



DECISION MAKING PROBLEM IN PENTAGONAL INTUITIONISTIC FUZZY NUMBER USING TOPSIS METHOD

K.Selvakumari¹, B.Ummusalma²

¹Assistant Professor, Department of Mathematics, Vels University, Chennai, Tamil Nadu

²Research Scholar, Department of Mathematics, Vels University, Chennai, Tamil Nadu

ABSTRACT:- Decision making problem is the process of finding the best option from all the feasible alternatives. In this paper a solution methodology for Multi Criteria Decision making problems in pentagonal intuitionistic fuzzy numbers. Therefore the objective of this paper is to select a best student on the basis of the student's performance using ranking techniques for pentagonal intuitionistic fuzzy numbers can be converted into crisp value with the help of this value using TOPSIS method we get the best student of college.

KEYWORDS: Fuzzy numbers, intuitionistic fuzzy numbers, pentagonal intuitionistic fuzzy number, accuracy function, topsis.

1. INTRODUCTION

In 1986 Intuitionistic fuzzy set was introduced by Atanassov. In this paper we introduced pentagonal Intuitionistic Fuzzy Number which is included membership & non-membership value. Under some condition, the accuracy function is defuzzified into crisp value by adding both membership & non-membership values, which is less than one. Technique for order performance by similarity to ideal solution (Topsis), one of knows classical MCDM method was first developed by Hwang & yoon for solving MCDM problem. The basic concept of topsis method is to select the best alternatives. C.T. Chen extended the concept of topsis to develop a methodology for solving MCDM. In this paper, the objective is to select a best student on the basis of the student's performance using ranking techniques for pentagonal intuitionistic fuzzy numbers then the value can be converted into crisp value, with the help of this value using TOPSIS method we get the best student of college.

2. PRELIMINARIES

2.1 Definition (Fuzzy Set) [7]

Let A be a classical set, $\mu_A(x)$ be a function from A to $[0, 1]$. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by $A^* = \{(x, \mu_A(x)); x \in A \text{ and } \mu_A(x) \in [0, 1]\}$.

2.2 Definition (Intuitionistic Fuzzy Set)[2]

Let X is a non empty set. An intuitionistic fuzzy set A in X is an object having the form A is equal to $\{(x, \mu_A(x), \nu_A(x)); x \in X\}$, where the function $\mu_A(x), \nu_A(x): X \rightarrow [0, 1]$ define respectively, the degree of membership and the degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

2.3 Definition (Fuzzy Number) [7]

A fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by $\mathcal{F}$.

2.4 Definition (Pentagonal Fuzzy Number) [5]

A pentagonal fuzzy number (PFN) of a fuzzy set A is defined as $A_p = \{a, b, c, d, e\}$, and its membership function is given by

$$\mu_{Ap}(x) = \begin{cases} 0 & \text{for } x < a, \\ \frac{(x-a)}{(b-a)} & \text{for } a \leq x \leq b \\ \frac{(x-b)}{(c-b)} & \text{for } b \leq x \leq c \\ 1 & x = c \\ \frac{(d-x)}{(d-c)} & \text{for } c \leq x \leq d \\ \frac{(e-x)}{(e-d)} & \text{for } d \leq x \leq e \\ 0 & \text{for } x > e \end{cases}$$

2.5 Definition (Pentagonal Intuitionistic Fuzzy Number)[5,2]

A pentagonal intuitionistic fuzzy number A^1 of an intuitionistic fuzzy set is defined as $A^1 = \{(a_1, b_1, c_1, d_1, e_1) | (a_2, b_2, c_2, d_2, e_2)\}$ where all $a_1, b_1, c_1, d_1, e_1, a_2, b_2, c_2, d_2, e_2$ are real numbers and its membership function $\mu_{A^1}(x)$, non-membership function $\vartheta_{A^1}(x)$, are given by

