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OPTIMIZATION OF A PRODUCT MIX IN A PAPER MILL – A CASE STUDY

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Abstract:- *The main thrust of the study is to determine the best product mix in M/s Delta Paper Mills, Vendra by using one of the Optimization Techniques i.e. Linear Programming Problem. The company is producing mainly three kinds of papers based on GSM (Grams/sq. meter) namely lower GSM, medium GSM and Higher GSM. Presently the company is accepting orders at random, without any specific regard to the above aspect of optimization. The reason for carrying out this analysis is that the company would get lesser returns for the same work if it does not adhere to any systematic procedure, which is the case now. The objective of carrying out the case study is to make higher profits for the same work and from the same resources; further to reduce the unnecessary wastage of time & materials. About 80% of the net profit of the company is from the medium GSM Paper and most of the remaining from lower GSM Paper. The best product mix is determined using Linear Programming Problem (LPP) and estimated that the total profit of the company is increased by 30%.*

Keywords *- Optimization, Paper Processing, Product mix, Linear Programming Problem, Constraints*

I. INTRODUCTION

A wide range of fibrous raw materials, chemicals, fillers & additives, water, energy and labour are utilised during the pulping and papermaking process. Their availability, price and quality to a large extent determine the cost of production. The industry has to strictly monitor these basic inputs for its market positioning and competitive advantage. Better utilization of the inputs and reuse of by-products / wastes is a key factor which controls the economics of pulp and paper manufacturing [1,2]. The paper making process is shown in Fig 1. As compared to other industries like the cement and chemical industry, the input consumption in Pulp and Paper industry is very high and the output is comparatively very low. The input-output ratio is 8:1. Fibre, energy, water and chemicals are the important inputs in paper manufacture[3]. Both from the quality aspect and also from the cost of production point of view, it is very important to have some norms of the basic inputs, so that the cost of production is maintained at minimum level without sacrificing the quality of production [1,4].

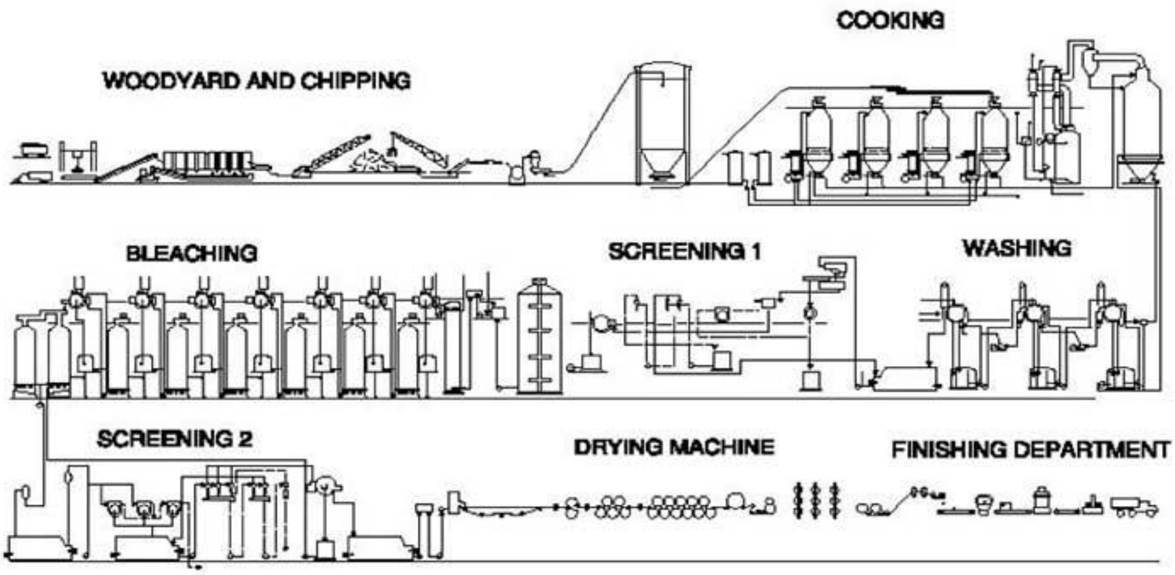

Fig 1. Paper Making Process

The delta Paper Mills Limited was established in Vendra, near Bhimavaram (West Godavari District of Andhra Pradesh State) in the year 1978. Initially, it had a capacity to produce 30 tons of paper/day and employed as many as 800 workers. Seven years later in 1985 the mill was augmented to produce a total of 60- 65 tons/day on an average. Despite doubling production, the number of employees remained the same. All papers produced by the mill can be classified into two broad groups namely Cultural papers and Industrial Papers.

