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Impact of Common Fixed Point in 2- Complete Fuzzy Metric Spaces Using Different Property

Seema devi[1], Kaur Kamalprit[2]

[1]*Department Of Mathematics, GMN College A/Cantt, Haryana, India,*
[2]*Department of Mathematics, P.U. Patiala, Patiala, Punjab,India*

Abstract: *In the present paper we find the impact of common fixed point in 2-complete Fuzzy Metric. This theorem is a version of many fixed point theorems in complete metric spaces, given by many authors announced in the literature .Fixed point theory on fuzzy metric space is combination of fuzzy set theory and fixed point theory. Fuzzy set theory has very wide range applications due to concept of fuzzy set and fuzzy logic. The present paper is aimed at obtaining a new common fixed point theorem on 2-complete fuzzy metric space which satisfies a contractive condition. We are using the concept of complete metric spaces and some property in this theorem.*

1 Introduction

The concept of 2-metric space is a natural generalization of the metric space. Initially, it has been investigated by S. Gähler [7,8] in 1960.After this, number of fixed point theorems have been proved by Many researchers like H.K. Pathak [1] and M.S. Khan [4] for 2-metric spaces by introducing compatible mappings, which are more general than commuting and weakly commuting mappings. K. Iseki [3] is well-known in this literature which also includes J. Matkowski et.al. [2], M. Imdad et.al. [5], P.P.Murthy et.al. [6], S.V.R.Naidu and J.R. Prasad,et.al. [9]. Commutability of two mappings was weakened by Sessa with weakly commuting mappings. Some researchers also prove some common fixed point theorems for a class of A-contraction on 2-metric space. Jungck and Rhoades defined the concepts of d-compatible and weakly compatible mappings as extensions of the concept of compatible mapping for single-valued mappings on metric spaces. Several authors used these concepts to prove some common fixed point theorems. Jungck extended the class of non-commuting mappings by compatible mappings and extended the class of non-commuting mappings by compatible mappings.

Fuzzy metric space is a generalization of metric space. The study on uncertainty and on randomness began to explore with the concept of fuzziness in mathematics. Fuzzy set is used in fuzzy metric space, which is initiated by Lofti. A. Zadeh[10]. After that Kramosil and Michalek[11] introduced the concept of fuzzy metric space. A very important notion of fuzzy metric space with continuous t-norm is laid by Georage and Veeramani[12]. Grabiec[13] extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Compatible mapping is generalized from commutatively mappings by Jungck .After that Jungck and Rhodes initiated the notion of weak compatible and proved that compatible maps are weakly compatible but converse is not true. A common E.A property is the generalization of the concept of non compatibility is introduced under strict contractive conditions by Aamri and El.Moutawakil.

In present time, Fuzzy set theory and Fuzzy logic is not only active field of research in mathematics but also in other field of engineering, medicine, communication, physics, biology etc. are field in which the applicability of fuzzy theory was accepted. Since, Many authors regarding the theory of fuzzy sets and its applications have developed a lot of literature. In this paper, we prove a new fixed point theorem on fuzzy metric space by using above results.

E.A. property replaced the condition of completeness of space by natural condition of closeness of range. Wutiphol introduced the new property which is so called "common limit in the range of g" which does not require the condition of closeness of range.

In the present work we find fixed point in 2-complete fuzzy metric spaces for a class maps using some different property.

2 Main Results

Theorem2.1: Let F, G, S and T be four self mappings of a 2-complete Fuzzy metric space (X, d) satisfying the following conditions that the pair (F, S) and (G, T) are weakly compatible and

$$d(Fx,Gy,t) \le \phi[\min\{d(Sx,Ty,t), d(Fx,Sx,t), d(Gy,Ty,t), d(Fx,Ty,t), d(Sx,Gy,t)\}]$$

Where ϕ is a contractive modulus.

Then the pair F, G, S and T has a unique common fixed point in X.

