

**Constituent gluons in hybrid mesons: A group theoretical analysis**Tapashi Das¹¹Department of Physics, Gauhati University, Guwahati-781 014, India.

Abstract:—Hybrids are predicted states composed of a pair of quark-antiquark and a gluon in Quantum Chromodynamics. In this work the application of group theoretical tool of Young Tableau in $SU(3)_C$ is shown and infer the number of constituent gluons in such hadronic states having upto seven constituent gluons.

Keywords: QCD; Young Tableau; Hybrid Meson

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1 Introduction

Quantum Chromodynamics (QCD) is the theory of the hadronic interactions. Gluonic hadrons falls into two categories: glueballs and hybrids. The properties of glueballs and hybrids are determined by the long distance features of QCD and their study will yield fundamental insight into the structure of the QCD vacuum [1, 2]. In QCD, hybrid mesons are states with a valence color octet $q\bar{q}$ pair neutralized in color by an excited gluon field [3]. The existence of hybrid meson was suggested in 1976 by Jaffe and Johnson [4] and Vainshtein and Okun [5]. Often symbolic notation $q\bar{q}g$ is used which reminds of the excitation of the gluon field between the $q\bar{q}$ pair [6]. Hybrids can decay into two mesons with one of them having one unit of angular momentum [7].

Hybrid mesons have not been an easy subject to study due to the lack of phenomenological support and therefore much debate has been associated with their properties.

The forthcoming experiment FAIR has PANDA as detector [8], specifically designed to detect glueballs, hybrid mesons and charmonium spectroscopy. In the proposed experiment the search for hybrids are restricted to the mass region below $2.2\text{GeV}/c^2$ [9].

Theoretical calculations show that hybrids should exist at energy ranges accessible with current collider technology [10].

In the present paper, the results for hybrids using Young Tableau [11, 12] are reported. In the previous publications [13, 14, 15], the corresponding results for pent quark multiplets and scalar glueballs are presented.

In this work, the notion of hybrid mesons having upto seven constituent gluons ($q\bar{q}ng, 1 \leq n \leq 7$) will be generalized even though in literature hybrid meson constituting only one gluon is discussed.

2 Formalism and results

In this section, the results of Young Tableau of Group Theory as applied for $SU(3)_C$ are summarized to find the maximum number of constituent gluons from the multiplicity of the experimentally observed hybrid mesons. The young tableau gives a definite way of obtaining direct sum of various irreducible representations [11, 12].

The results for hybrid mesons upto 7 constituent gluons are shown in Table-1. Only condition is that each of the hybrids are color singlet.

The n multiplicity of m dimensional irreducible representation of $SU(3)_C$ are denoted as $n(m)$ ie eg. $3(10)=10+10+10$ and so on.

Table 1: Number of color singlet hybrids upto seven constituent gluons

Number of constituent gluons	Direct product	Direct sum	Number of Color Singlet hybrids
1	$3_C \times 3_C \times 8_C$	$1+3(8)+10+10+27$	1
2	$3_C \times 3_C \times 8_C \times 8_C$	$3(1)+10(8)+19(10)+5(10)+34(27)+4(28)+30(35)+12(64)+6(81)+125$	3
3	$3_C \times 3_C \times 8_C \times 8_C \times 8_C$	$10(1)+40(8)+26(10)+22(10)+39(27)+4(28)+34(35)+13(64)+6(81)+125$	10
4	$3_C \times 3_C \times 8_C \times 8_C \times 8_C \times 8_C$	$40(1)+177(8)+139(10)+101(10)+213(27)+44(28)+230(35)+106(64)+10(80)+78(81)+21(125)+8(154)+216$	40
5	$3_C \times 3_C \times 8_C \times 8_C \times 8_C \times 8_C \times 8_C$	$177(1)+847(8)+759(10)+491(10)+1079(27)+362(28)+1460(35)+10(55)+754(64)+150(80)+702(81)+235(125)+148(154)+18(162)+31(216)+10(260)+343$	177
6	$3_C \times 3_C \times 8_C \times 8_C \times 8_C \times 8_C \times 8_C \times 8_C$	$847(1)+4300(8)+4245(10)+2517(10)+6669(27)+2674(28)+9100(35)+178(55)+5084(64)+1540(80)+5502(81)+2105(125)+1708(154)+354(162)+456(216)+249(260)+42(343)+28(280)+12(405)+572+273+330$	847
7	$3_C \times 3_C \times 8_C \times 8_C \times 8_C \times 8_C \times 8_C \times 8_C \times 8_C$	$4300(1)+22878(8)+24314(10)+13486(10)+38584(27)+18816(28)+56644(35)+2100(55)+33544(64)+13496(80)+40404(81)+28(91)+16960(125)+616(143)+16184(154)+4440(162)+5018(216)+3600(260)+56(270)+700(280)+805(343)+390(405)+40(440)+57(512)+14(595)+729$	4300

