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FULLY DEVELOPED FREE CONVECTIVE FLOW OF A JEFFREY FLUID IN A CIRCULAR PIPE

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Abstract- *Free convection flow of a Jeffrey fluid in a circular pipe has been investigated. A non-linear density temperature relationship is taken to express the body force term as buoyancy term. Applying perturbation method, the nonlinear governing equations are solved and the expressions for the velocity field and the temperature distribution are obtained. The rate of heat transfer from the pipe wall to the fluid is determined. It is observed that the velocity increases with increasing NDT parameter, γ, or Jeffrey parameter, $\lambda 1$, whereas the temperature decreases with increasing γ or $\lambda 1$. The results have been compared with the corresponding case of linear density temperature variation. The Nusselt number has also been plotted against the free convection parameter, K, for various values of γ and it is observed that the Nusselt number increases with increasing K..*

Key words: *Fully Developed, Free Convection flow, Jeffrey fluid, Circular pipe.*

I. INTRODUCTION

In fluids, heat convection takes place by forced convection and natural (or free) convection. Forced convection is a mechanism, or type of transport in which fluid motion is generated by an external force (like a pump, fan, suction device, etc.). So it is used to increase the rate of heat exchange. Free convection flow, on the other hand, results from the action of body forces on the fluid, that is, forces which are proportional to the mass or the density of the fluid. In this case, the flow patterns are determined by the buoyant force on the heated fluid.

Ostrach [1, 2] has analyzed the effect of the frictional heating and the heat sources in the fluid, on the fully developed laminar convection flow between two parallel vertical plates when the wall temperatures are either constant or varying linearly along the plate length. Following this linear density temperature (LDT) variation, several convection problems are investigated. Water is a liquid which does not behave like normal liquids. The volume of water increases, if we heat it or cool it, provided initially the water is at 4^0c. This is known as anomalous expansion of water. In such cases, the density and temperature, relationship is modeled as quadratic density temperature (QDT) variation. Goren [3] has obtained a similarity solution of the boundary layer equations of the free convection flow from a semi-infinite plate of uniform temperature to water at 4^0c. In this study he has established the necessity of using QDT variation. Sinha [4] has investigated the fully developed laminar convection flow between two parallel vertical plates, assuming QDT variation. Barrow, Seetharama Rao [5] and Brown [6] examined the flows with the LDT relation on laminar free convection. Using the QDT relationship, Sinha [4], Agarwal & Upmanyu [7], and Balakrishan et al. [8] discussed free convective flows of Newtonian fluids in tubes. Bhargava and Agarwal [9] have investigated the fully developed free convective flow of a Newtonian fluid in a circular pipe.

Nonlinear density temperature (NDT) relation accommodates the LDT relation and QDT relation to some extent. Furthermore, this relation takes care of the linear temperature-dependence of $\Delta\rho$ used by earlier researchers. Gilpin [10] has used a density-temperature relation, which is similar to the NDT relation and which has been introduced by Vanier and Tien [11] with a view to predict the heat-transfer results in the case of water for temperatures between 0^0 and 20^0C. Sastri and Vajravelu [12] considered the problem of free convection between vertical walls. Using NDT relation they investigated the fully developed free convection flow and heat transfer between two long parallel vertical walls kept at constant temperatures. Krishna Gopal Singha [13] investigated analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. Hayat et al. [14] investigated the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. Vajravelu et al. [15] examined the free convection flow of Jeffrey fluid in a vertical porous stratum under peristalsis.

A nonlinear density temperature variation can be defined as

$$\Delta\rho = -\beta_0\rho\left(T-T_s\right)-\beta_1\rho\left(T-T_s\right)^2 \qquad (1)$$

where β_0, β_1 are the constants and T_s is the temperature in hydrostatic condition,

This reduces to LDT variation when β_1=0 and QDT variation when β_0=0.