Proof: let Y_n be a sequence in X such that $Y_n = Fx_n = Tx_{n+1}$ and $Y_{n+1} = Gx_{n+1} = Sx_{n+2}$

$d(Y_n, Y_{n+1}, t) = d(Fx_n, Gx_{n+1}, t)$
$\leq \phi[\min\{d(Sx_n, Tx_{n+1}, t), d(Fx_n, Sx_n, t), d(Gx_{n+1}, Tx_{n+1}, t), d(Fx_n, Tx_{n+1}, t), d(Sx_n, Gx_{n+1}, t)$
$\leq \phi[\min\{d(Y_{n-1}, Y_n, t), d(Y_n, Y_{n-1}, t), d(Y_{n+1}, Y_n, t), d(Y_n, Y_n, t), d(Y_{n-1}, Y_{n+1}, t)\}]$
$\leq \phi[\min\{d(Y_{n-1}, Y_n, t), d(Y_n, Y_{n+1}, t)\}] \leq \phi[d(Y_n, Y_{n+1}, t)] \Rightarrow d(Y_n, Y_{n+1}, t) \leq \phi[d(Y_n, Y_{n+1}, t)]$
But ϕ is a contractive module
$\Rightarrow \phi[d(Y_n, Y_{n+1}, t)] < d(Y_n, Y_{n+1}, t)$ this is possible only if $\lim_{n\to\infty} d(Y_n, Y_{n+1}, t) = 0$.
Now we show that Y_n is a Cauchy sequence in X. then $\exists\ \varepsilon > 0$ such that $m < n < N, d(Y_n, Y_m, t) \geq \varepsilon$
but $d(Y_{n-1}, Y_m, t) < \varepsilon$ and $\varepsilon \leq d(Y_m, Y_n, t) = d(Fx_m, Gx_n, t)$
$\leq \phi[\min\{d(Sx_m, Tx_n, t), d(Fx_m, Sx_m, t), d(Gx_n, Tx_n, t), d(Fx_m, Tx_n, t), d(Sx_m, Gx_n, t)\}]$
$\leq \phi[\min\{d(Y_{m-1}, Y_{n-1}, t), d(Y_m, Y_{n-1}, t), d(Y_n, Y_{n-1}, t), d(Y_m, Y_{n-1}, t), d(Y_{m-1}, Y_n, t)\}] \leq \phi[\min\{\varepsilon, \varepsilon, 0, \varepsilon, \varepsilon\}]$
$\Rightarrow \varepsilon \leq \phi(\varepsilon)$
But ϕ is a contractive module $\Rightarrow \phi(\varepsilon) < \varepsilon \Rightarrow \varepsilon < \varepsilon$ which is a contradiction hence Y_n is a Cauchy sequence.
Since X is complete $\exists$ a point z in X. $s.t.\ \lim_{n\to\infty} Y_n = z \Rightarrow \lim_{n\to\infty} Gx_n = \lim_{n\to\infty} Sx_n = z = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} T_n$ since
$F(X) \subseteq T(X)$ $\exists$ a point $\alpha \in X$ $\alpha \in X$ $s.t.\ z = T\alpha$ if z≠Gα ,
Then we get $d(G\alpha, z, t) = d(G\alpha, Fx_n, t)$
$\leq \phi[\min\{d(Sx_n, T\alpha, t), d(Fx_n, Sx_n, t), d(G\alpha, T\alpha, t), d(Fx_n, T\alpha, t), d(Sx_n, G\alpha, t)$
$\leq \phi[\min\{d(z, z, t), d(z, z, t), d(G\alpha, z, t), d(z, z, t), d(z, G\alpha, t) \leq \phi[d(G\alpha, z, t)]$
$\Rightarrow d(G\alpha, z, t) \leq \phi[d(G\alpha, z, t)]$
But ϕ is a contractive modulus $\Rightarrow \phi[d(G\alpha, z, t)] < d(G\alpha, z, t)$ which is a contradiction so $G\alpha = z$ i.e.
$G\alpha = z = T\alpha \Rightarrow \alpha$ is a co-incidence point of G and T and (G,T) is weakly compatible.
$\Rightarrow GT\alpha = TG\alpha \Rightarrow Gz = Tz$ Now $G(X) \subseteq S(X)$ $\exists$ a point $w \in X$ $s.t.\ Sw = z$ if $Fw \neq z$ using (3)
$d(Fw, z, t) = d(G\alpha, Fw, t) \leq \phi[\min\{d(Sw, T\alpha, t), d(Fw, Sw, t), d(G\alpha, T\alpha, t), d(Fw, T\alpha, t), d(Sw, G\alpha, t)$
$\leq \phi[\min\{d(z, z, t), d(Fw, z, t), d(z, z, t), d(Fw, z, t), d(z, z, t) \leq \phi[d(Fw, z, t)]$
$\Rightarrow d(Fw, z, t) \leq \phi[d(Fw, z, t)]$
But ϕ is a contractive modulus $\Rightarrow \phi[d(Fz, z, t)] < d(Fz, z, t)$ which is a contradiction,
so $Fw = z = Sw$ hence w is a co-incidence point of F and S and (F,S) is weakly compatible .
And $Gz = Tz \Rightarrow d(z, Gz, t) \leq \phi[d(z, Gz, t)]$ and ϕ is a contractive modulus therefore
$\phi[d(z, Gz, t)] < d(z, Gz, t)$
which is a contradiction. So $Gz = z = Tz$
Hence we have $Gz = Tz = Fz = Sz = z$ So F, S, T, G has a unique common fixed point in X.
Now we prove uniqueness let there be another point say w s.t. w≠z then
$d(Fz, Gw, t) \leq \phi[\min\{d(Sz, Tw, t), d(Fz, Sz, t), d(Gw, Tw, t), d(Fz, Tw, t), d(Sz, Gw, t)\}]$
$d(z, w, t) \leq \phi[\min\{d(z, w, t), d(z, z, t), d(w, w, t), d(z, w, t), d(z, w, t)\}]$
$\Rightarrow d(z, w, t) \leq \phi[d(z, w, t)]$ and ϕ is a contractive modulus
$\Rightarrow \phi[d(z, w, t)] < d(z, w, t)$
which is a contradiction $z = w$ and hence the uniqueness.