All writing and printing papers, viz., lower GSM, medium GSM and higher GSM papers are called cultural papers.

Wrapping, packing, photographic 6t other functional papers may be called industrial papers. But the major production (about 90%) of the mill deals with cultural papers and that is why the optimization is concerned with cultural papers only. The paper making process in the paper mill is shown in Fig 2.

There are six departments in Delta Paper Mills through which paper manufacturing takes place successfully. They are

(1) Pulping
(2) Stock preparation
(3) Paper rnakmg
(4) Converting & Finishing
(5) Process & quality control department
(6) Stores

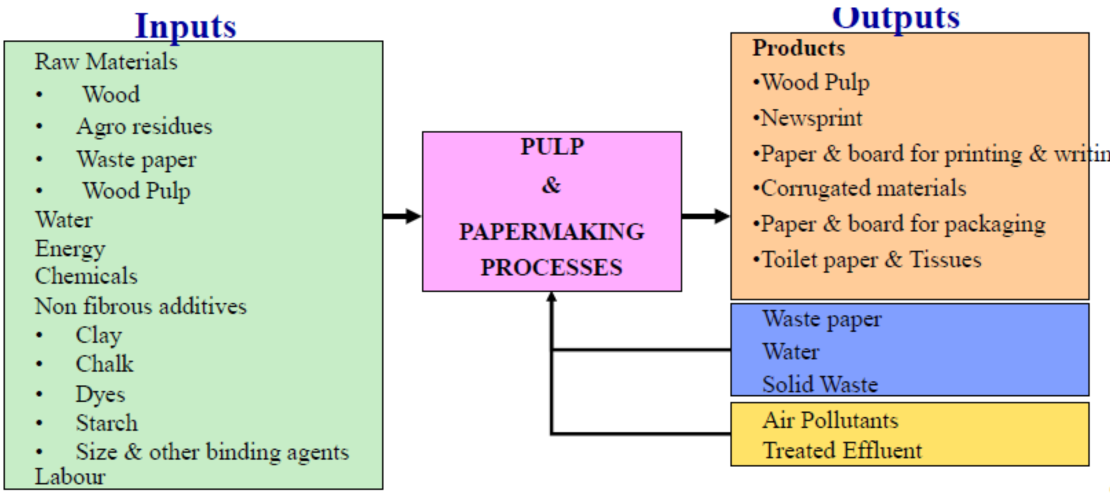

Fig 2. Flow chart of Paper Making Process in a Paper Mills

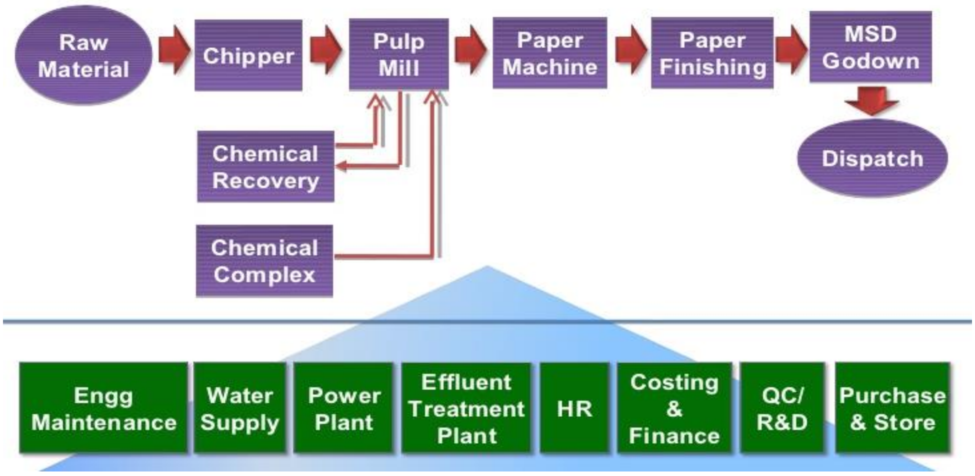

Fig 3. Flow diagram of paper mill

The usually used raw materials in Delta Paper Mills are:

(1) Paddy straw
(2) Begasse
(3) Cotton linters
(4) Grade-I cuttings
(5) Exercise note books
(6) Jute yarn
(7) Gunny cuttings
(8) Waste Papers, etc.