In this paper, we discuss the problem of fully developed laminar free convection flow of a Jeffrey fluid in a circular pipe, implementing the NDT relationship defines above. The flow and heat transfer both depend upon a new parameter $\gamma=(\beta_1/\beta_0)\Delta T$ in addition to the heat source parameter α and the free convection parameter K. The velocity field, the temperature distribution and Nusselt number are obtained and the results are discussed through graphs.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the fully developed steady laminar free convection flow of a Jeffrey fluid in a circular pipe. In the cylindrical coordinate system (r,ϕ,z), let u,v,w be the velocity components. The motion being rotationally symmetric and assuming that the flow is fully developed, all the physical quantities will be independent of ϕ and z. The radial and tangential components of velocity are zero. The corresponding equations of continuity, motion and energy are

$$\frac{\partial w}{\partial z}=0 \qquad (2)$$

$$0=-\frac{\partial p}{\partial z}+\rho f_z+\frac{\mu}{1+\lambda_1}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right] \qquad (3)$$

$$0=K_1\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\mu\left(\frac{\partial w}{\partial r}\right)^2+Q \qquad (4)$$

Where, Q a constant, denotes the heat added due to heat sources, f_z the generating body force, K_1 the coefficient of thermal conductivity and p the pressure.
The boundary conditions are

$$\text{at}\quad r=0,\ \frac{dw}{dr}=0,\ \frac{dT}{dr}=0 \qquad (5)$$

$$\text{at}\quad r=a,\ w=0,\ T=T_w \qquad (6)$$

Following Ostrach [1], the body force term in (3) can be expressed as buoyancy term. In the hydrostatic condition equation (3) gives

$$\rho_s f_z-\frac{\partial p_s}{\partial z}=0 \qquad (7)$$

and hence

$$\begin{aligned}\rho f_z-\frac{\partial p}{\partial z}&=(\rho-\rho_s)f_z+\rho_s f_z-\frac{\partial p}{\partial z}\\&=(\rho-\rho_s)f_z+\frac{\partial p_s}{\partial z}-\frac{\partial p}{\partial z}\\&=(\rho-\rho_s)f_z-\frac{\partial p_D}{\partial z}\end{aligned} \qquad (8)$$

where $p_D=p-p_s$. (9)

From (1), we get

$$\Delta\rho=-\beta_0\rho\theta-\beta_1\rho\theta^2 \qquad (10)$$

where $\theta=T-T_s$

Now using relation (9) and (10), equations (2)-(5) lead to

$$\frac{d^2 w}{dr^2}+\frac{1}{r}\frac{dw}{dr}-\frac{\rho}{\mu}(1+\lambda_1)\left(\beta_0\theta+\beta_1\theta^2\right)f_z=0 \qquad (11)$$

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}+\frac{\mu}{K_1}\left(\frac{dw}{dr}\right)^2+\frac{Q}{K_1}=0 \qquad (12)$$

3. NON-DIMENSIONALIZATION OF THE FLOW QUANTITIES

We introduce the following non-dimensional quantities:

$$\eta = \frac{r}{a}, \theta^* = \frac{K\theta}{\theta_w}, w^* = \frac{Kw}{W},$$

where

$$\theta_w = T_w - T_s, \quad W = \frac{f_z \beta^2 a^2 \theta_s^2}{v}, \quad K = \frac{f_z^2 \beta^2 \rho^2 a^4 \theta_s^3}{\mu K_1} \tag{13}$$

Equations (11) and (12) reduce to

$$\frac{d^2 w^*}{d\eta^2} + \frac{1}{\eta}\frac{dw^*}{d\eta} - (1+\lambda_1)\left(\theta^* + \frac{\theta^{*2}}{K}\gamma\right) = 0 \tag{14}$$

$$\frac{d^2 \theta^*}{d\eta^2} + \frac{1}{\eta}\frac{d\theta^*}{d\eta} + \left(\frac{dw^*}{d\eta}\right)^2 + \alpha K = 0 \tag{15}$$

where $\alpha\left(= Qa^2/\theta_\omega K_1\right)$ is the heat source parameter. The parameter K can also be expressed as $K = Gr(\mathrm{Pr}) f_z a / C_p$ in which $Gr\left(= \beta_0 f_z a^3 \theta_\omega^2 / v^2\right)$ is the Grashoff number and Pr is the Prandtl number.