$$\mu_{A^1}(x) = \begin{cases} 0 & \text{for } x < a_1, \\ \frac{(x-a_1)}{(b_1-a_1)} & \text{for } a_1 \leq x \leq b_1 \\ \frac{(x-b_1)}{(c_1-b_1)} & \text{for } b_1 \leq x \leq c_1 \\ 1 & x = c_1 \\ \frac{(d_1-x)}{(d_1-c_1)} & \text{for } c_1 \leq x \leq d_1 \\ \frac{(e_1-x)}{(e_1-d_1)} & \text{for } d_1 \leq x \leq e_1 \\ 0 & \text{for } x > e_1 \end{cases}$$

$$\vartheta_{A^1}(x) = \begin{cases} 1 & \text{for } x < a_1 \\ \frac{(b_2-x)}{(b_2-a_2)} & \text{for } a_2 \leq x \leq b_2 \\ \frac{(c_2-x)}{(c_2-b_2)} & \text{for } b_2 \leq x \leq c_2 \\ 0 & x = c_1 \\ \frac{(x-c_2)}{(d_2-c_2)} & \text{for } c_2 \leq x \leq d_2 \\ \frac{(x-d_2)}{(e_2-d_2)} & \text{for } d_2 \leq x \leq e_2 \end{cases}$$

3. ACCURACY FUNCTION OF AN INTUITIONISTIC PENTAGONAL FUZZY NUMBER

Accuracy function of a pentagonal intuitionistic fuzzy number $A^1 = \{(a_1, b_1, c_1, d_1, e_1), (a_2, b_2, c_2, d_2, e_2)\}$ is defined as $H(A^1) = (a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2 + e_1 + e_2)/5$.

4. PROCEDURE

STEP 1

Using accuracy function of Pentagonal Intuitionistic fuzzy number, fuzzy value can be converted into crisp value

STEP 2

Construct the decision matrix. The first step of the TOPSIS method involves the construction of a decision matrix (DM)

$$DM = \begin{matrix} & \begin{matrix} C_1 & \dots & C_n \end{matrix} \\ \begin{bmatrix} X_{11} & \dots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{m1} & \dots & X_{mn} \end{bmatrix} \end{matrix}$$

Where 'i' is the criterion index $i = (1, \dots, m^2)$, m is the number of potential sites.

'j' is the alternative index $(j=1, \dots, n)$ the elements $c_1, c_2, \dots, c_n$ refer to criteria.

While $L_1, L_2, \dots, L_n$ refer to the alternative locations.

STEP 3

Construct to the normalized decision matrix.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^m x_{ij}^2}}; i=1, 2, \dots, n \text{ and } j = 1, 2, \dots, m$$

Where x_{ij} and r_{ij} are original and the normalized score of decision matrix respectively.

STEP 4

Construct the weighted normalized decision matrix by normalized decision matrix by multiplying the weights w_i of evaluation criteria with the normalized decision matrix r_{ij} .

$$V_{ij} = w_j \times r_{ij} \quad \begin{matrix} j=1, 2, 3, \dots, m \\ i=1, 2, 3, \dots, n \end{matrix}$$

STEP 5

Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

$$A^+ = \{V_1^+, V_2^+, \dots, V_n^+\} \text{ maximum values}$$

Where $V_i^+ = \{\max(V_{ij}) \text{ if } j \in J^+; \min(V_{ij}) \text{ if } j \in J^-\}$

$$A^- = \{V_1^-, V_2^-, \dots, V_n^-\} \text{ minimum values}$$

Where $V_i^- = \{\min(V_{ij}) \text{ if } j \in J^+; \max(V_{ij}) \text{ if } j \in J^-\}$