Theorem 2.2*:* Let A, B, C and D be self mappings of fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ and $a * a \geq a, \forall a \in (0,1]$ satisfying the following conditions:

(i) $A(X) \subset D(X), B(X) \subset C(X)$

(ii) For all $x, y \in X, k \in (0,1)$ and $t > 0$ such that

$M(Ax, By, kt) * [M(Cx, Ax, kt) \times M(Dy, By, kt)]$

$$\geq \begin{Bmatrix} M(Cx, Ax, t) * \\ M(Cx, Dy, t) * \\ M(Cx, By, t) \end{Bmatrix}$$

Then (A, C) and (B, D) have coincident point.

Further, if (A, C) and (B, D) are weakly compatible then A, B, C and D have unique fixed point in X.

Proof: By using condition the pair (B, D) satisfies E.A property,

then there exit a sequence $\{x_n\} \in X$, such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Dx_n = p, p \in X$ (1)

In *(i)* condition, $B(X) \subset C(X)$, then there exit a sequence $\{y_n\} \in X$,

such that $Bx_n = Cy_n$ (2)

From(1) and (2) ,
we have $\lim_{n\to\infty} Cy_n = p$

Now, we will show that, $\lim_{n\to\infty} Ay_n = p$

Taking $x = y_n, y = x_n$ in *(ii)*,
we have

$M(Ay_n, Bx_n, kt) * [M(Cy_n, Ay_n, kt) \times M(Dx_n, Bx_n, kt)]$

$$\geq \begin{Bmatrix} M(Cy_n, Ay_n, t) * M(Cy_n, Dx_n, t) \\ * M(Cy_n, Bx_n, t) \end{Bmatrix}$$

