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Parametric Study on Response of Railway Tracks Under Moving Loads

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Abstract —*This paper addresses the dynamic response of a railway tracks under moving loads. In the present analysis, an infinite Euler–Bernoulli beam of constant cross-section resting on an elastic foundation is considered. The beam and foundation are assumed to be homogeneous and isotropic. The foundation is modeled using one and two parameters with damping. The beam is subjected to a constant point load moving with a constant speed along the beam. An effort has been made to find the solution of the governing differential equation analytically. In this parametric study we find that the deflection and bending moment response of the beam is symmetric with respect to point load in the case of without damping and asymmetric in the case of damping. The maximum displacement as well as maximum bending moment both occurs under the moving load for both the cases with or without damping.*

Keywords*-Bending Moment, Deflection, Damping, Moving load, Railway track dynamics*

I. INTRODUCTION

Railway transportation is world’s most efficient mean of transportation of passengers and goods. It is safest, most economical and environment friendly in all the transportation systems. India has a very vast network of railway tracks. There is around 60,000 kilometer railway line in our country. The department of Indian Railways tries its best to make railway system more secure, safer, economical and environmentally responsible.
The design of railway tracks is a vast topic involving study of underlying soil layers, ballasts, sleepers, rail pads, rails, their behavior under moving load, and climatic conditions. Through the statistical analysis of historical data, researchers were able to extract desired and alternate design features and the development of new models and methods by observing what goes wrong in tracks. The successful and economical design of railway track is dependent on many parameters like the accuracy, availability, level of collection of data required and details and the methodology which is used to forecast the track response to the moving loads. Hence the response of deflection and bending moment of the track needs a proper attention while developing the economical design of new tracks and analysis of existing tracks.
Researches indicate that the resonant behavior of railway track is a major cause of its damage. When the train is passing over it, it is vibrates for a short period due to loading of the moving train and the energy waves are created in the soil medium. The track vibrations can crack the sleepers or fasteners or made the cause of failure of existing soil foundation. It indicates that the study of response of tracks is very much critical for determine the strength of railway tracks. A critical speed existing for moving train at which resonance behavior is occurred. In the recent years, due to the design of fast moving trains, the study of the dynamic response of railway tracks has received significant attention. Most of researchers idealized the track as a beam in their analysis. They found suitable various subgrade models like one parameter foundation model (Winkler's soil medium) and two-parameter foundation model (Pasternak model soil medium) for simulating the supporting soil foundation. By most of the researchers the track-soil foundation system represented by a beam resting on either the one parameter, Winkler's model soil medium or two parameter viscoelastic foundation model. The effects of inertia forces of the traveling vehicles in these dynamic studies were neglected.
As the soil is a non linear material, thus considerable error is found in the analysis when the soil is modeled as a linear material. The non-linear stress-strain relationship is described by Kondner (1963) in the form of a hyperbolic curve. In the railway track analysis, considering nonlinear behavior of soil, only a few researches have been extended. In the Pasternak model, the foundation is modeled by spring and end of these springs are connected by a layer. Exponential decay functions (Bettess, 1977) are used to model the infinite domain.
The main aim of this analysis is to describe the response of deflection and bending moment of a Euler-Bernoulli beam lying on both one parameter foundation model (Winkler-type elastic foundation) and two parameter foundation model (Paternak’s model) subjected to a moving load for both the cases with and without damping and then compare the results for each case and show the difference of modeling by both methods. The effect of damping on different velocity ratios is also shown.

1.1 Subgrade

Soil below the ballasts is referred as a subgrade soil. The velocity of Rayleigh surface waves which are most damaging waves propagating in the ground depends upon the type of soil and soil conditions. In the evaluation of the response of the track to the moving load or moving force, the knowledge of complete stress-strain characteristics of the soil below the track is necessary. There is a large variety of soils in nature and soil conditions are also different place to place in real so theoretically it is not possible to generalize a complete stress-strain relationship for any type of soil. Because of this difficulty in the behavior of actual soils, a lot of idealized models are developed to analyze soil-structures. The commonly employed theories such as the classical theories of elasticity and plasticity are the theories for idealizations in the analysis in soil mechanics. Some models like Winkler model shows purely elastic characteristics and gives linear relationship between the applied loads and the corresponding resulting displacements.

1.1.1 Winkler Model (1864)

According to Winkler's model, settlement (w) of the foundation at any point is directly proportional to applied load (P), and it is independent of load applied at any another points. This can be expressed in mathematical form as

$$P(x,y)=kw(x,y) \quad \ldots (1.1)$$

Where 'k' is proportionality constant, termed as the modulus of sub grade reaction having units as $kN/m^2/m$

