

Similarity measures of generalised vague soft sets and its application in medical diagnosis

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Abstract —The soft set theory was initiated by Molodtsov and has been successful used as an effective mathematical tool for dealing with vagueness and uncertainties. In this paper, we first introduce the concept of generalised vague soft set. Then we propose a similarity measure between two generalised vague soft sets and study its basic properties. Finally, an application of generalised vague soft sets in medical diagnosis is given to demonstrate the feasibility and validity of the proposed similarity measure.

Keywords- soft set; vague soft set; generalised vague soft set; similarity measures; medical diagnosis

I. INTRODUCTION

In order to deal with various uncertainties in real life, many mathematical tools, such as theory of probability, fuzzy sets, rough sets and vague sets have been developed. Unfortunately, all these theories have their inherent difficulties. To do this, Molodtsov [1] proposed the soft sets theory, which is a new method for modeling vagueness and uncertainty. Since its appearance, the soft set has attracted the attention of many scholars in different fields, and lots of results on soft set have obtained in theory and application. For example, Maji and Biswas et al. [2] defined some algebraic operations of soft sets. Ali et al. [3,4] further developed some new operations on soft sets and studied some important properties associated with these new operations. They also discussed semiring(hemiring) structures of soft sets and showed that soft sets with fixed set of parameters are MV algebras and BCK algebras. Yang and Guo [5] introduced the notions of anti-reflexive kernel, symmetric kernel, reflexive closure, and symmetric closure of a soft set relation. Feng et al. [6] proposed the new concept of soft rough set and presented important properties of soft rough approximations based on soft approximation spaces. Maji et al. [7] proposed the concept of fuzzy soft sets by combining fuzzy sets and soft sets. Maji et al. [8,9] and Yang et al. [10] further extended fuzzy soft sets to intuitionistic fuzzy softs and interval-valued fuzzy soft sets, respectively. Majumdar and Samanta [11] gave the concept of generalized fuzzy soft sets by fuzzifying the parameters. Zhou and Li [12] further extended generalized fuzzy soft set to generalized vague soft set. Some other extended models can be found in [13-15].

The similarity measure is an important tool for determining the degree of similarity between two objects. Measures of similarity between sets, as an important topic in the theory of sets, have gained attention for their wide applications in various fields, such as pattern recognition, machine learning, decision making and medical diagnosis. The similarity measures of fuzzy sets and some extended models have been studied by many researchers [16-20]. Recently, The similarity measures of soft sets has also received much attention by many scholars. Majumdar and Samanta [21], Kharal[22] and Min [23] discussed the similarity measures of soft sets from different perspectives. Majumdar and Samanta et al. [11,24,25] further studied the similarity measures of fuzzy soft sets, intuitionistic fuzzy soft sets and generalized fuzzy soft sets and applied them to deal with medical diagnosis problems. Wang and Qu [26] developed the similarity measures, entropy and distance measure of vague soft sets and investigated their relations. The main goal of this paper is to discuss the similarity measures of generalized vague soft sets and apply it to medical diagnosis.

The rest of this paper is organized as follows. The following section briefly reviews some basic notions of soft sets, fuzzy soft sets, generalized fuzzy soft sets and so on. Section 3 presents a kind of method of similarity measure between generalised vague soft sets. In section 4, An application based on the proposed similarity measure in medical diagnosis problem is shown, and some illustrative examples are also employed to show that the method presented here are not only reasonable but also efficient in practical applications. The conclusion is given finally in section 5.

II. PRELIMINARY

In this section, we will briefly recall some basic concepts on soft sets, fuzzy soft sets, generalized fuzzy soft sets and generalized vague soft sets. Further detailed information can be referred to [1,7,11,12].

Definition 1. [1] Let $P(U)$ denotes the power set on U . A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 2. [7] Let $\tilde{P}(U)$ is the set of all fuzzy subsets of U . A pair (F, A) is called a fuzzy soft set over U , where F is a mapping given by $F : A \rightarrow \tilde{P}(U)$.

From the above definition, it is clear that $F(e)$ is a fuzzy set on U , for any $e \in A$, and the membership function of $F(e)$ is denoted by $F_e : U \rightarrow [0,1]$.