As taking $n \to \infty$, we obtain

$M(Ay_n, p, kt) * [M(p, Ay_n, kt) \times M(p, p, kt)]$

$$\geq \begin{Bmatrix} M(p, Ay_n, t) * M(p, p, t) \\ * M(p, p, t) \end{Bmatrix}$$

$\Rightarrow M(Ay_n, p, kt) \geq \{M(Ay_n, p, t)$

we get

$\lim_{n\to\infty} Ay_n = p$ and $\lim_{n\to\infty} Ay_n = p = \lim_{n\to\infty} Dy_n$

Let suppose $C(X)$ is a complete subspace of X , then $p = C(q)$,for some $q \in X$

$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Cy_n = \lim_{n\to\infty} Bx_n$
$= \lim_{n\to\infty} Dx_n = p = C(q)$ (3)

We will claim that $A(q) = C(q)$

Taking $x = q, y = x_n$ in *(ii)*

$M(Aq, Bx_n, kt) * [M(Cq, Aq, kt) \times M(Dx_n, Bx_n, kt)]$

$$\geq \begin{Bmatrix} M(Cq, Aq, t) * M(Cq, Dx_n, t) \\ * M(Cq, Bx_n, t) \end{Bmatrix}$$

As $n \to \infty$, and from (3), we get

$M(Aq, q, kt) * [M(Cq, Aq, kt) \times M(p, p, kt)]$

$$\geq \begin{Bmatrix} M(Cq, Aq, t) * M(Cq, p, t) \\ * M(Cq, p, t) \end{Bmatrix} \quad \Rightarrow M(Aq, Cq, kt) \geq \{M(Aq, Cq, t)$$

$\Rightarrow A(q) = C(q)$. (4)

This implies (A, C) have coincident point $q \in X$.

By using *given* conditions, the weak compatibility of (A, C) implies that $AC(q) = CA(q)$

$\Rightarrow AA(q) = AC(q) = CA(q) = CC(q)$ (5)

From *(i)* condition, $A(X) \subset D(X)$, there exists $r \in X$ such that $A(q) = D(r)$

$\Rightarrow Cq = Aq = Dr$ (6)

We claim that $D(r) = B(r)$

Taking $x = q, y = r$ in *(ii)*, we get

$M(Aq, Br, kt) * [M(Cq, Aq, kt) \times M(Dr, Br, kt)]$

$\geq \begin{Bmatrix} M(Cq, Aq, t) * M(Cq, Dr, t) \\ * M(Cq, Br, t) \end{Bmatrix}$

From (6) we have

$D(r) = B(r)$ (7)

By using (6) and(7) we get

$\Rightarrow Cq = Aq = Dr = Br$ (8)

Again by using the definition of weak compatibility of (B, D) implies that $BDr = DBr$

$\Rightarrow BDr = DBr = BBr = DDr$

We will prove that Aq is the common fixed point of A, B, C and D.

From (ii),by taking $x = Aq, y = r$

$M(AAq, Br, kt) * [M(CAq, AAq, kt) \times M(Dr, Br, kt)]$

$\geq \begin{Bmatrix} M(CAq, AAq, t) * M(CAq, Dr, t) \\ * M(CAq, Br, t) \end{Bmatrix}$

From equation (5), (8) and lemma (9),we have $AAq = Br = Aq$.

This implies $Aq = AAq = CAq$ is common fixed point of A and C.

Similarly, we prove that Br is the common fixed point of B and D.

By using (8), $Aq = Br$, Aq is the fixed point of A, B, C and D.

Finally, we show the uniqueness of the common fixed point. If possible, let x' and y' be two fixed point of A, B, C and D .Then by taking $x = x', y = y'$

By using definition of fixed point and fuzzy metric spaces, we get $x' = y'$.

Thus, the mapping A, B, C and D have a unique common fixed point.

