th} level like explicit system in finite difference. We get

$$\frac{(P_{i,j+1} - P_{i,j})}{\Delta t} = mS''_{i,j} \tag{2}$$

Where $S''_{i,j} = S''(x_i)$ at j^{th} level. Now substitute values of $S''(x_i)$ from (2) into below equation [5]. We get,

$$S''(x_{i-1}) + 4S''(x_i) + S''(x_{i+1}) = \frac{6}{h^2}\{f(x_{i-1}) - 2f(x_i) + f(x_{i+1})\} \tag{3}$$

$$\frac{(P_{i-1,j+1} - P_{i-1,j})}{\Delta tm} + 4\frac{(P_{i,j+1} - P_{i,j})}{\Delta tm} + \frac{(P_{i+1,j+1} - P_{i+1,j})}{\Delta tm} = \frac{6}{h^2}\left(P_{i-1,j} - 2P_{i,j} + P_{i+1,j}\right) \tag{4}$$

$$\left(P_{i-1,j+1} - P_{i-1,j}\right) + 4\left(P_{i,j+1} - P_{i,j}\right) + \left(P_{i+1,j+1} - P_{i+1,j}\right) = r\left(6P_{i-1,j} - 12P_{i,j} + 6P_{i+1,j}\right) \tag{5}$$

Here $r = \frac{\Delta tm}{h^2}$,

$$P_{i-1,j+1} + 4P_{i,j+1} + P_{i+1,j+1} = (1+6r)P_{i-1,j} + (4-12r)P_{i,j} + (1+6r)P_{i+1,j} \tag{6}$$

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Where $i = 1,2,\ldots$ as $P_{i,j=0}$ will be obtain from initial condition of equation (1).
This set of simultaneous equations can be solved equation (6) recognized as cubic spline explicit formula to solve equation (1). Now $P_{0,j+1}$ is known due to the given initial conditions. The set of simultaneous equations obtained in explicit system contains only (n – 1) unknowns. These (n – 1) equations from (6) in (n – 1) unknown can be solved by any standard method. Furthermore, the values of P are known at $(j+1)^{th}$ level, we can continue to calculate the next level (j + 2) by repeating the same process. For each set of simultaneous equations in (n – 1) unknowns give tri-diagonal matrix. It can be calculated by any standard method, thus the method can continue by steps. The convergence and stability of these methods totally depends upon the values of r.

3.2. Implicit Method [4][5]

The finite difference replacement of (1) corresponding to implicit system is

$$\frac{(P_{i,j+1}-P_{i,j})}{\Delta t} = m\frac{(S''_{i,j}+S''_{i,j+1})}{2} \tag{7}$$

Where $S''_{i,j}, S''_{i,j+1}$ denote second derivatives at $x = x_i$ at the time level j and j + 1 respectively. However, second derivatives at $(j+1)^{th}$ level cannot be computed as the values of P are not known. We use the relationship of the equation $S''(x_i)$ at j^{th} and $j+1^{th}$ level and rewrite it in the following forms.

$$S''_{i-1,j} + 4S''_{i,j} + S''_{i+1,j} = \frac{6}{h^2}\{P_{i-1,j} - 2P_{i,j} + P_{i+1,j}\} \tag{8}$$

$$S''_{i-1,j+1} + 4S''_{i,j+1} + S''_{i+1,j+1} = \frac{6}{h^2}\{P_{i-1,j+1} - 2P_{i,j+1} + P_{i+1,j+1}\} \tag{9}$$

$i = 1,2,3,\ldots$ as $P_{i,j=0}$ will be obtain from initial condition of equation (1).
So from equation (7) we have,

$$\left.\begin{aligned} S''_{i-1,j+1} &= \frac{2}{\Delta tm}[P_{i-1,j+1} - P_{i-1,j}] - S''_{i-1,j} \\ S''_{i,j+1} &= \frac{2}{\Delta tm}[P_{i,j+1} - P_{i,j}] - S''_{i,j} \\ S''_{i+1,j+1} &= \frac{2}{\Delta tm}[P_{i+1,j+1} - P_{i+1,j}] - S''_{i+1,j} \end{aligned}\right\} \tag{10}$$