STEP 6

Calculate the separation measures of each alternative from PIS and NIS

$$s_i^+ = \sqrt{\sum_{j=1}^n (V_j^+ - V_{ij})^2} \quad i=1, 2, 3, \dots, m$$

$$s_i^- = \sqrt{\sum_{j=1}^n (V_j^- - V_{ij})^2} \quad i=1, 2, 3, \dots, m$$

STEP 7

Calculate the relative closeness coefficient to the ideal solution of each alternative

$$C_i = \frac{s_i^-}{(s_i^+ + s_i^-)} \quad , \quad 0 \leq C_i \leq 1$$

$$i = 1, 2, \dots, m$$

STEP 8

Based on the decreasing values of closeness coefficient, alternatives are ranked from most valuable to worst. The alternative having highest closeness coefficient (C_i) is selected.

5. NUMERICAL EXAMPLE

A teacher is desirable to select the best student on the basis of choice parameter. After pre-evaluation four students A_1 ($i=1, 2, 3, 4$) have remind as alternatives for further evaluation. Four criteria are considered as B_1 ; attendance B_2 ;

communication skill B₃; percentage B₄; sports skill. (Whose weighting vector is completely unknown) they construct the following table

PERFORMANCE STUDENTS	ATTENDENCE	COMMUNICATION SKILLS	PERCENTAGE	SPORTS SKILLS
ABDUL	$\{(3,4,5,6,7); (4,5,6,7,8)\}$	$\{(4,6,7,8,9); (7,9,12,13,15)\}$	$\{(2,10,12,13,14); (7,10,11,12,14)\}$	$\{(2,4,6,8,9); (7,8,9,10,12)\}$
ILYAAS	$\{(2,3,5,6,7); (3,4,5,6,9)\}$	$\{(8,9,10,11,12); (9,11,13,15,17)\}$	$\{(6,8,14,15,16); (8,9,13,15,16)\}$	$\{(4,5,6,7,8); (5,6,7,8,9)\}$
SHEIK ISMAIL	$\{(5,7,10,11,12); (6,7,8,9,10)\}$	$\{(1,4,5,6,8); (8,9,10,11,13)\}$	$\{(7,9,12,13,14); (4,6,10,12,13)\}$	$\{(2,4,7,8,9); (7,8,9,10,11)\}$
SULTHAN	$\{(5,6,7,8,9); (10,13,15,17)\}$	$\{(2,3,4,5,6); (5,6,7,8,9)\}$	$\{(7,11,13,14,16); (10,12,13,14,15)\}$	$\{(4,8,10,11,12); (7,8,9,10,11)\}$

STEP 1

Using accuracy function of Pentagonal Intuitionistic fuzzy number, fuzzy value can be converted into crisp value.

PERFORMANCE STUDENTS	ATTENDANCE	COMMUNICATION SKILLS	PERCENTAGE	SPORTS SKILLS
ABDUL	11	18	21	15
ILYAAS	10	23	24	13
SHEIK ISMAIL	17	15	20	15
SULTHAN	18	11	25	18
WEIGHT	0.1	0.2	0.3	0.4

STEP 2

Calculate $(\sum x_{ij}^2)^{1/2}$ for each column and divide each column by that to get r_{ij}

PERFORMANCE ATTENDANCE COMMUNICATION SKILLS PERCENTAGE SPORTS SKILLS				
ABDUL	0.38	0.51	0.46	0.48
ILYAAS	0.31	0.66	0.53	0.42
SHEIK ISMAIL	0.58	0.43	0.44	0.48
SULTHAN	0.62	0.31	0.55	0.58
WEIGHT	0.1	0.2	0.3	0.4

Then it is multiplied weight criteria. Therefore it is

$$V_{11} = 0.1 \times 0.38 = 0.038$$

$$V_{12} = 0.2 \times 0.51 = 0.102$$

$$V_{13} = 0.3 \times 0.46 = 0.138$$

$$V_{14} = 0.4 \times 0.48 = 0.192$$

STEP 4

Find the positive ideal solution A^+ .