Definition 3. [11] Let $F : E \rightarrow P(U)$ and μ a fuzzy subset of E , i.e. $\mu : E \rightarrow [0,1]$. A function $F_\mu : E \rightarrow P(U) \times [0,1]$ defined as $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in \tilde{P}(U)$, is called a generalised fuzzy soft set over soft universe (U, E) .

Definition 4. [12] Let $F(U)$ is the power set of vague sets on U , $F : A \rightarrow F(U)$ is a function, and α a vague sets of A , i.e. $\alpha : A \rightarrow [0,1]^2$. A function $F_\alpha : A \rightarrow F(U) \times [0,1]^2$ defined as $F_\alpha(e) = (F(e) = \{h / \mu_{F(e)}(h)\}, \alpha(e))$, where $\mu_{F(e)}(h) = [t_{F(e)}(h), 1 - f_{F(e)}(h)]$, $\alpha(e) = [t_{\alpha(e)}, 1 - f_{\alpha(e)}]$, is called generalised vague soft set (in short, GVS set) over soft universe (U, E) .

Here for each parameter e , $F_\alpha(e)$ indicates not only the degree of belongingness of elements of U in $F(e)$, but also the degree of preference of such belongingness which is represented by $\alpha(e)$.

Definition 5 [12] Let $A \subseteq E$. A GVS set F_α is said to be relative absolute GVS set, if $t_{F(e)}(h) = t_{\alpha(e)} = 1$ and $f_{F(e)}(h) = f_{\alpha(e)} = 0$ for all $h \in U, e \in A$. Usually, we denote it as Ω_A .

Definition 6. [12] Let $A \subseteq E$. A GVS set F_α is said to be relative null GVS set, if $t_{F(e)}(h) = t_{\alpha(e)} = 0$ and $f_{F(e)}(h) = f_{\alpha(e)} = 1$ for all $h \in U, e \in A$. Usually, we denote it as Φ_A .

III. SIMILARITY MEASURES OF GENERALISED VAGUE SOFT SETS

In this section, we will introduce a similarity measure of GVS sets based on the set-theoretic approach.

Let $U = \{h_1, h_2, \dots, h_n\}$ be a finite universal, $E = \{e_1, e_2, \dots, e_m\}$ a finite set of parameters, and $F_\delta = (F(e_j), \delta(e_j))$ and $G_\eta = (G(e_j), \eta(e_j))$ two GVS sets over (U, E) , where $F(e_j)$ and $G(e_j)$ are vague sets over U for every $e_j \in E$, δ and η are the vague subsets of E . That is to say, $F_\delta = (F(e_j) = \{h_i, \mu_{F(e_j)}(h_i)\}, \delta(e_j))$ and $G_\eta = (G(e_j) = \{h_i, \mu_{G(e_j)}(h_i)\}, \eta(e_j))$ for all $e_j \in E, h_i \in U$, where $\mu_{F(e_j)}(h_i) = [t_{F(e_j)}(h_i), 1 - f_{F(e_j)}(h_i)]$, $\mu_{G(e_j)}(h_i) = [t_{G(e_j)}(h_i), 1 - f_{G(e_j)}(h_i)]$, $\delta(e_j) = [t_{\delta(e_j)}, 1 - f_{\delta(e_j)}]$, $\eta(e_j) = [t_{\eta(e_j)}, 1 - f_{\eta(e_j)}]$, $i = 1, \dots, n$ and $j = 1, \dots, m$.

For convenience, two families of vague soft sets $\{F(e_j), j = 1, \dots, m\}$ and $\{G(e_j), j = 1, \dots, m\}$ are written as $\ddot{F}$ and $\ddot{G}$, respectively. Then the similarity measure between the two GVS sets F_δ and G_η is defined as follows:

$$S(F_\delta, G_\eta) = S_D(\ddot{F}, \ddot{G}) \cdot S_d(\delta, \eta),$$

where $S_D(\ddot{F}, \ddot{G})$ and $S_d(\delta, \eta)$ indicate the similarity degrees of between $\ddot{F}$ and $\ddot{G}$ and between δ and η respectively. Formally they are defined as follows:

$$S_D(\ddot{F}, \ddot{G}) = \max\{S_D^j(\ddot{F}, \ddot{G}), j = 1, \dots, m\},$$

$$S_d(\delta, \eta) = 1 - \frac{\sum_{j=1}^m |\delta(e_j) - \eta(e_j)|}{\sum_{j=1}^m |\delta(e_j) + \eta(e_j)|}$$