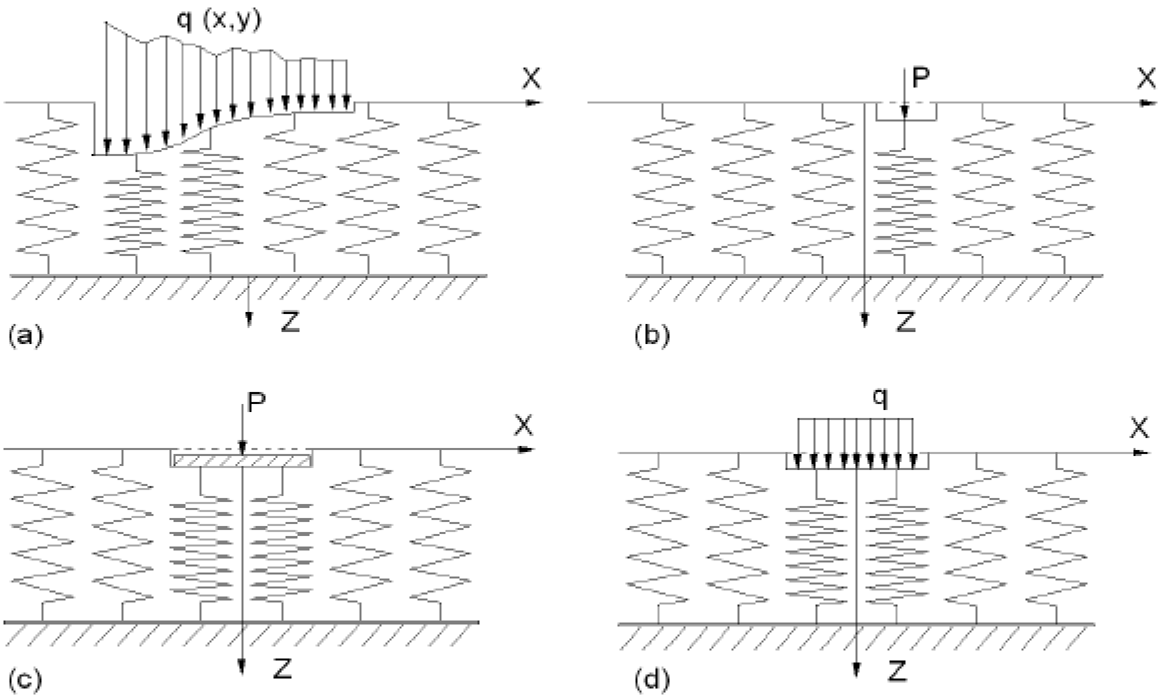


Figure 1 Settlement of surface in Winkler model (a) Varying load, (b) Point load, (c) Rigid load, (d) Uniform flexible load (Selvadurai, 1979)

In the Winkler model, a number of mutually independent spring elements as shown in Fig. 1are considered as an idealization of the soil medium, with spring constant 'k'. This model is quite simple with simple numerical calculations. It is easy to idealize any railway track problem by Winkler model, but it can be applied only for the soil media which does not possess any interaction between soil particles like non-cohesive soils. Thus for cohesive soils, the two parameter elastic models are developed by researchers.

1.1.2 Two Parameter Elastic Models

As the Winkler model is not capable to give the interaction between the closely spaced vertical springs, many other soil response models have been developed. As these models have two independent elastic constant, they are called two-parameter models. Two different approaches are followed for developing these models. The first approach proceeds from the discontinuous Winkler model and eliminates its discontinuous behavior by introducing some interaction between the springs. The second approach starts from the elastic continuum and made some simplifying assumptions for distribution of stresses [Reissner (1958), Vlasov and Leontiev (1966)].

1.1.3 Pasternak Model (1954)

In the Pasternak model all the springs are interconnected by a layer consisting of incompressible elements which deform by transverse shearing to provide interaction between the springs. Shear modulus is interaction parameter which represents the interaction due to shear action among the spring elements.

II. PERAMETRIC STUDIES

To get the dynamic response of moving load on railway track visually on the graph sheets, to better understand the nature of deflection and bending moment and also to compare the different cases and different models parametric studies are done for both the models one parameter as well as two parameters and also for both the cases with or without damping. For this numerical computation some assumed values are used. These parameters satisfy most of the railway tracks and material properties of railway track in India typically. By using these parameters deflection and bending moment have been calculated by mentioned equations earlier. After plotting the graph of deflection and bending moment with respect to new co-ordinate system ξ for all the cases, deflection and bending moment variations are also compare for different cases.
Parameters which are assumed for computation of deflection and bending moment are listed here in following table 1.

Table 1 Properties of Beam and Soil

Properties	Assumed Values
ρ (kg/m)	245
EI (Nm^2)	1.75×10^6
K	40.78×10^5
k_1	666875
P (N)	93360
E_s (N/m^2)	3.73×10^6
v_s	0.4

III. RESULTS AND DISCUSSION

Based on the above equations and assumed parametric values numerical calculations are done to get the response of dynamic deflection of the track for a moving point load. In this analysis we use the Maxwell Reciprocal theorem. According to Maxwell the displacement at point B due to load at point A is equal to the displacement of point A due to same load at point B. so if we calculate the deflection from $\xi = -\infty$ to ∞ when load is at $\xi = 0$. It is equal to the deflection at $\xi = 0$ when load moves from $\xi = -\infty$ to ∞.
Here we consider that the load is moving with a constant velocity v with respect to coordinate system x and also consider a moving coordinate system ξ moving with same constant velocity v. So point load P with respect to new coordinate system ξ always seems at a fix location say at $\xi = 0$.And by the given above equations we can calculate the deflection for $\xi = -\infty$ to ∞. So according to Maxwell theorem it is equal to deflection at$\xi = 0$ when load P moves from $\xi = -\infty$ to ∞.