For the sake of convenience, dropping the stars, eqns. (14) and (15) finally are

$$\frac{d^2 w}{d\eta^2} + \frac{1}{\eta}\frac{dw}{d\eta} - (1+\lambda_1)\left(\theta + \frac{\gamma}{K}\theta^2\right) = 0 \tag{16}$$

$$\frac{d^2 \theta}{d\eta^2} + \frac{1}{\eta}\frac{d\theta}{d\eta} + \left(\frac{dw}{d\eta}\right)^2 + \alpha K = 0 \tag{17}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &at\, \eta = 0, \frac{dw}{d\eta} = 0, \frac{d\theta}{d\eta} = 0 \\ &at\, \eta = 1, w = 0, \theta = K. \end{aligned}\right\} \tag{18}$$

4. SOLUTION OF THE PROBLEM

Equations (16) and (17) are coupled nonlinear differential equations which cannot be solved for exact solution. Applying perturbation method, we write

$$w = Kw_0 + K^2 w_1 + K^3 w_2 + \ldots.. \tag{19}$$

$$\theta = K\theta_0 + K^2\theta_1 + K^3\theta_2 + \ldots.. \tag{20}$$

Substituting (19) and (20) into (16) and (17) and equating the coefficients of like powers of K on either side of the equations thus obtained, we get the following set of equations

$$\left.\begin{aligned} &w''_0 + \frac{1}{\eta}w'_0 - (1+\lambda_1)\left(\theta_\theta + \gamma\theta_0^2\right) = 0 \\ &w''_1 + \frac{1}{\eta}w'_1 - (1+\lambda_1)\left(\theta_1 + 2\gamma\theta_0\theta_1\right) = 0 \\ &w''_2 + \frac{1}{\eta}w'_2 + (1+\lambda_1)\left(\theta_2 + \gamma\left(2\theta_0\theta_2 + \theta_1^2\right)\right) = 0 \end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned}&\theta''_0+\frac{1}{\eta}\theta'_0+\alpha=0\\&\theta''_1+\frac{1}{\eta}\theta'_1+w'^2_0=0\\&\theta''_2+\frac{1}{\eta}\theta'_2+2\gamma w'_0 w'_1=0\end{aligned}\right\}\tag{22}$$

The boundary conditions (18) then reduce to

at $$\left.\begin{aligned}&\eta=0,\ w'_0=w'_1=w'_2=0\\&\theta'_0=\theta'_1=\theta'_2=0\end{aligned}\right\}\tag{23}$$

at $$\left.\begin{aligned}&\eta=1,\ w_0=w_1=w_2=0\\&\theta_0=1,\theta_1=\theta_2=0\end{aligned}\right\}\tag{24}$$

Solving eqns. (21) and (22) under (23) and (24), we get

$$W_0=D_0+A_1\eta^2-A_2\eta^4+A_3\eta^6\tag{25}$$

$$\theta_0=1+\frac{\alpha}{4}-\frac{\alpha}{4}\eta^2\tag{26}$$

$$\theta_1=D_1-A_4\eta^{12}+A_5\eta^{10}-A_6\eta^8+A_7\eta^6-A_8\eta^4\tag{27}$$

$$W_1=E_1+E_2\eta^{16}-E_3\eta^{14}+E_4\eta^{12}-E_5\eta^{10}+E_6\eta^8-E_7\eta^6-E_8\eta^4+E_9\eta^2\tag{28}$$

where $A_1=\left(\frac{1+\lambda_1}{4}\right)\left[1+\frac{\alpha}{4}+\gamma+\frac{\alpha\gamma}{2}+\frac{\alpha^2\gamma}{16}\right]$, $A_2=\left(\frac{1+\lambda_1}{16}\right)\left[\frac{\alpha}{4}+\frac{\alpha^2\gamma}{8}+\frac{\alpha\gamma}{2}\right]$

$A_3=\left(\frac{1+\lambda_1}{36}\right)\frac{\alpha^2 v}{16}$, $D_0=-A_1+A_2-A_3$, $A_4=\frac{36A_3^2}{144}$, $A_5=\frac{48A_2A_3}{165}$

$A_6=\frac{16A_2^2}{64}+\frac{24A_1A_2}{64}$, $A_7=\frac{16A_1A_2}{36}$, $A_8=\frac{4A_1^2}{16}$, $D_1=A_4-A_5+A_6-A_7+A_8$

$E_2=\frac{(1+\lambda_1)A_4\alpha v}{512}$, $E_3=\frac{(1+\lambda_1)\left(A_4\left(1+2v+\frac{\alpha v}{2}\right)+A_5\frac{\alpha v}{2}\right)}{196}$