Substituting the value of (10) into equation (9). We get,

$$\frac{2}{\Delta tm}[P_{i-1,j+1} - P_{i-1,j}] - S''_{i-1,j} + 4\left[\frac{2}{\Delta tm}[P_{i,j+1} - P_{i,j}] - S''_{i,j}\right] + \frac{2}{\Delta tm}[P_{i+1,j+1} - P_{i+1,j}] - S''_{i+1,j} = \frac{6}{h^2}\{P_{i-1,j+1} - 2P_{i,j+1} + P_{i+1,j+1}\} \tag{11}$$

$$2[P_{i-1,j+1} - P_{i-1,j}] - \Delta tm \cdot S''_{i-1,j} + 4[2[P_{i,j+1} - P_{i,j}] - \Delta tm \cdot S''_{i,j}] + 2[P_{i+1,j+1} - P_{i+1,j}] - \Delta tm \cdot S''_{i+1,j} = \frac{6\,\Delta tm}{h^2}\{P_{i-1,j+1} - 2P_{i,j+1} + P_{i+1,j+1}\} \tag{12}$$

$$2P_{i-1,j+1} + 8P_{i,j+1} + 2P_{i+1,j+1} - 6rP_{i-1,j+1} + 12rP_{i,j+1} - 6rP_{i+1,j+1} = 2P_{i-1,j} + 8P_{i,j} + 2P_{i+1,j} + \Delta tm \cdot [S''_{i-1,j} + 4S''_{i,j} + S''_{i+1,j}] \tag{13}$$

Here $r = \frac{\Delta tm}{h^2}$

From (8) we have; $S''_{i-1,j} + 4S''_{i,j} + S''_{i+1,j} = \frac{6}{h^2}\{P_{i-1,j} - 2P_{i,j} + P_{i+1,j}\}$

Substitute above equation in (13), we get

$$2P_{i-1,j+1} + 8P_{i,j+1} + 2P_{i+1,j+1} - 6rP_{i-1,j+1} + 12rP_{i,j+1} - 6rP_{i+1,j+1} = 2P_{i-1,j} + 8P_{i,j} + 2P_{i+1,j} + \Delta tm \cdot \frac{6}{h^2}\{P_{i-1,j} - 2P_{i,j} + P_{i+1,j}\} \tag{14}$$

$$2P_{i-1,j+1} + 8P_{i,j+1} + 2P_{i+1,j+1} - 6rP_{i-1,j+1} + 12rP_{i,j+1} - 6rP_{i+1,j+1} = 2P_{i-1,j} + 8P_{i,j} + 2P_{i+1,j} + 6r\{P_{i-1,j} - 2P_{i,j} + P_{i+1,j}\} \tag{15}$$

$$(1-3r)P_{i-1,j+1} + (4+6r)P_{i,j+1} + (1-3r)P_{i+1,j+1} = (1+3r)P_{i-1,j} + (4-6r)P_{i,j} + (1+3r)P_{i+1,j} \tag{16}$$

Where, $i = 1,2,3,\ldots$. Equation (16) is recognized as cubic spline implicit method to solve equation (1).
Now $P_{0,j+1}$ is again known due to the given boundary conditions. The set of simultaneous equations obtained in implicit system contains also (n–1) unknowns. Here also we can solve (n-1) equation by any standard method and by the calculated value from (j+1) level we can derived the further values for j+2, j+3,… level. In this case, again the value of r depends on the convergence and stability of the method same as explicit method. Values much larger than unity are not desirable.

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IV. EXPERIMENTAL RESULT

4.1. Explicit Method Case-(a)

Let us consider the equation (1) with two different cases-(a) & (b).

$$\frac{\partial P}{\partial t} = m\frac{\partial^2 P}{\partial x^2}\ ; m > 0, 0 < x < 10, t > 0$$

For case (I), Let the population in the different location be distributed as $P(x,t)$. The initial condition is $P(x,0) = C_0 + C_1 e^{-\alpha x}$, here C_0 , C_1 and α are constants and in this case let us assumes the value of $C_0 = 500$, $C_1 = 100$ and $\alpha = 0.4$. Furthermore the boundary condition at particular time is $P(0,t) = C_0 + C_1$. Let migration rate (m) be taken as 0.15. Now, we shall determine the solution of the equation (1) which satisfying above conditions.

