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# SHEAR LAG EFFECT IN MULTI-CELL BOX GIRDERS

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**Abstract** — *When the axial load is applied into a wide flange by shear from the webs the flange distorts in its plane and plane sections do not remain plane. The resulting stress distribution in the flange is not uniform. In very wide flanges, shear lag effects have to be taken into account for the verification of stresses, especially for short spans, since it causes the longitudinal stress at a flange-web intersection to exceed the mean stress in the flange.The distribution of bending stress in thin-walled box girder at any transverse section is non-uniform and in general at web-flange junction the stresses reach their maximum, decreasing towards the middle point of the top and bottom slabs and cantilever flanges. This phenomenon of shear lag is being investigated by carrying out parametric analysis of three-cell box girder. It has been investigated that the span and the cantilever flange of three-cell box girder plays an important role on shear lag effect and it has been observed that shear lag effect becomes more severe as span length becomes larger and deck width becomes wider for box girder with non-uniform thickness of deck.*

***Keywords***- *Shear lag; Box girder; Cantilever; Deck*

## I. INTRODUCTION

In the elementary theory of bending it is assumed that the cross sections of girder which are plane before bending remain plane after bending. However, for a girder with wide flanges such as box girders, this assumption is not always justifiable. When a box girder is under load, the longitudinal normal stresses induced in the flange are assumed to be uniformly distributed across the flange width. However, in most cases, particularly in a wide flange, these stresses are not uniformly distributed due to the shear deformations of the flange plates. Thus, longitudinal stress on a flange near the web is much different than that far from the web. Under some circumstances, cross section distortion combined with shear lag effect may cause stresses significantly larger that those predicted by simple beam theory and torsion theory. When the longitudinal stress on a flange near the web is much larger than that far from the web is known as positive shear lag .In the converse case, it is referred to as negative shear lag, in which the stress near the web is much smaller than that far from the web .Chang [1] derived formulas for evaluating shear lag coefficient in a simply supported prestressed box girder under dead load. In case of prestressed tendons having parabolic configurations, formulas to compute the shear lag effect were also developed. Shear lag effect caused by movable load is also analysed according to the eccentricity of the load to the half width ratio of box girder and charts were prepared to predict the shear lag coefficient for live load. Luo and Tang [2] developed a finite segment method for analyzing shear-lag effect in box girders. They assumed that the spanwise displacement of the flange plates are described by a third-power parabolic function. The results of parametric studies made by them confirm that the shear lag effect in box girders depends upon not only on the ratio of the flange width to span length (b/L) but also on height ratio of mid span to that at support (h/H). Zhou [3] analyzed the effect of prestressing on shear lag in box girders using finite element method based on variational principle. He concluded that shear lag coefficient of a box beam has a maximum value generally at locations where prestress tendons are anchored. Negative shear lag under the uniformly distributed load and prestressing may occur both at midspan of simply supported box girder and at the fixed end of a cantilever box girder. The same shall also occur at other cross-sections of simply supported or cantilever box girder according to the combination of distributed loads and prestressing. Shear lag and negative shear lag effect in cantilever box girder were analyzed through variation approach and finite element techniques by Zheng and Chang [4]. The complex distribution of bending stresses under symmetrical loading on flanges of cellular sections were also presented. The origin of negative shear lag was explained from physical point of view by Lee et al [5]. In their study, a numerical investigation was carried out to determine the origin of negative shear lag. The results of their study revealed that the origin of negative shear lag is positive shear lag. Positive shear lag creates negative shear lag which depends on the compatibility requirement. They concluded that at any given location negative shear lag can take place whenever the portion of shear flow acting along the flange edges is larger than the remaining portion of shear flow caused by positive shear lag. A finite difference solution was presented for shear lag effect on the box girder with varying depth by Chang and Yun [6]. The analysis is carried out according to Reissner's method in which the spanwise displacement is assumed to be a quartic parabolic curve on the cross-section instead of a quadratic one. They found that with same length to width ratio, the shear lag on box girder with varying depth is less than on box girder with constant depth. Experimental studies on shear lag of box girders were carried out by Luo and Li [7]. They concluded that shear lag effect is related to width of the box girder, the larger the width, the more severe the shear lag effect. Under concentrated load shear lag effect is more severe than under distributed load.