$E_4=\frac{(1+\lambda_1)\left(A_5\left(1+2v+\frac{\alpha v}{2}\right)+A_6\frac{\alpha v}{2}\right)}{144}$, $E_5=\frac{(1+\lambda_1)\left(A_6\left(1+2v+\frac{\alpha v}{2}\right)+A_7\frac{\alpha v}{2}\right)}{100}$

$E_6=\frac{(1+\lambda_1)\left(A_7\left(1+2v+\frac{\alpha v}{2}\right)+A_8\left(\frac{\alpha v}{2}\right)\right)}{64}$, $E_7=\frac{(1+\lambda_1)\left(A_8\left(1+2v+\frac{\alpha v}{2}\right)\right)}{36}$

$E_8=\frac{(1+\lambda_1)\alpha\gamma D_1}{32}$, $E_9=\frac{(1+\lambda_1)D_1\left(1+2v+\frac{\alpha v}{2}\right)}{4}$

Similarly we can obtain the solutions for θ_2 and W_2. The velocity and temperature functions are then obtained from (19) and (20). The rate of heat transfer from the pipe wall to the fluid per unit area of the pipe surface is given by

$$q=\frac{K_1\theta}{aK}\left(\frac{\partial\theta}{\partial\eta}\right)_{n=1}=\frac{K_1\theta}{a}\left(\frac{-\alpha}{2}+K\left(-12A_4+10A_5-8A_6+6A_7-4A_8\right)\right)\tag{29}$$

The Nusselt number is therefore

$$Nu = \frac{qa}{\theta_{\omega} K_1} = \frac{1}{K}\left(\frac{\partial \theta}{\partial \eta}\right)_{n=1} = \frac{-\alpha}{2} + K\left(-12A_4 + 10A_5 - 8A_6 + 6A_7 - 4A_8\right) \qquad (30)$$

IV. ANALYSIS AND RESULTS

In this paper, fully developed free convective flow of a Jeffrey fluid in a circular pipe is investigated and the results are discussed for various physical parameters. We note that when the Jeffrey parameter λ_1 is taken as zero, the results agree with the corresponding ones of Bhargava and Agarwal [9].

Flow solutions are depicted graphically to study the parameters α, γ, λ_1 and K on the velocity, the temperature and the Nusselt number.

Taking γ=0 in equations (21) and (22) under (23) and (24), we obtain velocity, temperature corresponding to LDT case of Jeffrey fluid flow in a circular pipe and they are given by

$$W_0 = D_0 + A_1\eta^2 - A_2\eta^4 + A_3\eta^6 \qquad (31)$$

$$\theta_0 = 1 + \frac{\alpha}{4} - \frac{\alpha}{4}\eta^2 \qquad (32)$$

$$\theta_1 = D_1 - A_4\eta^{12} + A_5\eta^{10} - A_6\eta^8 + A_7\eta^6 - A_8\eta^4 \qquad (33)$$

$$W_1 = E_1 + E_2\eta^{16} - E_3\eta^{14} + E_4\eta^{12} - E_5\eta^{10} + E_6\eta^8 - E_7\eta^6 - E_8\eta^4 + E_9\eta^2 \qquad (34)$$

where $A_1 = \left(\frac{1+\lambda_1}{4}\right)\left(1+\frac{\alpha}{4}\right)$, $A_2 = \left(\frac{1+\lambda_1}{16}\right)\frac{\alpha}{4}$, $A_3 = \left(\frac{1+\lambda_1}{36}\right)\frac{\alpha^2 v}{16}$

$$D_0 = -A_1 + A_2 - A_3,\; A_4 = \frac{36A_3^2}{144},\; A_5 = \frac{48A_2A_3}{165},\; A_6 = \frac{16A_2^2}{64} + \frac{24A_1A_2}{64}$$

$$A_7 = \frac{16A_1A_2}{36},\; A_8 = \frac{4A_1^2}{16},\; D_1 = A_4 - A_5 + A_6 - A_7 + A_8,\; E_2 = \frac{(1+\lambda_1)A_4\alpha v}{512}$$