3.1 Deflection and Bending Moment on One Parameter Foundation

For one parameter foundation model critical velocity is calculated by equation (2) and it is got 147.67 m/sec.

$$V_{cr}=\sqrt{(b/a)} \qquad (2)$$

We consider here only the sub critical velocities for moving load so$v<v_{cr}$_crHere we take x = 20meter for cvr = 0.25, x = 40 meter for cvr = 0.5, x = 60 meter for cvr = 0.75 and x =73 meter for cvr = 0.99. Here x is nothing but shows the load position att = 0and ξ denotes the location of load at time t second. We also change t from 0 to 1 second so that we can get the overall response when load is moving with a constant velocity.
Some of the results for one parameter foundation model are tabulated here.

Table 2 Response for One Parameter Foundation Model

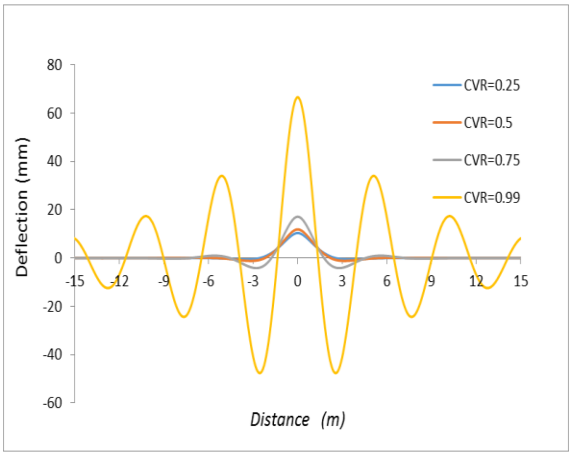
Critical Velocity Ratio	Max. +ve Deflection (mm)	Distance from load (m)
0.25	10.39	0
0.50	11.9	0
0.75	17.16	0
0.99	70.72	0

Critical Velocity Ratio	Max. -ve Deflection (mm)	Distance from load (m)
0.25	0.56	3.56
0.50	1.17	3.12
0.75	4.14	2.76
0.99	51.38	2.6

Critical Velocity Ratio	Max. +ve B.M.(N-mm)	Distance from load(m)
0.25	3.44	1.84
0.50	4.56	1.92
0.75	9.29	2.04
0.99	80.16	2.34

Critical Velocity Ratio	Max. -ve B.M.(N-mm)	Distance from load(m)
0.25	15.86	0
0.50	18.16	0
0.75	26.19	0
0.99	107.96	0

Then we plot the response of deflection and bending moment with respect to distance shown on below.



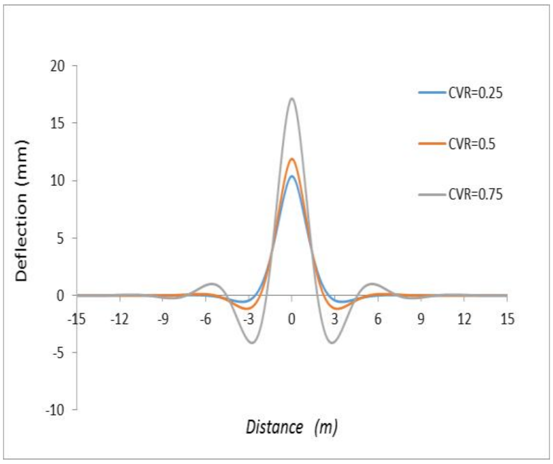
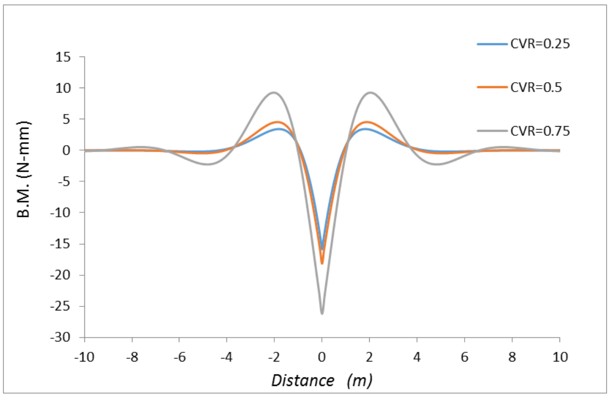


Figure 2 Deflection V/s Distance for One Parameter Foundation Model (CVR=0.25 to 0.99)

Figure 3: Deflection V/S Distance for One Parameter Foundation Model (CVR=0.25 to 0.75)



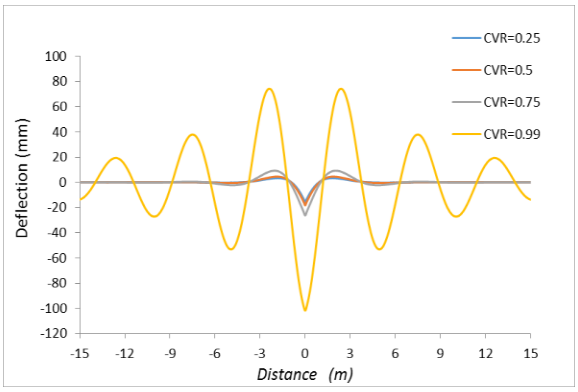


Figure 4 B.M. V/S Distance for One Parameter Foundation Model (CVR=0.25 to 0.75)

Figure 5 B. M. V/S Distance for One Parameter Foundation Model (CVR=0.25 to 0.99)

Figure: 2 and 5 shows the variation of deflection and bending moment with respect to distance $\xi = x - vt$ for critical velocity ratios 0.25 to 0.99. Figure 2and 5 shows an enlarge view of deflection and banding moment v/s distance for velocity ratio 0.25to 0.75.