$$= 1 - \frac{\sum_{j=1}^m |t_{\delta(e_j)} - t_{\eta(e_j)}| + \sum_{j=1}^m |f_{\delta(e_j)} - f_{\eta(e_j)}|}{\sum_{j=1}^m |t_{\delta(e_j)} + t_{\eta(e_j)}| + \sum_{j=1}^m |2 - f_{\delta(e_j)} - f_{\eta(e_j)}|},$$

where

$$S_D^j(\ddot{F}, \ddot{G}) = 1 - \frac{\sum_{i=1}^n |\mu_{F(e_j)}(h_i) - \mu_{G(e_j)}(h_i)|}{\sum_{i=1}^n |\mu_{F(e_j)}(h_i) + \mu_{G(e_j)}(h_i)|}$$

$$= 1 - \frac{\sum_{i=1}^n |t_{F(e_j)}(h_i) - t_{G(e_j)}(h_i)| + \sum_{i=1}^n |f_{F(e_j)}(h_i) - f_{G(e_j)}(h_i)|}{\sum_{i=1}^n |t_{F(e_j)}(h_i) + t_{G(e_j)}(h_i)| + \sum_{i=1}^n |2 - f_{F(e_j)}(h_i) - f_{G(e_j)}(h_i)|}.$$

When the universe U and the parameter set E are continuous, $S_D(\ddot{F}, \ddot{G})$ and $S_d(\delta, \eta)$ can be written as follows:

$$S_D(\ddot{F}, \ddot{G}) = \max\{S_D^j(\ddot{F}, \ddot{G}), j = 1, \dots, m\},$$

$$S_d(\delta, \eta) = 1 - \frac{\int_E |t_{\delta(e_j)} - t_{\eta(e_j)}| + \int_E |f_{\delta(e_j)} - f_{\eta(e_j)}|}{\int_E |t_{\delta(e_j)} + t_{\eta(e_j)}| + \int_E |2 - f_{\delta(e_j)} - f_{\eta(e_j)}|},$$

Where

$$S_D^j(\ddot{F}, \ddot{G}) = 1 - \frac{\int_U |t_{F(e_j)}(h_i) - t_{G(e_j)}(h_i)| + \int_U |f_{F(e_j)}(h_i) - f_{G(e_j)}(h_i)|}{\int_U |t_{F(e_j)}(h_i) + t_{G(e_j)}(h_i)| + \int_U |2 - f_{F(e_j)}(h_i) - f_{G(e_j)}(h_i)|}.$$

To illustrate the above method, let us consider the following example.

Example 1. Let $U = \{h_1, h_2, h_3\}$ be the universe and $E = \{e_1, e_2, e_3\}$ the set of parameters. We define two GVS sets F_δ and G_η as follows:

$$F_\delta(e_1) = (\{h_1 / [0.1, 0.2], h_2 / [0.8, 0.9], h_3 / [0.3, 0.7]\}, [0.1, 0.2]),$$

$$F_\delta(e_2) = (\{h_1 / [0.6, 0.8], h_2 / [0.1, 0.3], h_3 / [0.4, 0.8]\}, [0.3, 0.5]),$$

$$F_\delta(e_3) = (\{h_1 / [0.2, 0.3], h_2 / [0.6, 0.8], h_3 / [0.2, 0.9]\}, [0.2, 0.5]),$$

$$G_\eta(e_1) = (\{h_1 / [0.4, 0.6], h_2 / [0.3, 0.9], h_3 / [0.4, 0.7]\}, [0.6, 0.7]),$$

$$G_\eta(e_2) = (\{h_1 / [0.5, 0.8], h_2 / [0.2, 0.5], h_3 / [0.5, 0.6]\}, [0.4, 0.8]),$$

$$G_\eta(e_3) = (\{h_1 / [0.3, 0.7], h_2 / [0.4, 0.8], h_3 / [0.3, 0.8]\}, [0.3, 0.6]).$$

Then $S_D^1(\ddot{F}, \ddot{G}) \approx 0.79$, $S_D^2(\ddot{F}, \ddot{G}) \approx 0.88$, $S_D^3(\ddot{F}, \ddot{G}) \approx 0.86$ and $S_d(\delta, \eta) \approx 0.69$. Hence