$$E_3 = \frac{(1+\lambda_1)\left(A_4\left(1+2v+\frac{\alpha v}{2}\right)+A_5\frac{\alpha v}{2}\right)}{196},\; E_4 = \frac{(1+\lambda_1)\left(A_5\left(1+2v+\frac{\alpha v}{2}\right)+A_6\frac{\alpha v}{2}\right)}{144}$$

$$E_5 = \frac{(1+\lambda_1)\left(A_6\left(1+2v+\frac{\alpha v}{2}\right)+A_7\frac{\alpha v}{2}\right)}{100},\; E_6 = \frac{(1+\lambda_1)\left(A_7\left(1+2v+\frac{\alpha v}{2}\right)+A_8\left(\frac{\alpha v}{2}\right)\right)}{64}$$

$$E_7 = \frac{(1+\lambda_1)\left(A_8\left(1+2v+\frac{\alpha v}{2}\right)\right)}{36},\; E_8 = 0,\; E_9 = \frac{(1+\lambda_1)D_1\left(1+2v+\frac{\alpha v}{2}\right)}{4}$$

The variation of velocity with η is calculated, from equations (25) and (28), for different values of γ and λ_1 and is shown in Figures 1-4, for fixed K. We observe that the velocity increases with the increase in the NDT parameter γ, heat source parameter α and Jeffrey parameter λ_1.

From the equations (26) and (27), we have calculated the temperature as a function of η, for fixed K, for different values of γ, λ_1 and α and is shown in Figures 5-8. We observe that the temperature decreases with the increase in the parameters γ, α and λ_1.

From the equation (30) we have calculated the Nusselt number as a function of K for fixed λ_1 and for different values of NDT parameter γ and α and is shown in Figures 9 and 10. We observe that the Nusselt number increases with the increase in the NDT parameter γ and α.

We have calculated the Nusselt number as a function of K and γ for fixed α, and for different values of Jeffrey parameter λ_1 and is shown in Figures 11 and 12.We observe that the Nusselt number increases with the increase in the parameters λ_1 and γ.

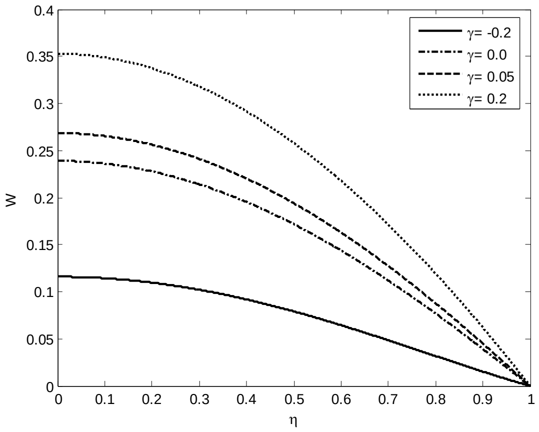

Fig.1 Velocity distribution for various values of γ for fixed α=5, K=0.5 and λ1 =0.01.

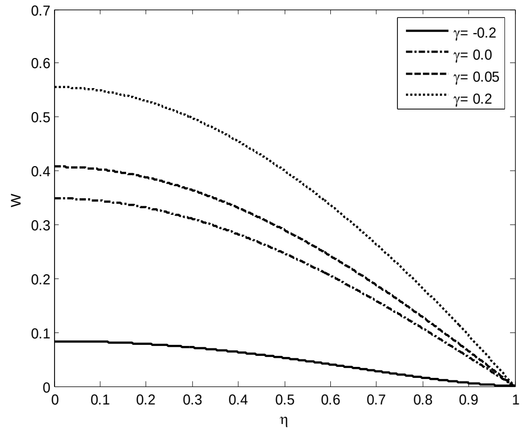

Fig.2 Velocity distribution for various values of γ for fixed α=10, K=0.5 and λ_1 =0.01.