By the analysis of table 2 and above figures we observed that:-

1. The deflection is maximum under the point load and it rapidly decrease with increase in distance from point load at any instant. Same results are obtained for bending moment. Maximum negative banding moment occurs under the point load. It decreases more rapidly than deflection with increase in distance from point load at any instant.
2. Both deflection and bending moment are symmetric with respect to point load.
3. At some distance from the load there is a small negative deflection then positive deflection then again negative deflection and so on. The maximum absolute valuesof these deflections decrease with increase in distance from load. Negative deflection means an upward displacement and positive deflection means downward displacement. In the case of bending moment negative bending moment refers to tension at bottom of beam.
4. The distance of maximum negative deflection from load is decrease with increasing in critical velocity ratio whereas in the case of bending moment the distance of maximum positive bending moment increases with increasing in critical velocity ratio.
5. Maximum deflection and maximum bending moment are increase with increase in critical velocity ratio.

3.2 Deflection and Bending Moment on Two Parameter Foundation Model

By the same procedure based on the above equations and assumed parametric values numerical calculations are done to get the response of dynamic deflection and bending moment of the track for a moving point load. Here critical velocity will be got by equation $V_{cr} = \surd(b+c\_1/a$ and it is 156.62 m/sec.

Results are shown in tabular form.

Table 3 Response for One Parameter Foundation Model

Critical Velocity Ratio	Max. +ve Deflection (mm)	Distance from load (m)	Critical Velocity Ratio	Max. -ve Deflection (mm)	Distance from load (m)
CVR=0.25	9.75	0	CVR=0.25	0.36	3.76
CVR=0.5	10.97	0	CVR=0.5	0.77	3.28
CVR=0.75	14.67	0	CVR=0.75	2.61	2.88
CVR=0.99	66.79	0	CVR=0.99	47.58	2.48

Critical Velocity Ratio	Max. +ve B.M.(N-mm)	Distance from load(m)	Critical Velocity Ratio	Max. -ve B.M.(N-mm)	Distance from load(m)
CVR=0.25	3.01	1.76	CVR=0.25	14.89	0
CVR=0.5	3.86	1.84	CVR=0.5	16.74	0
CVR=0.75	6.9	2.04	CVR=0.75	22.4	0
CVR=0.99	74.43	2.33	CVR=0.99	101.95	0

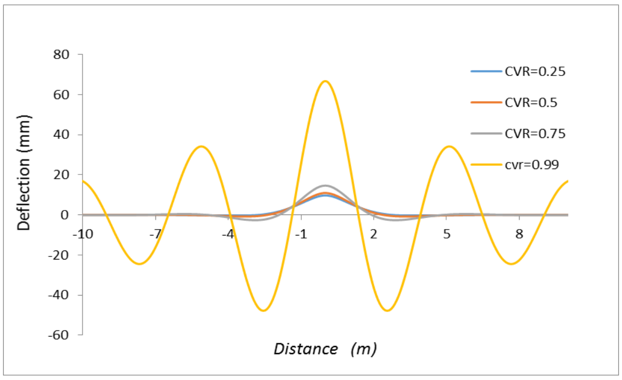


Figure 6 Deflection V/s Distance for Two Parameter Foundation Model (CVR=0.25 to 0.99)

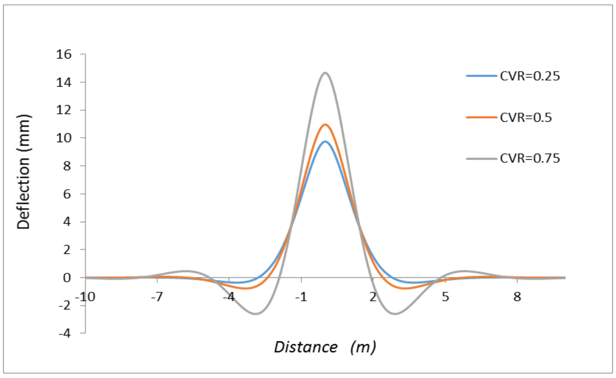


Figure 7 Deflection V/S Distance for Two Parameter Foundation Model (CVR=0.25 to 0.75)

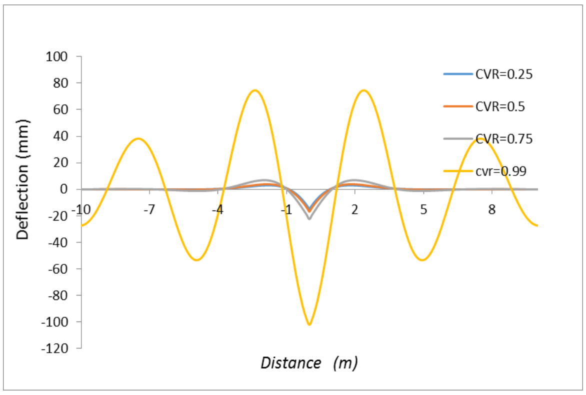


Figure 8 B.M. V/S Distance for Two Parameter Foundation Model (CVR=0.25 to 0.99)