$$A^+ = \{0.062, 0.132, 0.165, 0.232\}$$

PERFORMANCE ATTENDANCE COMMUNICATION SKILLS PERCENTAGE SPORTS SKILLS				
STUDENTS				
ABDUL	0.0005	0.0009	0.0007	0.0016
ILYAAS	0.0009	0	0.00003	0.0040
SHEIK ISMAIL	0.00001	0.0021	0.0010	0.0016
SULTHAN	0	0.0049	0	0

STEP 5

Find the negative ideal solution A^- .

$$A^- = \{0.031, 0.062, 0.132, 0.168\}$$

PERFORMANCE STUDENTS	ATTENDENCE	COMMUNICATION SKILLS	PERCENTAGE	SPORTS SKILLS
ABDUL	0.007	0.0016	0.00003	0.0005
ILYAAS	0	0.0049	0.0007	0
SHEIK ISMAIL	0.0007	0.0005	0.0007	0.0005
SULTHAN	0.0009	0	0.0010	0.004

STEP 6(a)

Determine the separation from ideal solution S_i^+

	$\sum (v_j^+ - v_{ij})^2$	$S_i^+ = [\sum (v_j^+ - v_{ij})^2]^{1/2}$
ABDUL	0.0038	0.061
ILYAAS	0.0050	0.071
SHEIK ISMAIL	0.0048	0.069
SULTHAN	0.0049	0.07

STEP 6 (b)

Determine the separation from ideal solution S_i^-

	$\sum (v_j^- - v_{ij})^2$	$S_i^- = [\sum (v_j^- - v_{ij})^2]^{1/2}$
ABDUL	0.0092	0.095
ILYAAS	0.0056	0.075
SHEIK ISMAIL	0.0026	0.051
SULTHAN	0.0061	0.078

STEP 7

Calculate the relative closeness to the ideal solution $C_i^+ = S_i^- / (S_i^+ + S_i^-)$

	$S_i / (S_i^+ + S_i^-)$	C_i^+
ABDUL	0.095/0.156	0.6089 → BEST
ILYAAS	0.075/0.146	0.513
SHEIK ISMAIL	0.051/0.12	0.425
SULTHAN	0.078/0.148	0.527

Therefore, we concluded that abdul is the best student for this TOPSIS method.

6. CONCLUSION

In this paper focus on MCDM Intuitionistic Fuzzy environment problems. In which the rating of alternatives are represented as pentagonal intuitionistic fuzzy numbers. The accuracy of ranking method is developed for the MCDM and applied to TOPSIS method with pentagonal intuitionistic fuzzy numbers. With the help of derived result we conclude the teacher can select the best student by using their choice factors.

7. REFERENCES

- [1] Amit kumar, Anila gupta and amarpreet kumar, method of solving fully fuzzy assignment problems using triangular fuzzy numbers, international journal of computer and information engineering 3:4 2009
- [2] Atanassov K.T., Intuitionistic fuzzy sets, fuzzy sets and systems, 1986, 2087-96.
- [3] Deng-Feng-Li, A ratio ranking method of triangular Intuitionistic fuzzy number and its application to MADM problems, computers mathematics with applicatios, 2010, 60, 1557-1570.
- [4] Chen S.M and Tan J.M 1994. Handling Multi Criteria fuzzy Decision Making Problems Based on vague set theory, Fuzzy Sets and Systems, 67, Pp.163-172.
- [5] Ponnivalavan K. and Pathinathsn T. "Intuitionistic Pentagonal Fuzzy Number", ARPN Journal of Engineering and Applied Sciences, vol.10, no.12, ISSN 1819-6608, Pp: 5446-5450, (2015).
- [6] L.Shen, H.Wang and X.Feng, ranking methods of Intuitionistic fuzzy numbers in multi criteria decision making , 2010-3rd International conference of information management, innovation management and industrial engineering.
- [7] L.A. Zadeh, fuzzy sets information and control, 1965, 8, 338- 353