Let the length of the area (x) is divided into 10 equal subintervals x_0 to x_{10}. We have length of subinterval $h = \Delta x = 1$. Also let time interval $\Delta t = 1$ and $m = 0.15$ then we get

$r = \frac{\Delta t m}{h^2} = \frac{1*0.15}{(1)^2} = 0.15$, Which gives, $1 + 6r = 1 + 0.9 = 1.9$ and $4 - 12r = 4 - 1.8 = 2.2$

Now substituting the values of 1 + 6r and 4 – 12r in equation (6) for j = 0 and i = 1,2,3, ...

$$i = 1\ , P_{0,1} + 4P_{1,1} + P_{2,1} = (1.9)P_{0,0} + (2.2)P_{1,0} + (1.9)P_{2,0}$$
$$i = 2\ , P_{1,1} + 4P_{2,1} + P_{3,1} = (1.9)P_{1,0} + (2.2)P_{2,0} + (1.9)P_{3,0}$$
$$i = 3\ , P_{2,1} + 4P_{3,1} + P_{4,1} = (1.9)P_{2,0} + (2.2)P_{3,0} + (1.9)P_{4,0} \tag{17}$$

Similarly for $i = 4$ to 10 find out other equations. Since $P_{9,1}$ and $P_{11,1}$ are symmetric. We get,

$$i = 10\ , P_{9,1} + 4P_{10,1} + P_{11,1} = (1.9)P_{9,0} + (2.2)P_{10,0} + (1.9)P_{11,0}$$
$$\therefore\ 2 * P_{9,1} + 4P_{10,1} = 2 * (1.9)P_{9,0} + (2.2)P_{10,0}$$

Here we get 10 algebraic equations in 10 unknowns with tri-diagonal matrix. Similarly for j = 1, 2,..we get another 10 algebraic equations. Proceeding in this way, the results obtained by explicit method are shown in Table 1 and plotted in Figure 1 & 2.

Table 1. Population Growth: PDE-Cubic Spline-Explicit Method-Case-(a)

Location (x)	Time t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00
1	567.03	569.70	571.02	572.28	573.16	574.04	574.70	575.38	575.93	576.47	576.94
2	544.93	546.01	547.87	549.13	550.55	551.61	552.75	553.66	554.62	555.41	556.23
3	530.12	531.03	531.66	532.86	533.73	534.86	535.75	536.77	537.62	538.54	539.33
4	520.19	520.75	521.43	521.84	522.61	523.18	523.97	524.60	525.37	526.04	526.77
5	513.53	513.92	514.29	514.78	515.09	515.59	515.97	516.48	516.93	517.45	517.94
6	509.07	509.33	509.59	509.87	510.16	510.43	510.73	511.02	511.34	511.67	512.00
7	506.08	506.25	506.47	506.57	506.81	506.97	507.21	507.39	507.64	507.84	508.11
8	504.08	504.22	504.22	504.47	504.51	504.76	504.84	505.09	505.20	505.45	505.57
9	502.73	502.69	503.00	503.01	503.31	503.34	503.61	503.67	503.92	504.00	504.24
10	501.83	502.34	502.37	502.71	502.73	503.03	503.06	503.33	503.38	503.63	503.71

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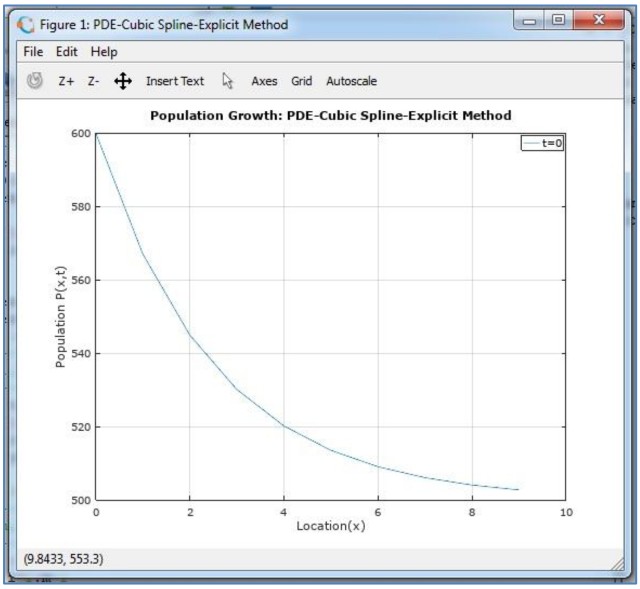