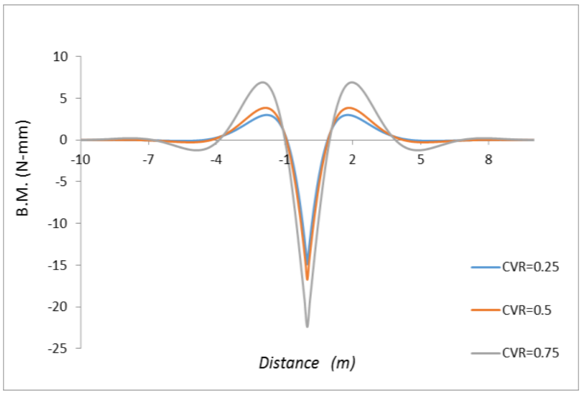


Figure 9 B. M. V/S Distance for Two Parameter Foundation Model (CVR=0.25 to 0.75)

In Figure: 6 to 9 the variation of deflection and bending moment respect to parameter $\xi = x - vt$ are shown.

By the observation of tables and graphs some results are obtained with are listed below.

1. The deflection is maximum under the point load and it rapidly decrease with increase in distance from point load at any instant. Same results are obtained for bending moment. Maximum negative banding moment occurs under the point load. It decreases more rapidly then deflection with increase in distance from point load at any instant.
2. Both deflection and bending moment are symmetric with respect to point load.
3. At some distance from the load there is a small negative deflection then positive deflection then again negative deflection and so on. The maximum absolute values of these deflections decrease with increase in distance from load. Negative deflection means an upward displacement and positive deflection means downward displacement. In the case of bending moment negative bending moment refers to tension at bottom of beam.
4. The distance of maximum negative deflection from load is decrease with increasing in critical velocity ratio whereas in the case of bending moment the distance of maximum positive bending moment increases with increase in critical velocity ratio.

5. Maximum deflection and maximum bending moment are increase with increasing in critical velocity ratio.The process of the development of the concrete for strength aspects in various proportions varying from 0%, 10%, 15%, 20% and 25% marble dust as a replacement of cement along with fly ash. The main aim of the study was to identify the best proportion of marble dust with fly ash, which can be replaced with cement to get the desired strength. The proportion of fly ash was taken as 30% by weight of cement decided from previous studies published in various journals. The effect of marble dust is evaluated by performing the different tests on the cubes and cylinders to know its compressive strength at different intervals of days, splitting tensile strength. Tests of these specimens were conducted at 7, 28 and 56 days after casting.

3.3 Comparisons of Deflection and Bending Moment Between and Two Parameter Foundation Model
We get the deflection and bending moment response for both models one parameter foundation model as well as two parameter foundation model for the case of without damping for various velocities ratios.

Table 3 Comparison of Response for One and Two Parameter Foundation Model

C V R	Max. +ve Deflection (mm)		Max. -ve Deflection (mm)		Max. +ve B.M.(N-mm)		Max. -ve B.M.(N-mm)	
	one parameter model	Two parameter model	one parameter model	Two parameter model	one parameter model	Two parameter model	one parameter model	Two parameter model
0.25	10.39	9.75	0.56	0.36	3.44	9.75	15.86	14.89
0.5	11.9	10.97	1.17	0.77	4.56	10.97	18.16	16.74
0.75	17.16	14.67	4.14	2.61	9.29	14.67	26.19	22.4
0.99	70.72	66.79	51.38	47.58	80.16	66.79	107.96	101.95

The response obtained for one parameter foundation model and two parameter foundation model have been compared in table 4 for various velocity ratios. Comparison of both the models is also shown in Figure 10 and 11 at velocity ratio0.99. The maximum deflection and bending moment for two parameter foundation model is smaller in comparison to one parameter model response. This is only due to an additional parameter which is shear interaction in two parameter foundation model. So we can easily understand the difference between one and two parameter foundation model.