Theorem 2.3 Let f and g are weakly compatible self mappings of a fuzzy metric space (*Y, M, R*) satisfying following property satisfying inequality:

1. $M(fx, fy, kt) \geq M(gx, gy, t)$,k>0
2. $M(fx, ffx, t) \geq \max\{M(gx, gfx, t), M(fx, gx, t), M(ffx, gfx, t), M(fx, gfx, t), M(gx, ffx, t)\}$ whenever $fx \neq ffx$ if the range of f and g are subspace of Y, then f and g have a common fixed point.

Proof: since f and g satisfy above property there exists a sequence $\{x_n\}$ in Y such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx$ for some x in Y.

There exists some $u \in X$ $fu = gx$

Now we claim that $fu = gu$

If $fu \neq gu$ then $M(fx_n, fu, kt) \geq M(gx_n, gu, t)$

Letting $n \to \infty$ we have $M(gx, fu, kt) \geq M(gx, gu, t)$ which implies $fu = gu$

Since f and g are weakly compatible then they must commute at their coincidence points which implies $fgu = gfu$

Now $M(fu, ffu, t) \geq \max\{M(gu, gfu, t), M(fu, fu, t), M(ffu, fgu, t), M(fu, fgu, t), M(fu, ffu, t)\}$

$M(fu, ffu, t) \geq \max\{1, M(fu, fu, t), M(ffu, fgu, t), M(fu, fgu, t), M(fu, ffu, t)\}$

$M(fu, ffu, t) \geq 1$ a contradiction implies $fu = ffu$.

Hence $fu = ffu = fgu = gfu = ggu$

Hence fu is a common fixed point of f and g. Hence the theorem.

3 CONCLUSION

Our result extends and generalizes some result in fuzzy metric spaces. The purpose of this paper is to utilize the different property to prove fixed point theorem for compatible maps in 2-complete fuzzy metric without using continuity completeness and closeness of the space.

REFERENCES

[1] H.K. Pathak,Kang S.M. andBackJ.H., Weak compatible mappings of Type (A)and common fixed points, *Kyungbook Math.J*.,35(1995),345- 359.

[2] J. Matkowski, Fixed point theorems for mappings with a contractive iterate at a point, Proc. Amer. Soc. 62(2)(1977), 344-348.

[3] K. Iseki, Fixed point theorem in 2-metric spaces, *Math. Sem. Notes, Kobeuni*,3(1)(1975) 133-136.

[4] M.S. Khan, Fixed point theorem by altering distance between the points, Bull. Austral. Math. Soc., 30(1984), 1-9.

[5] M. Imdad, M.S. Kumar and M.D. Khan, A Common fixed point theorem in 2-Metric spaces. Math., Japonica 36(5)(1991), 907-914.

[6] P.P. Muthuy, S.S. Chang, Y.J.Cho and B.K.Sharma, Compatible mappings ofType (A) and common fixed point theorems , Kyungpook Math, J.,32(2) (1992), 203-216.

[7] S. Gahler, Uber die unifromisieberkeit 2-metrischer Raume, Math.Nachr. 28(1965), 235-136.

[8] S. Gahler, 2-metrische Raume und iher topologische structur.*Math.Nachr*,26(1963), 115-148.

[9] S.V.R. Naidu and J.R. Prasad, Fixed point theorem in 2- metric spaces, Indian J. Pure Appli. Math.17 (8) (1986), 974-993.

[10] L. A. Zadeh, "Fuzzy sets," *Inform. and Control*, vol. 8, 1965, pp. 338-353.

[11] I. Kramosil, J. Michalek, "Fuzzy metric and statistical metric spaces," *Kybernetica*, vol. 11, 1975, pp.326-334.

[12] A. George, P.Veeramani, "On some results in fuzzy metric spaces," *Fuzzy sets and systems*, vol.64, 1994, pp.336-344.

[13] M. Grabiec, "Fixed points in fuzzy metric Spaces," *Fuzzy sets and systems*, vol.27, 1988, pp.385-389.