Figure 1. Initial Population: PDE-Cubic Spline-Explicit Method-Case-(a)

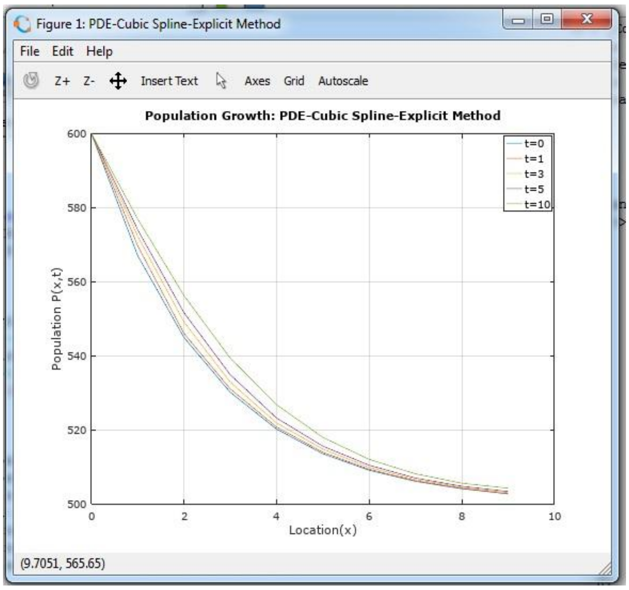

Figure 2. Population Growth: PDE-Cubic Spline-Explicit Method-Case-(a)

Figure 1 shows that initially (t=0) the population was distributed across the region is in exponentially decreasing order. As time changed the population will be grown as per the migration rate and the nature in which the location will be occupied by the population. It is also observed that at every period of time the population at initial location (x=0) will be remain constant as per the boundary condition taken. Now, the said growth in population at remaining locations (x=1 to 10) in various time (t=1, t=3, t=5, t=10) will be seen in Figure 2.

4.2. Explicit Method Case-(b)

In second case, Let us assume the condition $P(x,0) = C_0 + C_1 e^{-\alpha x}$, with the boundary condition $P(0,t) = C_0 + C_1 e^{-\alpha t}$. Again considering the values of $C_0 = 500$, $C_1 = 100$, $\alpha = 0.4$. and migration rate is m=0.15. We should follow the similar process as per explicit method for case-(a).

Here also we get 10 algebraic equations in 10 unknowns with tri-diagonal matrix. Similarly for j = 1, 2,..we get another 10 algebraic equations. Proceeding in this way, the results are shown in Table 2 and plotted in Figure 3 & 4.

Table 2. Population Growth: PDE-Cubic Spline-Explicit Method-Case-(b)

Location (x)	Time t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.00	567.03	544.93	530.12	520.19	513.53	509.07	506.08	504.08	502.73	501.83
1	567.03	578.53	569.25	562.33	552.84	546.53	539.52	535.16	530.23	527.44	523.90
2	544.93	543.64	552.77	551.19	552.26	548.28	547.01	542.65	541.04	537.00	535.71
3	530.12	531.66	529.16	534.69	534.12	537.39	535.81	537.44	535.07	535.92	533.13
4	520.19	520.58	522.42	520.12	523.44	522.39	525.54	524.39	526.98	525.36	527.56
5	513.53	513.97	513.94	515.73	514.02	516.30	514.91	517.51	516.15	518.84	517.17
6	509.07	509.32	509.71	509.44	510.93	509.67	511.49	510.04	512.25	510.61	513.24
7	506.08	506.25	506.43	506.75	506.39	507.56	506.57	508.16	506.72	508.82	506.93
8	504.08	504.22	504.23	504.40	504.71	504.38	505.36	504.45	506.02	504.48	506.69
9	502.73	502.69	503.00	503.04	503.21	503.58	503.20	504.23	503.18	504.92	503.14
10	501.83	502.34	502.37	502.69	502.79	502.84	503.43	502.79	504.09	502.73	504.80