These raw materials are treated in pulping departments to get raw pulp, which is bleached, by treating with chemicals. This treated pulp is sent to the stock preparation department for further purification. The output pulp is supplied to the papermaking department where water is removed from the pulp and paper is sent on to the rollers to be made into the required GSM. This is sent into the converting section where the paper is made into rolls and sheets either of standard size or according to the orders. The converted sheets are finished and packed in the finishing department. Inspection of raw materials, pulp and strength of paper is done by the process & quality control department at regular intervals of time. The stores will supply necessary materials to various departments for maintenance and other handling of the plant.

II. ORGANIZATION & METHODOLOGY

The delta paper mill produces three types of paper based on GSM i.e. lower GSM (47 to 50 GSM), medium GSM (50 to 60 GSM) and higher GSM (above 70 GSM) papers.
The furnish (% of raw materials & their losses) for the above 3 kinds of paper is as given in Table 1.

Table 1. Raw materials used in preparation of paper

Raw materials	Lower GSM	Medium GSM	Higher GSM
Straw	35%	45 %	50 %
Begasse	30%	28 %	25 %
Gunny cuttings & cotton linters	10 %	07 %	05 %
Waste Paper	25%	20 %	20 %

Table 2. Raw materials and their loss percentage during preparation of paper

% Loss when straw is treated	64%
% Loss when begasse is treated	50%
% Loss when gunny + Cotton is treated	30%
% Loss when waste paper is treated	25%

No. of cooks per day is 150 tons with straw and begasse, where as 40 tons of cooking from gunny, cotton and waste paper. From the above data, the net losses on an average of various papers are obtained and as shown on schematic diagram in Table 3:

Table 3. Processing of Paper – Expected output from one ton of input

	Lower GSM (tons)			Medium GSM (tons)			Higher GSM (tons)		
	Input	Losses	Output	Input	Losses	Output	Input	Losses	Output
Input	1	-	-	1	-	-	1	-	-
Pulping	1	50 %	0.5	1	53 %	0.47	1	55 %	0.45
Bleaching	0.5	0.2 %	0.499	0.47	0.2 %	0.449	0.45	0.2 %	0.449
Stock preparation	0.499	17 %	0.414	0.449	16 %	0.382	0.449	15 %	0.382
Converting & finishing	0.414	0 %	0.414	0.382	0 %	0.382	0.382	0 %	0.382
Output	-	-	0.414	-	-	0.382	-	-	0.382

The yield per ton of raw material is 406.7 kg for lower GSM, 386.9 kgs for medium GSM and 374.8 kgs for higher GSM paper form one ton of raw material. The capacities of processes in tons/day are in given in Table 4.

Table 4. Capacity of the Plant

Pulping process	190 tons/day
Bleaching Process	80 tons/day
Stock preparation	75 tons/day
Converting & finishing	70 tons/day

The maximum input of the three kinds of paper due to their demand is shown in Table 5.

Table 5. Constraint on each Paper

Lower GSM paper	110 tons/day
Medium GSM paper	154 tons/day
Higher GSM paper	100 tons/day

Based on the restrictions and the objective function, the problem is formulated as Linear Programming Problem.

III. FORMULATION OF LINEAR PROGRAMMING PROBLEM

The decision variables are x_1, x_2 and x_3 are input for lower GSM paper, input for medium GSM paper and input for higher GSM paper. The selling price is considered from the market. The selling prices of three kinds of paper are estimated based on the process. The production charges for making one ton of all kinds of paper are estimated from the processing and wastage charges. After adjusting for wastage the selling price and other expenses, the profit for each ton of the making three kinds of paper is Rs 1536.8 for lower GSM, Rs 1617 for medium GSM paper and Rs. 973.1 for higher GSM paper. The objective function is obtained as Maximize, Z = 1536.7 x_1 + 1617 x_2 + 973.1 x_3.

Taking into account of all the losses of weight at different stages, capacities, the availability of raw materials, the availability of chemicals, the constraints are formulated and are given below.