$S_D(\ddot{F}, \ddot{G}) = \max\{S_D^1(\ddot{F}, \ddot{G}), S_D^2(\ddot{F}, \ddot{G}), S_D^3(\ddot{F}, \ddot{G})\} = 0.88$. Therefore, we have

$$S(F_\delta, G_\eta) = S_D(\ddot{F}, \ddot{G}) \cdot S_d(\delta, \eta) = 0.88 \times 0.69 \approx 0.61.$$

Proposition 2. Let F_δ , G_η and H_σ be three GVS sets over (U, E) and S a similarity measure. Then the following hold:

- (1) $0 \leq S(F_\delta, G_\eta) \leq 1$.
- (2) $S(F_\delta, G_\eta) = S(G_\eta, F_\delta)$.
- (3) $S(F_\delta, G_\eta) = 1$ if $F_\delta = G_\eta$.
- (4) $S(F_\delta, G_\eta) = 0$ if $F_\delta \cap G_\eta = \Phi_E$.
- (5) $S(F_\delta, H_\sigma) \leq S(F_\delta, G_\eta)$ and $S(F_\delta, H_\sigma) \leq S(G_\eta, H_\sigma)$ if $F_\delta \subseteq G_\eta \subseteq H_\sigma$.

Proof. The proof is straightforward.

Majumdar and Samanta [21] defined the concepts of α -similar and significant similarity about soft sets. In a similar way, we give the notions about GVS set.

Definition 3. Let S be a similarity measure. Then two GVS sets F_δ and G_η over (U, E) are called to be κ -similar, denoted as $F_\delta \approx_\kappa G_\eta$, if $S(F_\delta, G_\eta) \geq \kappa$ for $\kappa \in (0, 1)$. Specially, F_δ and G_η are called to be significantly similar if $\kappa = 0.5$ (or $S(F_\delta, G_\eta) \geq 0.5$).

Proposition 4. $\approx_\kappa$ is reflexive and symmetric, but not transitive.

Proof. The reflexive and symmetry can be obtained from (2) and (3) of Proposition 2.

To illustrate that $\approx_\kappa$ is not transitive, we give an example as follows.

Example 5. Let $\kappa = 0.6$. We consider the GVS set F_δ and G_η given in Example 1, and define a GVS set H_σ as follows:

$$\begin{aligned} H_\sigma(e_1) &= (\{h_1 / [0.8, 0.9], h_2 / [0.1, 0.2], h_3 / [0.5, 0.7]\}, [0.8, 0.9]), \\ H_\sigma(e_2) &= (\{h_1 / [0.1, 0.3], h_2 / [0.8, 0.9], h_3 / [0.4, 0.9]\}, [0.7, 1.0]), \\ H_\sigma(e_3) &= (\{h_1 / [0.5, 0.6], h_2 / [0.1, 0.2], h_3 / [0.6, 0.8]\}, [0.6, 0.8]). \end{aligned}$$

Then $S(F_\delta, G_\eta) \approx 0.61 > \kappa$, $S(G_\eta, H_\sigma) \approx 0.63 > \kappa$, but $S(F_\delta, H_\sigma) \approx 0.34 < \kappa$. Hence $\approx_\kappa$ is not transitive.

Clearly, the GVS sets F_δ and G_η are significantly similar, but F_δ and H_σ are not significantly similar in the above example.

IV. AN APPLICATION IN MEDICAL DIAGNOSIS

The notion of similarity measure between two generalised fuzzy sets has been utilized to perform medical diagnosis in literature [11]. In this section, we give an example to show how to carry out medical diagnosis based on the proposed similarity measures for GVS sets.

Suppose that there is a patient with some visible symptoms and the symptoms of influenza have been obtained. Our purpose is to estimate the possibility that the patient is suffering from influenza. For that, we first construct the GVS set of symptoms for the patient and a model GVS set for influenza. Next we calculate the degree of similarity between these two GVS sets. If they are significantly similar, then we conclude that the patient is possibly suffering from influenza.

Let universal U contain only two elements, i.e. $U = \{h_1, h_2\}$, where h_1 and h_2 stand for "yes" and "no" respectively. And let the parameters set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$ be the set of visible symptoms, where $e_i (i=1, 2, \dots, 9)$ stand for the parameters "fever with chill", "fever with no chill", "headache", "sore throat", "body ache", "malaise", "breathing trouble", "cough with sputum" and "cough with no sputum", respectively.