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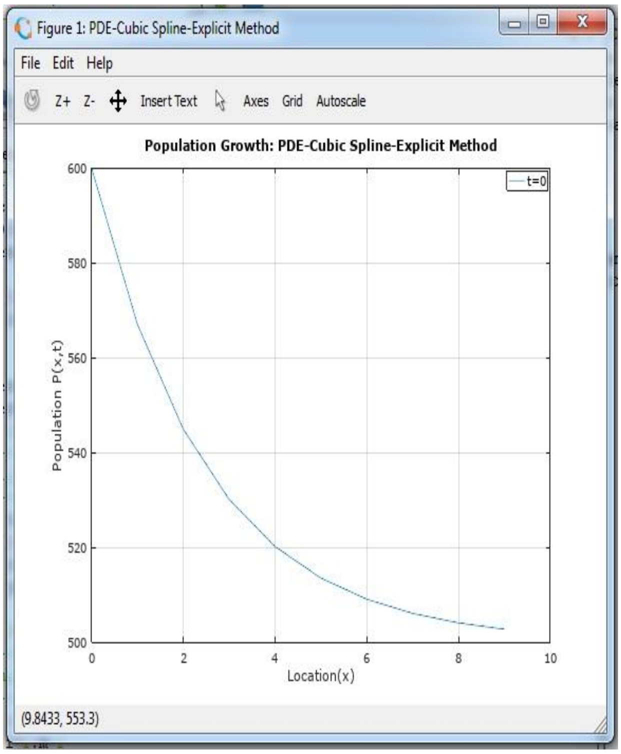

Figure 3. Initial Population: PDE-Cubic Spline-Explicit Method –Case-(b)

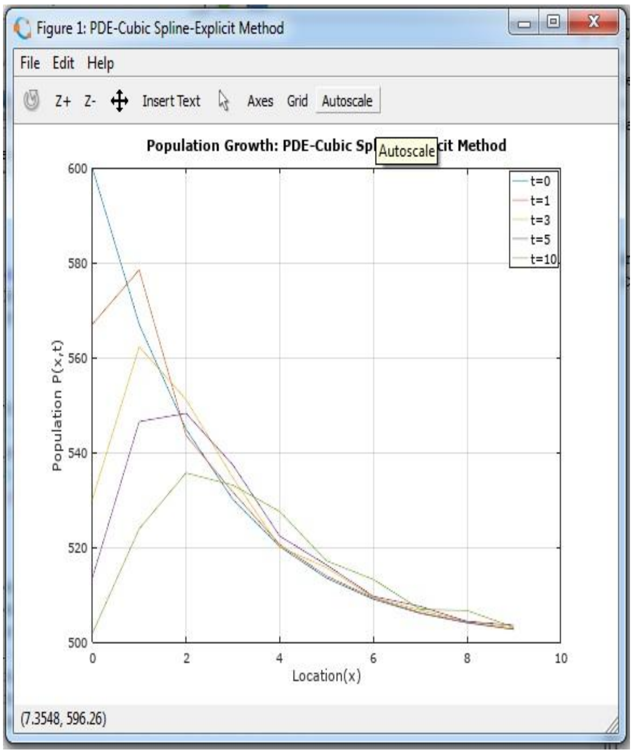

Figure 4. Population Growth: PDE-Cubic Spline-Explicit Method-Case-(b)

Figure 3 shows that initially (t=0) the population was distributed across the region is in exponentially decreasing order. As time changed the population growth will be go down as per the migration rate and the nature in which the location will be occupied by the population. It is also observed that at different time interval the population at initial location (x=0) will be highly decrease as per the boundary condition taken. Whereas the decrement in population growth at the nearby locations of (x=0) is comparatively lower than initial location. While decreasing population nature at the far locations viz. (x=4 to 10) in different time (t=1, t=3, t=5, t=10) will be seen in Figure 4.

