

**FUZZY NETWORK PROBLEMS USING JOB SEQUENCING
TECHNIQUE IN HEXAGONAL FUZZY NUMBERS**K.Selvakumari¹, G.Sowmiya²¹Assistant Professor, Department of Mathematics, Vels University, Chennai, Tamil Nadu, India²Research Scholar, Department of Mathematics, Vels University, Chennai, Tamil Nadu, India

ABSTRACT:-The critical path problem is a classical and important network optimization problem appearing in many applications. This paper presents a method for finding critical path in the fuzzy project network. A fuzzy network problem can be easily solved by job sequencing technique. In fuzzy project network, when the duration time of each activity represented by a hexagonal fuzzy number. In this paper, finding fuzzy shortest path in the network using Pascal's triangle graded mean integration for duration time can be converted into crisp value, we solve the problem and obtained the (n-1) sub sequence of the route using the proposed algorithm, we calculate the optimal sequence which leads to the shortest path of the network.

Keywords: Fuzzy set, Hexagonal fuzzy numbers, Job sequencing technique, Pascal's triangle graded mean approach.

1. INTRODUCTION

Zadeh introduced the concept of fuzzy set in 1965[14]. The critical path method was developed in 1957 by J.E.Kelly. Network analysis is a technique which determines the various sequences of jobs concerning a project and the completion time. This paper deals with the comparison for finding the fuzzy shortest path in the network using Pascal's triangle graded mean integration [8] with the help of hexagonal fuzzy numbers, defuzzified value can be converted into crisp value. Using Johnson's sequencing rule [13] we solved the problem and obtained the (n-1) subsequence of the route. We calculate the optimal sequence which leads to the shortest path of the network.

2. PRELIMINARIES**2.1 Definition: (Fuzzy Set) [3]**

Let X be a set. A fuzzy set \tilde{A} on X is defined to be a function $\tilde{A}:X \rightarrow [0,1]$ or $\mu_{\tilde{A}}: X \rightarrow [0,1]$ Equivalently, a fuzzy set \tilde{A} is defined to be the class of objects having the following representation $\tilde{A}=\{(x, \mu_{\tilde{A}}(x)): x \in X\}$ where $\mu_{\tilde{A}}:X \rightarrow [0,1]$, is a function called the membership function of \tilde{A} .

2.2 Definition: (Fuzzy Number)[13]

The fuzzy number \tilde{A} is a fuzzy set whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is piecewise continuous;
2. A fuzzy set \tilde{A} of the universe of discourse X is convex;
3. A fuzzy set of the universe of discourse X is called a normal fuzzy set if $\exists x_i \in X, \mu_{\tilde{A}}(x_i) = 1$.

2.3 Definition: (Hexagonal Fuzzy number) [10]

A fuzzy Number \tilde{A}_H is a hexagonal fuzzy number denoted by $\tilde{A}_H(a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{2} \left(\frac{x - a_1}{a_2 - a_1} \right) & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x - a_2}{a_3 - a_2} \right) & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x - a_4}{a_5 - a_4} \right) & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6 - x}{a_6 - a_5} \right) & \text{for } a_5 \leq x \leq a_6 \\ 0 & \text{for } x > a_6 \end{cases}$$

3. PASCAL'S TRIANGLE GRADED MEAN APPROACH [8]



Figure: PASCAL'S TRIANGLE

The following are the pascal's hexagonal approach:

Let $A = (a_1, a_2, a_3, a_4, a_5, a_6)$ are two hexagonal fuzzy numbers then we can take the coefficient of fuzzy numbers from pascal's triangles and apply the approach we get the following formula;

$$P(A) = \frac{a_1 + 5a_2 + 10a_3 + 10a_4 + 5a_5 + a_6}{32} \dots\dots\dots (1)$$

The coefficients of $a_1, a_2, a_3, a_4, a_5, a_6$ are 1, 5, 10, 10, 5, 1. This approach can be extended for n-dimensional Pascal's hexagonal fuzzy order also.

4. WORKING RULE

Step (1) Using Pascal's triangle graded mean approach Hexagonal fuzzy number can be converted into crisp values.

Step (2) Construct the NXN adjacency matrix. Where N is the node of the network.

Step (3) The NXN adjacency matrix split into NX2 sub matrix. The number of sub matrix will be N-1. Thus a network having 5 nodes then it will be 4 submatrixes.

Step (4) Apply Johnson's methods for n-jobs, 2 machine algorithm to the n-job 2-machine problem established and determine S_k .

Step (5) Select minimum total processing distance sequence as the optimal sequence. This optimal sequence is determining the shortest path of the given project network.