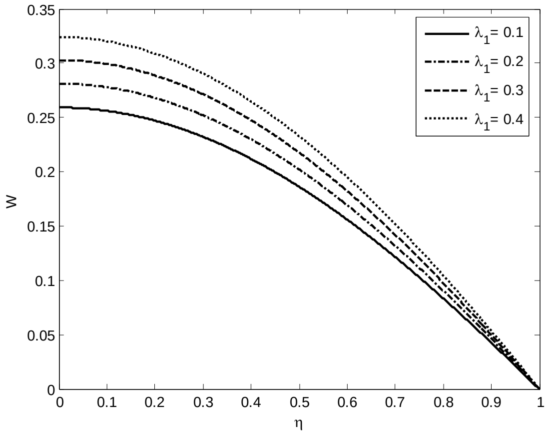

Fig. 3 Velocity distribution for various values of λ_1 for fixed α=5, K=0.5 and γ =0.

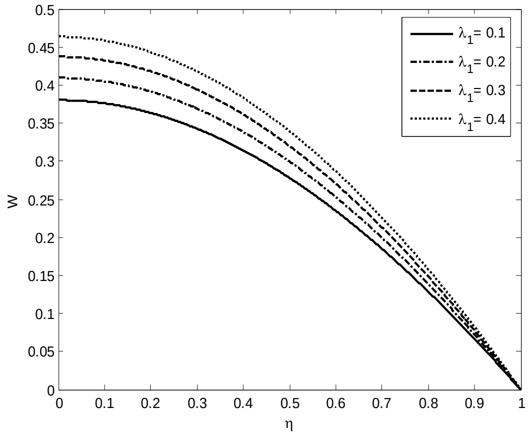

Fig. 4 Velocity distribution for various values of $\lambda_1$for fixed α=5, K=0.5 and γ =0.2.

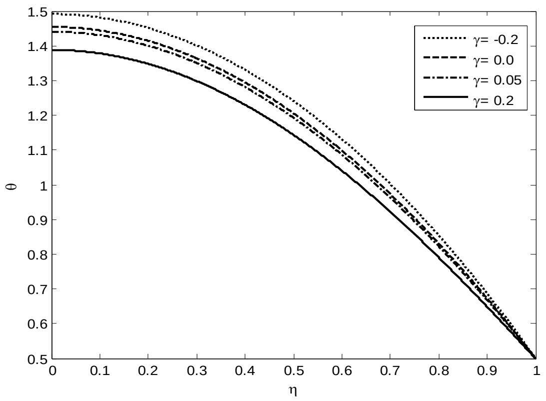

Fig.5 Temperature distribution for various values of γ for fixed α=7, K=0.5 and λ_1 =0.01.

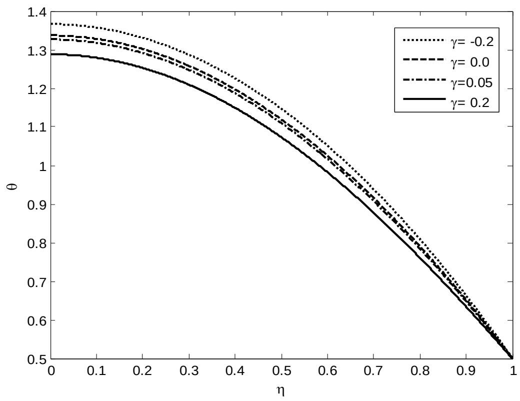

Fig.6 Temperature distribution for various values of γ for fixed α=8, K=0.5 and λ_1 =0.01.

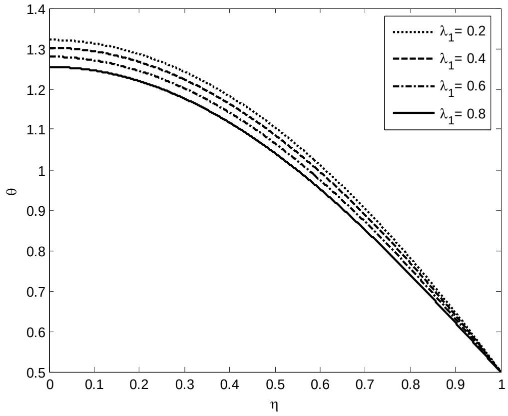

Fig.7 Temperature distribution for various values of λ_1 for fixed α=7, K=0.5 and γ =0.