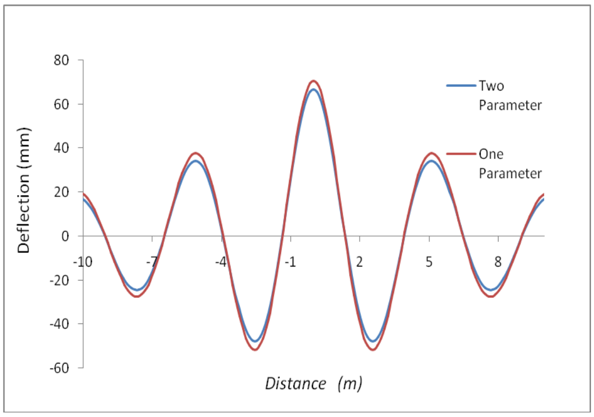


Figure 10 Comparison between Deflection of One and Two Parameter Modeling

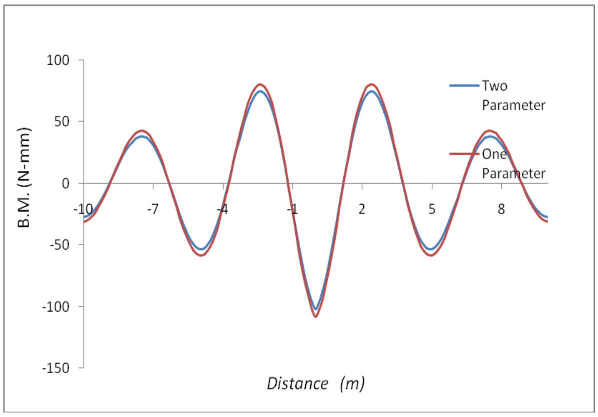


Figure 11 Comparison between B. M. of One and Two Parameter Modeling

Through the table 4 and figure 10 to 11 some results are observed in comparison of both foundation models are listed below:-

1. Maximum positive or negative deflection and maximum negative bending moment are low in case of two parameter foundation model as compare to one parameter foundation model
2. In two parameter foundation model positive bending moment is higher than the bending moment which obtained in one parameter foundation model.

3.4 Response of Deflection and Bending Moment with Damping

We have discussed earlier deflection and bending moment response for one and two parameter foundation models without damping case. Here we will discuss response for two parameter foundation model with damping. The governing differential equations are same as before. The only change is we take here $d \neq 0$ which shows damping. First we define critical damping as $d_{cr} = 2b\sqrt{2a}$, $d<d_{cr}$

Here we take the damping ratio η = 10%, 20% and 30%. So $d = \eta d_{cr}$. We can also take other values of damping ratios. For the numerical calculations we adopt again the same procedure here. By the assumed parametric values we prepare table for the variation between deflection and damping for different velocity ratios.

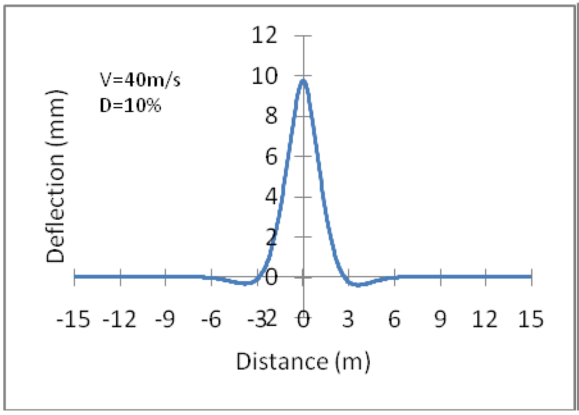
Table 4 Effect of Damping and Velocity of Load on Responses of Beam

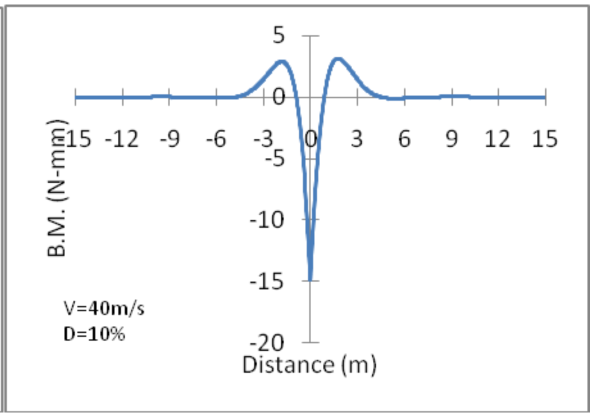
Maximum +ve Deflection				
v(m/s)	d=0%	10%	20%	30%
40	9.753	9.745	9.726	9.694
80	10.971	10.917	10.806	10.625
124.7	15.6	15.166	14.125	13.328

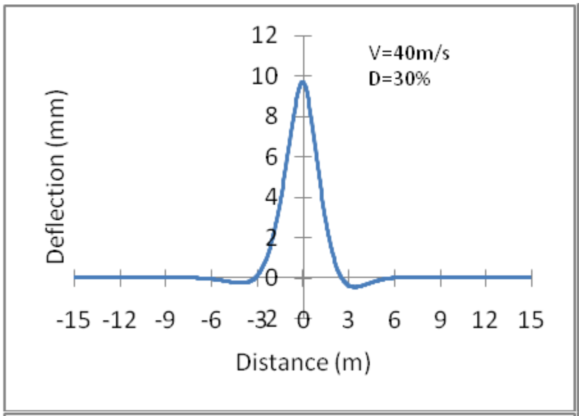
Maximum –ve Deflection				
v(m/s)	d=0%	10%	20%	30%
40	0.357	0.39	0.422	0.454
80	0.775	0.875	0.961	1.031
124.7	3.156	3.402	3.35	3.21