4.3. Implicit Method Case-(a)

The solution of equation (1) by Cubic spline implicit method,

Here $r = 0.15$

Now, Substitute the value of r in (16) we get,

$$1-3r = 0.55\,, 1+3r = 1.45, 4-6r = 3.1, 4+6r = 4.9$$

Substituting the values of $1-3r$, $1+3r$, $4+6r$ and $4 – 6r$ in below equation, we get

$$(1-3r)P_{i-1,j+1} + (4+6r)P_{i,j+1} + (1-3r)P_{i+1,j+1} = (1+3r)P_{i-1,j} + (4-6r)P_{i,j} + (1+3r)P_{i+1,j}$$

For $j = 0$

$$\begin{aligned} i = 1, (0.55)P_{0,1} + (4.9)P_{1,1} + (0.55)P_{2,1} &= (1.45)P_{0,0} + (3.1)P_{1,0} + (1.45)P_{2,0} \\ i = 2, (0.55)P_{1,1} + (4.9)P_{2,1} + (0.55)P_{3,1} &= (1.45)P_{1,0} + (3.1)P_{2,0} + (1.45)P_{3,0} \\ i = 3, (0.55)P_{2,1} + (4.9)P_{3,1} + (0.55)P_{4,1} &= (1.45)P_{2,0} + (3.1)P_{3,0} + (1.45)P_{4,0} \end{aligned} \tag{18}$$

Similarly for $i = 4,5, \ldots$ find out other equations.

Since $P_{9,1}$ and $P_{11,1}$ are symmetric. We get,

$$i = 10, (0.55)P_{9,1} + (4.9)P_{10,1} + (0.55)P_{11,1} = (1.45)P_{9,0} + (3.1)P_{10,0} + (1.45)P_{11,0}$$

$$2*(0.55)P_{9,1} + (4.9)P_{10,1} = 2*(1.45)P_{9,0} + (3.1)P_{10,0}$$

So we have 10 algebraic equations in 10 unknowns with tri-diagonal matrix. Similarly, applying above process are, we get the solution for $j = 1,2 \ldots$ .we get another 10 algebraic equations. This can be solved easily by above method. Proceeding in this way, the results obtained by implicit method are shown in Table 3 and plotted in Figure 5 & 6.

Table 3. Population Growth: PDE-Cubic Spline-Implicit Method-Case-(a)

Location (x)	Time t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00
1	567.03	569.23	570.74	571.93	572.91	573.75	574.49	575.14	575.73	576.27	576.76
2	544.93	546.20	547.65	549.01	550.26	551.41	552.47	553.44	554.34	555.18	555.97
3	530.12	530.99	531.87	532.81	533.78	534.75	535.70	536.62	537.50	538.36	539.18
4	520.19	520.77	521.37	521.98	522.62	523.28	523.96	524.65	525.34	526.03	526.71
5	513.53	513.92	514.32	514.74	515.16	515.60	516.05	516.52	516.99	517.48	517.98
6	509.07	509.33	509.60	509.88	510.16	510.46	510.76	511.07	511.39	511.72	512.07
7	506.08	506.26	506.44	506.62	506.81	507.01	507.22	507.43	507.66	507.89	508.14
8	504.08	504.20	504.30	504.44	504.58	504.74	504.90	505.08	505.25	505.44	505.63
9	502.73	502.78	502.93	503.09	503.24	503.40	503.56	503.72	503.88	504.05	504.22
10	501.83	502.21	502.42	502.61	502.78	502.95	503.11	503.27	503.43	503.60	503.76