Now assume the patient is having fever, headache, sore throat, body ache and so on. According to the symptoms of the patient, we can construct his GVS set F_δ which is given in Table 1.

Table 1. Tabular representation of the GVS set F_δ

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
h_1	[0.1,0.2]	[0.5,0.8]	[0.2,0.5]	[0.1,0.4]	[0.3,0.6]	[0.2,0.3]	[0.7,0.9]	[0.8,0.9]	[0.3,0.4]
h_2	[0.8,0.9]	[0.2,0.5]	[0.5,0.8]	[0.7,0.9]	[0.4,0.6]	[0.7,0.8]	[0.2,0.3]	[0.1,0.2]	[0.6,0.8]
δ	[0.1,0.2]	[0.3,0.6]	[0.2,0.5]	[0.5,0.8]	[0.4,0.6]	[0.1,0.5]	[0.4,0.5]	[0.2,0.4]	[0.3,0.4]

The model GVS set for influenza M_σ can easily be prepared with the help of a medical expert, and its tabular representation is given in Table 2.

Table 2. Tabular representation of the Model GVS set M_σ

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
h_1	[1,1]	[0,0]	[1,1]	[1,1]	[1,1]	[1,1]	[0,0]	[0,0]	[1,1]
h_2	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[1,1]	[1,1]	[0,0]
δ	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]

Then the similarity measure of M_σ and F_δ can be calculated as follows:

$$S(M_\sigma, F_\delta) = S_D(\ddot{M}, \ddot{F}) \cdot S_d(\sigma, \delta) \approx 0.26 < 0.5.$$

Hence the two GVS sets M_σ and F_δ are not significantly similar. So the patient is not possibly suffering from influenza.

Now, we again consider another patient having certain visible symptoms whose corresponding GVS set G_η is given in Table 3.

Table 3. Tabular representation of the GVS set G_η

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
h_1	[0.8,0.9]	[0.1,0.3]	[0.5,0.6]	[0.7,0.9]	[0.8,0.9]	[0.6,0.8]	[0.2,0.3]	[0.1,0.2]	[0.7,0.8]
h_2	[0.1,0.2]	[0.7,0.9]	[0.1,0.2]	[0.3,0.4]	[0.1,0.3]	[0.2,0.3]	[0.7,0.9]	[0.6,0.7]	[0.1,0.2]
δ	[0.8,0.9]	[0.7,1.0]	[0.6,0.8]	[0.5,0.8]	[0.9,1.0]	[0.7,0.9]	[0.8,1.0]	[0.7,0.9]	[0.7,0.8]

It is easy to obtain $S(M_\sigma, G_\eta) \approx 0.76 > 0.5$. Hence the two GVS sets are significantly similar. Therefore the patient is possibly suffering from influenza.

It is worth noting that the above similarity measure depends not only on $G(e)$ but also on $\eta(e)$. For instance, let us consider the patient whose corresponding GVS set H_ρ is given in Table 4, where $H(e) = G(e)$, but $\rho(e) \neq \eta(e)$ for each $e \in E$.

Table 4. Tabular representation of the GVS set H_ρ

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
h_1	[0.8,0.9]	[0.1,0.3]	[0.5,0.6]	[0.7,0.9]	[0.8,0.9]	[0.6,0.8]	[0.2,0.3]	[0.1,0.2]	[0.7,0.8]
h_2	[0.1,0.2]	[0.7,0.9]	[0.1,0.2]	[0.3,0.4]	[0.1,0.3]	[0.2,0.3]	[0.7,0.9]	[0.6,0.7]	[0.1,0.2]
δ	[0.3,0.4]	[0.1,0.2]	[0.0,0.1]	[0.1,0.3]	[0.2,0.3]	[0.4,0.5]	[0.0,0.2]	[0.2,0.4]	[0.1,0.3]

However $S(M_\sigma, H_\rho) = 0.32 < 0.5$, that is to say the two GVS sets M_σ and H_ρ are not significantly similar.

V. CONCLUSION

This paper can be viewed as a continuation of the study of Majumdar et al. [11] and Zhou et al. [12]. In this work, the concept of generalised vague soft set was introduced, and a measure of similarity method for generalised vague soft sets was developed. Based on the proposed similarity measure, an application in medical diagnosis was present, and an example was given to demonstrate its feasibility and validity. In the future, we will apply the proposed measure method to different soft models and various fields

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