Similarly in table-2, the correspondence between the observed number of hybrid mesons and the possible number of constituent gluons in them are shown. For example, if the number of observed hybrid meson is 40, possible number of constituent gluon will be 1,2,3,4.

The above analysis demonstrates that the Young Tableau calculation gives the possibility to infer the maximum number of constituent gluons from any number of observed hybrids. In table-2, we have confined the results for hybrids upto multiplicity of 4300.

Table 2: Possible number of constituent gluons corresponding to the number of hybrid mesons upto multiplicity 4300

Number of hybrids observed	Possible number of constituent gluons
1	1
3-9	1,2
10-39	1,2,3
40-176	1,2,3,4
177-846	1,2,3,4,5
847-4299	1,2,3,4,5,6
4300	1,2,3,4,5,6,7

The predictions discussed above do not give any information about the relationship between the number of constituent gluons and the masses of the observed hybrids since in exact $SU(3)_C$ symmetry, gluons are massless. However, there are specific models in literature, where the constituent gluons have effective mass of the order of 0.7 GeV [16,17] resulting in the following degenerate mass relation:

$$M_{\text{hybrid}(n)} = nm_g + m_{q\bar{q}} + M_H, \quad (1)$$

Where M_H is the degenerate mass of all hybrids due to possible dynamics of the quarks and gluons and n is the number of constituent gluons of mass m_g . The only possible prediction of masses of hybrids based on symmetries with massive gluon with effective mass m_g is a trivial lower bound

$$M_{\text{hybrid}} > nm_g$$

(2) As an illustration if a hybrid is observed at mass scale of the order of 4.2 GeV [18], number of constituent gluons cannot be more than 1 in case of charmonium hybrid.

If on the other hand search for hybrids are restricted to the mass region below 2.2 GeV [9], number of constituent gluons cannot be more than three for hybrid with light quarks. Charmonium hybrids will be beyond such experimental search.

In table-3, the maximum number of constituent gluon in u, d, s and c hybrid using table-2 are recorded. The masses are taken from the table-I of reference [24].

Table 3: Upper bounds on the number of constituent gluons and the hybrid multiplicity

Model [Ref.]	u/d hybrid (GeV)	No. of gluons	maximum allowed multiplicity	s hybrid (GeV)	No. of gluons	c hybrid (GeV)	No. of gluons	maximum allowed multiplicity
Lattice QCD [25 – 32]	1.7 - 2.1	3	10	1.9	2	4.2-4.4	2	3
Flux Tube [33 – 34]	1.8 - 2.1	3	10	2.1-2.3	2-3	4.1-4.5	2	3
Bag Model [35]	1.3-1.8	2-3	3-10	-	-	3.9	2	3

3 Conclusion

In the present work, an analysis of the possible number of constituent gluons as inferred from the observed multiplicity of hybrids having upto seven constituent gluons is shown. To that end, the group theoretical tool of Young Tableau is used. A very simple mass formula for hybrids is suggested neglecting the binding effects. With this level of approximation the suggested correspondence between the observed multiplicity and the number of constituent gluons in them will hopefully be useful as a simple constituent gluon counting rule for the hybrids to be discovered in the recently planned experiment like PANDA and future experiments.

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