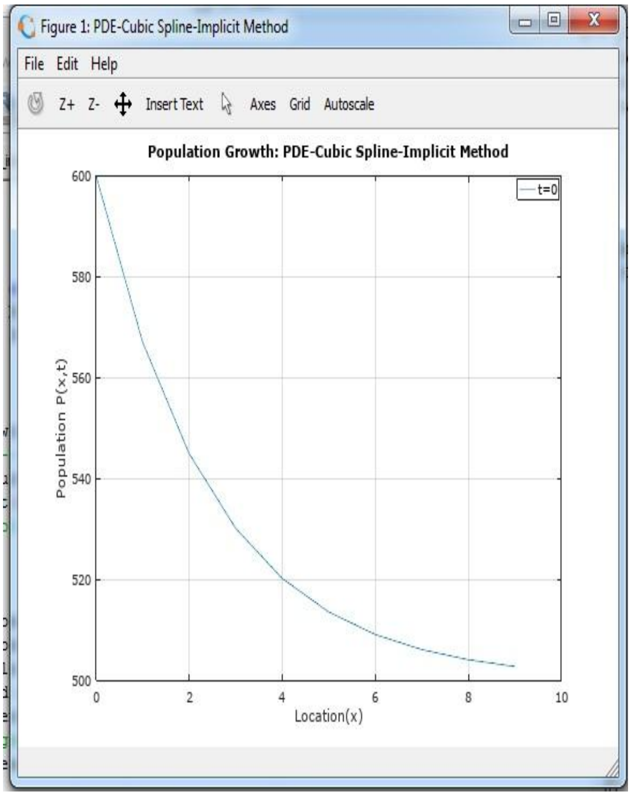

Figure 5: Initial Population: PDE-Cubic Spline-Implicit Method –Case-(a)

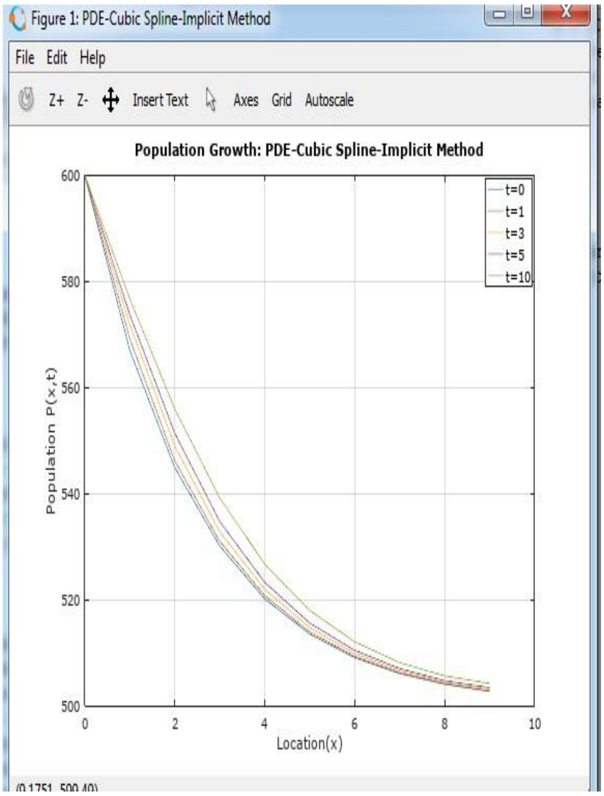

Figure 6: Population Growth: PDE-Cubic Spline-Implicit Method-Case-(a)

Hence it has been observed that, the resulting nature of population growth by implicit method follows the similar kind of nature we have observed in explicit method. The resulting graphs are seen in Figure 5 & Figure 6.

4.4. Implicit Method Case-(b)

Again consider equation (1) for case-(b). The same process we will follow which is already done in experimental results by implicit method. The values of $C_0 = 500$, $C_1 = 100$, $\alpha = 0.1$. and migration rate m=0.15. The results Table 4 for case-(b) by implicit method is as below,

Table 4. Population Growth: PDE-Cubic Spline-Implicit Method-Case-(b)

Location (x)	Time t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.00	567.03	544.93	530.12	520.19	513.53	509.07	506.08	504.08	502.73	501.83
1	567.03	572.24	567.28	559.85	552.32	545.52	539.65	534.72	530.60	527.19	524.34
2	544.93	545.93	548.74	549.61	548.83	547.01	544.63	542.03	539.40	536.87	534.49
3	530.12	531.01	531.70	533.00	534.19	534.93	535.19	535.02	534.52	533.78	532.87
4	520.19	520.77	521.39	521.92	522.61	523.38	524.11	524.73	525.18	525.46	525.59
5	513.53	513.92	514.32	514.75	515.15	515.58	516.06	516.56	517.05	517.52	517.94
6	509.07	509.33	509.60	509.88	510.17	510.46	510.75	511.06	511.39	511.74	512.09
7	506.08	506.26	506.44	506.62	506.81	507.01	507.22	507.43	507.66	507.89	508.14
8	504.08	504.20	504.30	504.43	504.58	504.74	504.90	505.08	505.25	505.44	505.63
9	502.73	502.78	502.93	503.09	503.24	503.40	503.56	503.72	503.88	504.05	504.22
10	501.83	502.21	502.42	502.61	502.78	502.95	503.11	503.27	503.43	503.60	503.76