Constraint of Raw Material is $x_1 + x_2 + x_3 \leq 190$
Constraint of bleaching process is $0.5x_1 + 0.469x_2 + 0.45\, x_3 \leq 80$
Constraint of Stock preparation is $0.499x_1 + 0.469x_2 + 0.449\, x_3 \leq 75$
Constraint of Rolling and Finishing is $0.413x_1 + 0.394x_2 + 0.381x_3 \leq 70$
Constraint of time is $0.132x_1 + 0.094x_2 + 0.1143x_3 \leq 24$
Constraint of input for lower GSM is $x_1 \leq 110$
Constraint of input for medium GSM is $x_2 \leq 154$
Constraint of input for higher GSM is $x_3 \leq 100$
Non negativity Constraints $x_1, x_2, x_3 \leq 0$

IV. MATHEMATICAL ANALYSIS & RESULTS

The simplex procedure is used to optimize the product mix as the problem is formulated as Linear Programming Problem (LPP). The simplex method is an iterative algorithm for efficiently solving LP problems. In the simplex method, the search usually starts at the origin and moves to that adjacent corner that increases (for maximization) or decreases (for minimization) the value of the objective function the most. The search moves to an even better corner adjacent to the new one. The process continues until no further improvement is possible. In each iteration, the objective function improves. The simplex procedure summarizes with the steps like standardize the problem, generate an Initial Solution, test for Optimality, identify the Incoming and Outgoing Variables, generate an Improved Solution and check for other Optimal Solutions [9].

The number of constraints are 8 with decision variables 3 and slack variables 8, the problem can be solved using simplex method. The iterations are summarized here from Table 6 to Table 7.

The initial simplex table is

Table 6. Initial Simplex Table

	Quantity	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Coefficients in the objective function		1536.7	1617	973	0	0	0	0	0	0	0	0
Constraint 1	190	1	1	1	1	0	0	0	0	0	0	0
Constraint 2	80	0.5	0.469	0.45	0	1	0	0	0	0	0	0
Constraint 3	75	0.499	0.469	0.449	0	0	1	0	0	0	0	0
Constraint 4	70	0.413	0.394	0.381	0	0	0	1	0	0	0	0
Constraint 5	24	0.132	0.094	0.1143	0	0	0	0	1	0	0	0
Constraint 6	110	1	0	0	0	0	0	0	0	1	0	0
Constraint 7	154	0	1	0	0	0	0	0	0	0	1	0
Constraint 8	100	0	0	1	0	0	0	0	0	0	0	1

Table 7. Iteration 1

C_j			1536.7	1617	973	0	0	0	0	0	0	0	0	
	Product Mix	Quantity	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	Replacement Ratio Ɵ
0	S_1	190	1	1	1	1	0	0	0	0	0	0	0	190
0	S_2	80	0.5	0.469	0.45	0	1	0	0	0	0	0	0	170.576
0	S_3	75	0.499	0.469	0.449	0	0	1	0	0	0	0	0	159.914
0	S_4	70	0.413	0.394	0.381	0	0	0	1	0	0	0	0	177.67
0	S_5	24	0.132	0.094	0.1143	0	0	0	0	1	0	0	0	255.32
0	S_6	110	1	0	0	0	0	0	0	0	1	0	0	-
0	S_7	154	0	1	0	0	0	0	0	0	0	1	0	154 (min) →
0	S_8	100	0	0	1	0	0	0	0	0	0	0	1	-
				↓										

Table 8. Iteration 2

C_j			1536.7	1617	973	0	0	0	0	0	0	0	0	
	Product Mix	Quantity	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	Replacement Ratio (Ө)
0	S_1	36	1	0	1	1	0	0	0	0	0	-1	0	36
0	S_2	7.774	0.5	0	0.45	0	1	0	0	0	0	-0.47	0	15.548
0	S_3	2.774	0.499	0	0.449	0	0	1	0	0	0	-0.46	0	5.56 (min) →
0	S_4	9.324	0.413	0	0.381	0	0	0	1	0	0	-0.39	0	22.576
0	S_5	9.524	0.132	0	0.1143	0	0	0	0	1	0	-0.09	0	72.1515
0	S_6	110	1	0	0	0	0	0	0	0	1	0	0	110
1617	X_2	154	0	1	0	0	0	0	0	0	0	1	0	-
0	S_8	100	0	0	1	0	0	0	0	0	0	0	1	-
	Z_j	249018	0	1617	0	0	0	0	0	0	0	1617	0	
	$C_j - Z_j$		1536.7	0	973	0	0	0	0	0	0	-1617	0	
			↓											