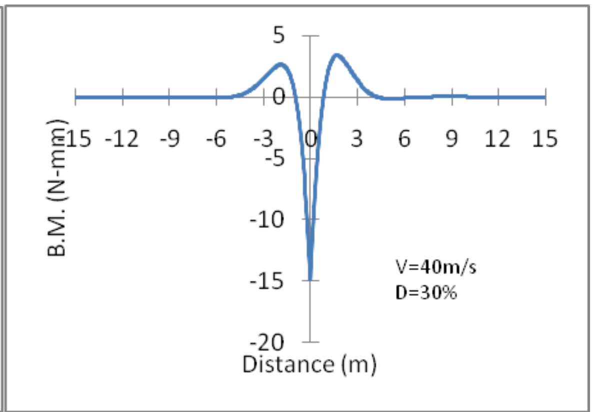
Maximum -ve B.M.				
v(m/s)	d=0%	10%	20%	30%
40	14.9	14.88	14.86	14.82
80	16.746	16.63	16.54	16.31
124.7	23.8	23.12	21.43	20.03

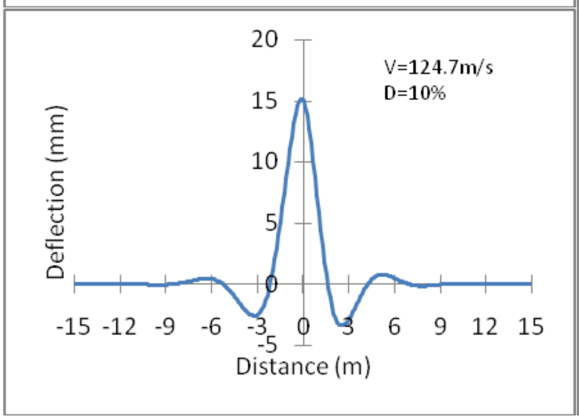
Maximum +ve B.M.				
v(m/s)	d=0%	10%	20%	30%
40	3.01	3.13	3.26	3.38
80	3.86	4.21	4.54	4.83
124.7	7.78	9.01	9.71	9.94