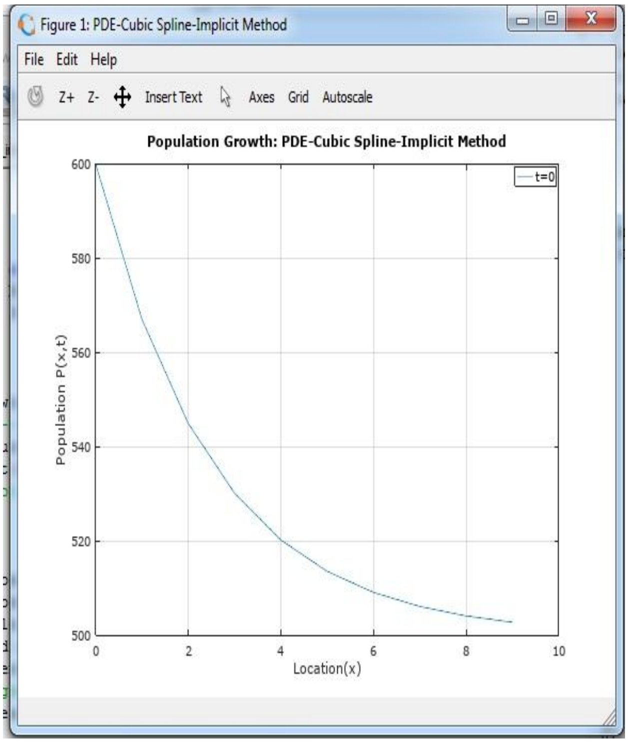

Figure 7: Initial Population: PDE-Cubic Spline-Implicit Method-Case-(b)

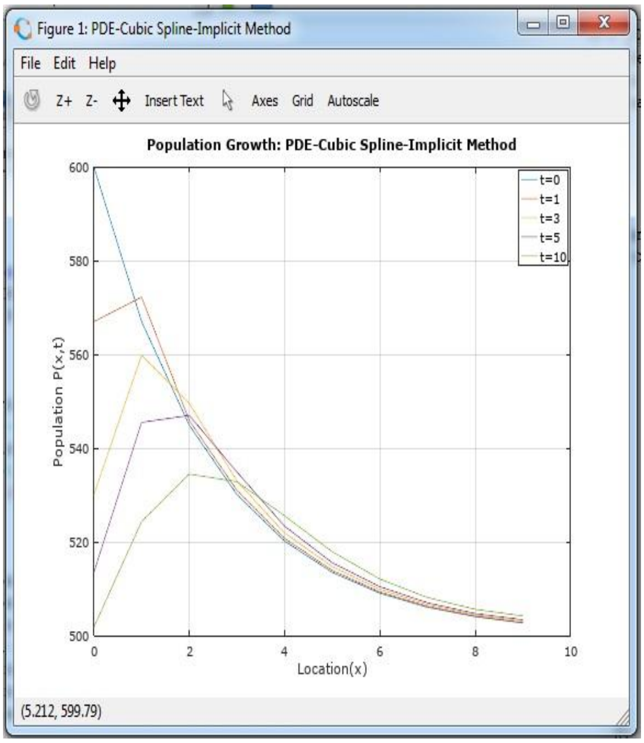

Figure 8: Population Growth: PDE-Cubic Spline-Implicit Method-Case-(b)

From the Figure 7 and Figure 8 shows the similar resulting nature as we have observed in explicit method for case-(b).

V. CONCLUSION

In the prediction of population growth with respect to time and locations with the numerical Cubic Spline approach of both explicit and implicit methods, with two different initial and boundary conditions having similar kind of prediction. As the initial and boundary conditions changed with the above numerical approach, the population growth will be varied and can be predicted accordingly. In both cases migration rate is the key factor for increasing or decreasing population.

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