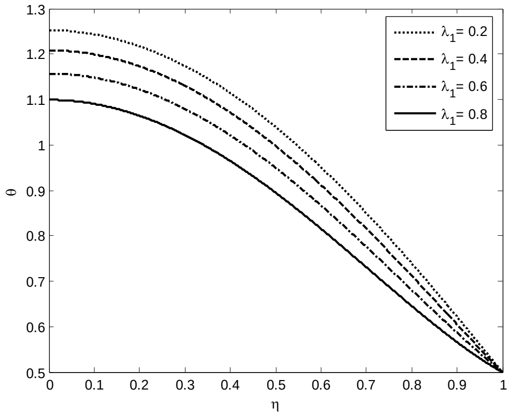

Fig.8 Temperature distribution for various values of λ_1 for fixed α=7, K=0.5 and γ =0.2.

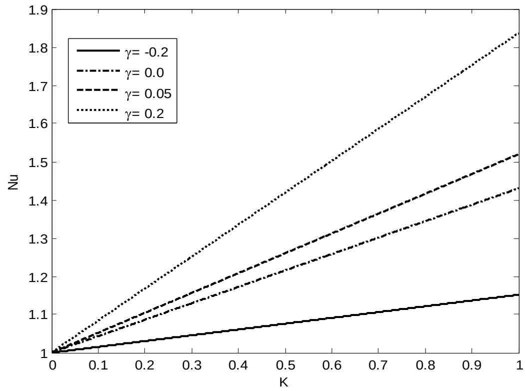

Fig.9 Nusselt number distribution for various values of γ for fixed α=5 and λ_1 =0.01.

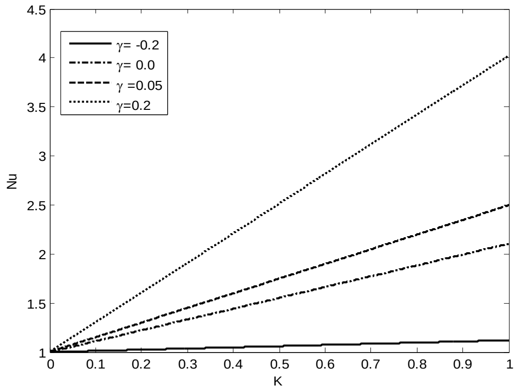

Fig.10 Nusselt number distribution for various values of γ for fixed α=10 and λ_1 =0.01.

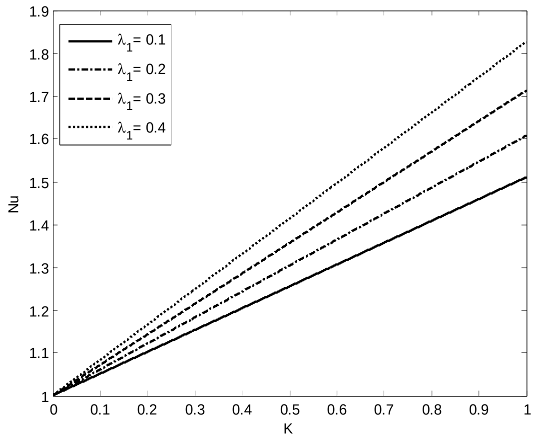

Fig.11 Nusselt number distribution for various values of λ_1 for fixed α=5 and γ =0.

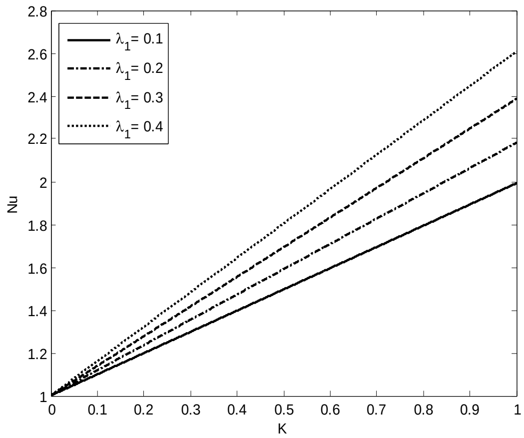

Fig.12 Nusselt number distribution for various values of λ_1 for fixed α=5 and γ =0.2.

V. CONCLUSIONS

- The velocity increases with the increase in the NDT parameter γ and heat source parameter α.
- The velocity increases with the increase in the Jeffrey parameter λ_1 and γ.
- The temperature decreases with the increase in the parameters γ and α.
- The temperature decreases with the increase in the parameters λ_1 and γ.
- The Nusselt number increases with the increase in the parameters γ, λ_1 and α.

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