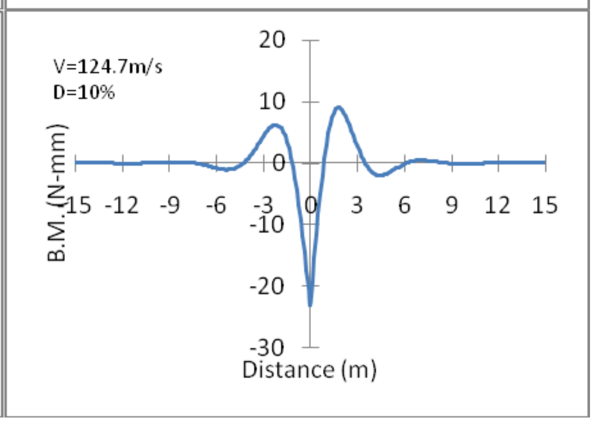












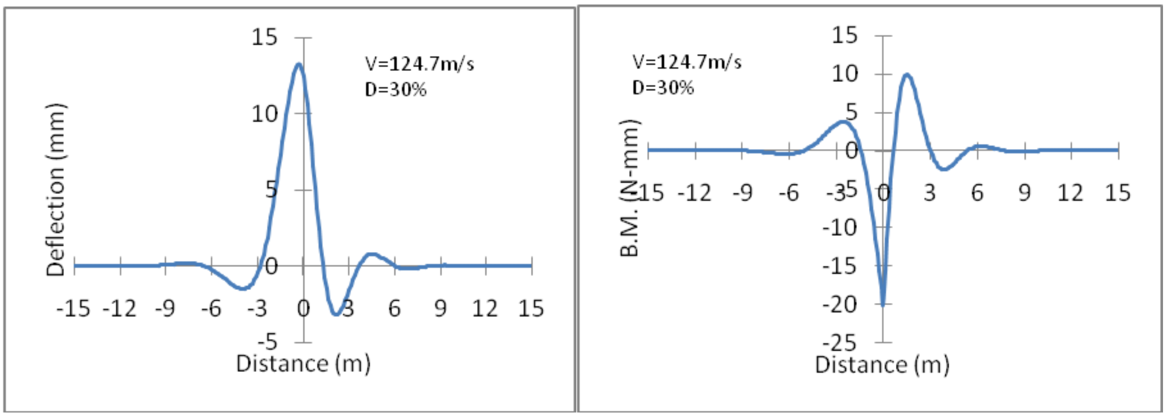


Figure 12: Response of Deflection and B.M. with Damping

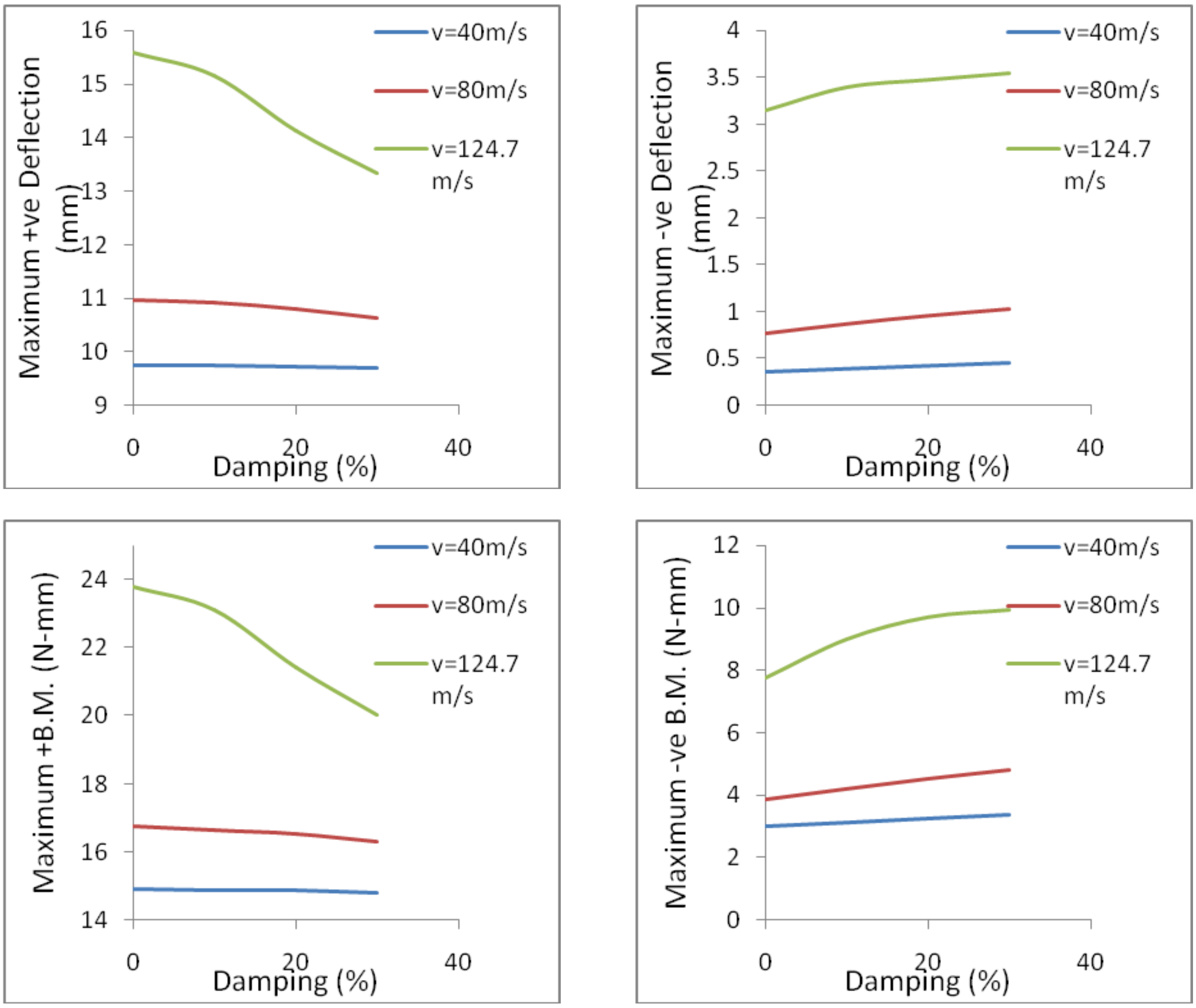


Figure13: Various Responses V/S Damping with varying Speed of Moving Load

After analyzing the table and graphs above we have observed some results that are listed below:-

1. Response of deflection and bending moment in case of damping are not symmetric about the load and asymmetry is increase with increase in damping and velocity of load.
2. Deflection and negative bending moment both are decrease as damping increase but uplift and positive bending moment both are increase as damping increase.
3. Distance of negative deflection and positive bending from point load is decrease as damping increase.
4. Maximum deflection and maximum bending moment both occur under the load.

3.5 Effect of Spring Constant and Shear Parameter on Responses

We compare response for different values of spring constant k and shear parameter k_1 which is tabular below:-

Table 5 Effect of Spring Constant and Shear Parameter on Deflection

Deflection in mm					Deflection in mm				
CVR	K1=0.8	K1=1	K1=1.2	K1=1.4	CVR	K=0.8	K=1	K=1.2	K=1.4
0.25	9.82	9.70	9.59	9.48	0.25	11.43	9.70	8.48	7.57
0.5	10.83	10.67	10.52	10.38	0.5	12.74	10.68	9.26	8.21
0.75	13.55	13.25	12.97	12.70	0.75	16.41	13.25	11.19	9.74
0.99	29.21	26.53	24.46	22.82	0.99	58.47	26.53	18.73	14.82

In table first we can see the effect of shear parameter and then the effect of second parameter spring constant when all other parameters are keep constant.
By observing the above table we can say that maximum deflection is decrease with increase in value of either shear parameter or spring constant or both.

V. CONCLUSION

A railway track which is represented by Bernoulli–Euler beam that placed on an elastic foundation subject to moving point loads is analyzed in this thesis. The response of deflection and bending moment has been studied and compared for one parameter and two parameter foundation models under damping and without damping.

The results and conclusions of this study are as follows:

- The deflection and bending moment response of the beam is symmetric with respect to point load in the case of without damping and asymmetric in the case of damping.
- The maximum displacement as well as maximum bending moment both occurs under the moving load for both the cases with or without damping.
- Both deflection and bending moment increase with increase in load velocity.
- The maximum displacement and bending moment both are less in case of two parameter foundation as compare to one parameter foundation model.
- Due to damping deflection and bending moment both are decrease with increase in damping.
- With increase in velocity of load or critical velocity ratio response shape get shaper and shaper.
- In the case of damping responses have high value behind the load as compare to ahead.
- With increase in parameters value like spring constant or shear modulus both the responses decrease.

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