5 NUMERICAL EXAMPLE

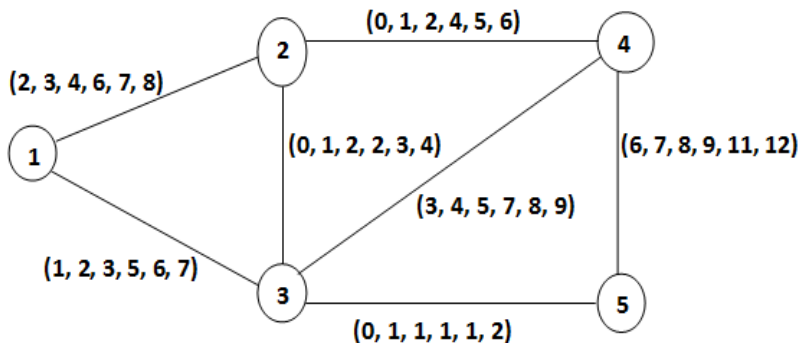


Figure 1: Project network

Node 1: Pondicherry; Node 2: Thindivanam; Node 3: Melmaruvathur;

Node 4: Chengalpattu; Node 5: Tambaram.

Find the shortest optimal path for finding minimum hours to deliver the courier service.

Step (1) Find the defuzzified value using equation (1)

NODE (I-J)	Fuzzy capacities	Crisp values using (PTGMA)
1-2	(2,3,4,6,7,8)	5
1-3	(1,2,3,5,6,7)	4
2-3	(0,1,2,2,3,4)	2
2-4	(0,1,2,4,5,6)	3
3-4	(3,4,5,7,8,9)	6
3-5	(0,1,1,1,1,2)	1
4-5	(6,7,8,9,11,22)	9

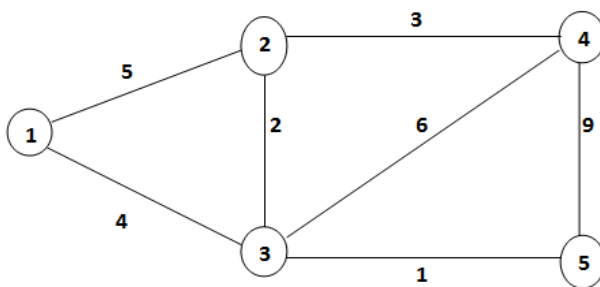


Figure 2: Project network

Step 2 The Adjacency matrix is,

Node	1	2	3	4	5
1	0	5	4	0	0
2	5	0	2	3	0
3	4	2	0	6	1
4	0	3	6	0	9
5	0	0	1	9	0

Step 3 The adjacency matrix split into sub matrix is,

Node	M_1	M_5	Optimal Sequence
1	0	0	1
2	5	0	4
3	4	1	5
4	0	9	3
5	0	0	2

Node	$M_1 + M_2$	$M_4 + M_5$	Optimal Sequence
1	5	0	5
2	5	3	4
3	6	7	3
4	3	9	2
5	0	9	1

Node	$M_1 + M_2 + M_3$	$M_3 + M_4 + M_5$	Optimal Sequence
1	9	4	5
2	7	5	3
3	6	7	4
4	9	15	2
5	1	10	1

Node	$M_1 + M_2 + M_3 + M_4$	$M_2 + M_3 + M_4 + M_5$	Optimal Sequence
1	9	9	1
2	10	5	4
3	12	9	5
4	9	18	3
5	10	10	2

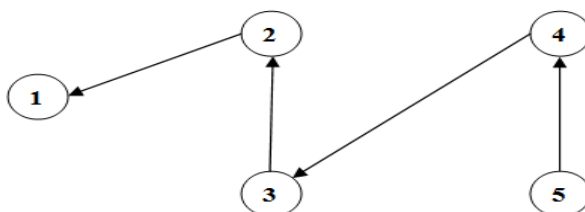
Step 4 The total time for each sub matrix is,

Sequence/Node	1	2	3	4	5
5	0	0	1	10	10
4	0	3	9	10	19
3	4	6	9	16	20
2	9	9	11	19	20
1	9	14	18	19	20

Similarly, other sequence of sub matrix 1-4-5-3-2=34; 5-3-4-2-1=23;1-4-5-3-2=34.

From all the total time of sub matrices **20** is minimum.

Therefore the optimal path is 5-4-3-2-1.



Sequence: 5-4-3-2-1=20

Figure 3: Optimal path

5. CONCLUSION

In this paper, a minimum shortest path is obtained using Pascal's triangle graded mean approach in a fuzzy network. The duration time are considered as hexagonal fuzzy number. A numerical example has particularly provided to explain the defuzzification for hexagonal fuzzy numbers. We solved the problem and obtained the (n-1) sub sequence of the route using the proposed algorithm, we calculated the optimal sequence which leads to the shortest path of the network.

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