Table 9. Iteration 3 (Final Iteration)

C_j			1536.7	1617	973	0	0	0	0	0	0	0	0	
	Product Mix	Quantity	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	Ө
0	S_1	30.44	0	0	0.1012	1	0	-2	0	0	0	0.94	0	
0	S_2	4.994	0	0	0.006	0	1	-1	0	0	0	-0.47	0	
1536.7	X_1	5.56	1	0	0.8998	0	0	2	0	0	0	-0.94	0	
0	S_4	7.028	0	0	0.0097	0	0	-0.83	1	0	0	-0.39	0	
0	S_5	8.790	0	0	-0.0043	0	0	0	0	-0.26	0	-0.12	0	
0	S_6	104.44	0	0	-0.898	0	0	-2	0	0	1	0.94	0	
1617	X_2	154	0	1	0	0	0	0	0	0	0	1	0	
0	S_8	100	0	0	1	0	0	0	0	0	0	0	1	
	Z_j	257562.072	1536.7	1617	1318.18	0	0	3078	0	0	0	173	0	
	$C_j - Z_j$		0	0	-408.18	0	0	-3078	0	0	0	-173	0	

All $C_j – Z_j$ values are either '0' or negative. No further profit improvement is possible. The optimum solution is obtained. The maximum profit obtained is Rs. 257562.072. The optimal solution is 5.56 tons / day for lower GSM paper, 154 tons / day for medium GSM paper and No ton for higher GSM paper. The slacks are 30.44, 4.994, 0, 7.028, 104.44, 0 and 100 tons per day obtained. Thus, there is still an improvement using the present available recourses. The sensitivity analysis may be applied to further improve the profit with the existing recourses.

V. DISCUSSION, CONCLUSIONS AND FUTURE SCOPE OF WORK

The main aim in doing this case study at M/s Delta Paper Mills, Vendra is to bring out the best product mix and there by to maximize the profits of the firm. The company is manufacturing papers based on the market requirements but not concentrating on profit oriented approach. An attempt has been made in this regard by adapting one of the optimization techniques LPP using simplex method.

It is observed from the results the input raw materials for lower GSM paper has to be 5.55 tons per day and that of medium GSM paper is 154 tons per day and higher GSM paper should not be manufactured at all, even though thick paper is making a net profit of Rs. 973 per ton.

But this is as far as the theoretical part of the problem is concerned. Practically speaking, it would be inevitable for the company to manufacture at least a limited quantity of thick paper as well, in order to have customer base and to maintain the goodwill among the customers.

At present the company is contributing Rs. 210000 per day. The amount may be increased to Rs 257000 per day if they follow the given product mix. Slack is not available in the stock preparation department and input capacity of medium GSM paper raw material, which are the real bottleneck in the problem. Thus, the profit can be improved if the capacity of the stock preparation unit and the cooking capacity of the medium GSM paper raw material are increased. This would be meaning that the company should acquire additional equipment for cooking purpose. The sensitivity analysis could be applied to improve the solution.

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REFERENCES

[1]. M. Bogdan, Some comments on a linear programming problem, 2017.
[2]. E.B. Breckner, De la poliedre la jocuri matriceale, Editura Fundat ̧iei pentru Studii Europene, Cluj-Napoca, 2007
[3]. G. Dantzig, Linear Programming and Extensions, Princeton University Press, 1963..
[4]. R. Robere, Interior Point Methods and Linear Programming,
[5]. Marcel Bogdan / Procedia Manufacturing 22 (2018) 1063–1068
[6]. Javad Tayyebi, Massoud Aman, On inverse *linear programming problems* under the bottleneck-type weighted Hamming distance Discrete Applied Mathematics, Volume 240, 11 May 2018, Pages 92-101.
[7]. X. Allamigeon, S. Gaubert, É. Goubault, The tropical double description method, in: J.-Y. Marion, Th. Schwentick (Eds.), Proceedings of the 27th International Symposium on Theoretical Aspects of Computer Science, STACS2010, in: Leibniz International Proceedings inInformatics (LPIcs),
[8]. Kellerer, H., & Strusevich, V.A. (2016) Optimizing the half-product and related quadratic Boolean functions: approximation and scheduling applications. Annals of Operations Research, 240, 39–94.
[9]. *Operations Research* Theory Methods & Applications by *SD Sharma*, 17 th Edition.