Transverse analysis of strutted box girder bridges was performed by Kenneth [8]. A program based on folded plate method was developed giving an indication to the severity of shear lag effect. It was observed that shear lag effect becomes more severe as the span length becomes shorter and deck width becomes wider.

A finite-element based parametric study was performed to derive a proposed simplified design equation to compute effective width as an indirect measure to account for shear lag effect in both positive and negative moment regions by Chen et al [9]. They found that all simple span bridges exhibit full effective width at all cross-sections of practical interest, except the ones with short spans, wide girder spacing and highly skewed supports. Almost all non-skewed multiple span continuous bridges similarly exhibit full width as effective width. Those with high skew angle produced smaller effective width.
In this paper, shear lag effect in three-cell box girder is been studied based on numerical model developed using concept of finite elements. The role of depth of box, its span and cantilever arm are being critically analyzed to understand the shear lag effect in multi-cell box girders.

## II. FINITE ELEMENT MODELLING

### 2.1 Box girder model

The box girder has been analysed as an area object using Bridge Modeller of SAP2000 as shown in Fig 1. The entire section of the box has been modelled using shell elements. As compared to the spine model the area model is preferable as it gives forces in the individual girder which is not the case with the spine model.

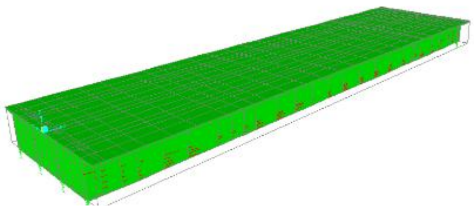


**Fig 1** Box girder model in SAP2000

**2.2 Geometry** The bridge cross-section is of three-cell hollow prestressed box with varying deck slab thickness along with haunches as shown in figure 2. It has span of 60m and width of 15 m. The dead weight of the box girder is taken into account by SAP 2000 Bridge Modeller by assigning the material property of concrete of density 25kN/m$^3$.

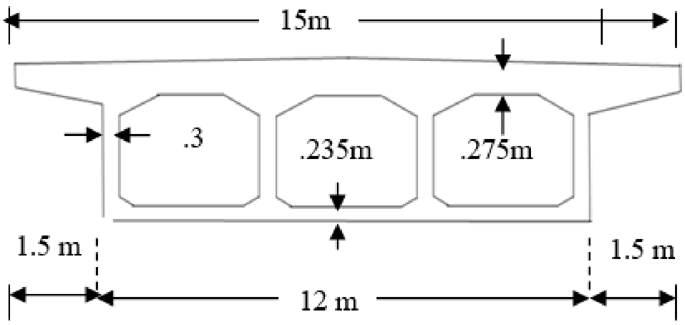



**Fig 2** Cross-section of three cell box girder

### 2.3 Load

Class 70R wheeled loading is considered for the parametric study. The vehicle has been idealized as concentrated forces moving along the deck. The vehicle has been assumed to move at constant velocity and the mass of vehicle with suspension is been ignored.

### 2.4 Boundary condition

The bridge has been modelled as simply supported at pier locations. Diaphragms are provided at ends and also in between at suitable spacing.

### 2.5 Materials

Material used for model is linear and isotropic. The structure has been assumed to be made of prestressed concrete. Properties of material are: Poisson's ratio=0.3, Modulus of elasticity, $E_c$=33540 MPa, Grade of concrete M45 and density =25kN/m$^3$

## III. PARAMETRIC STUDY

The behavior of longitudinal normal stress distribution across the outer web flange junction of box girder subjected to dead load and live load (IRC 70R) is been studied by varying various parameters namely the side cantilever length (a), the span (L) and the depth (D) of box girder.
The following values of various parameters are taken to carry out the analysis in SAP2000 as shown in Table 1

**Table 1:** Parameter adopted in the study

| Parameter | Range |
|---|---|
| Side Cantilever length (a) | 0.5, 1, 1.5, 2, 3, 3.5 & 4m |
| Span (L) | 20, 30, 40 to 80 m |
| Depth of Box Girder (D) | 3.4, 3, 2 &2.5m |

**3.1 EFFECT OF SHEAR LAG ON SPAN**

The following Figures 3, 4, 5 &6 modeled in SAP for cantilever length of 3.5m and depth 3.4m shows the variation of longitudinal normal stress distribution induced in flanges when the box girder is subjected to self-weight. It can be seen that for large cantilevers with increase in span length the severity of shear lag increases.

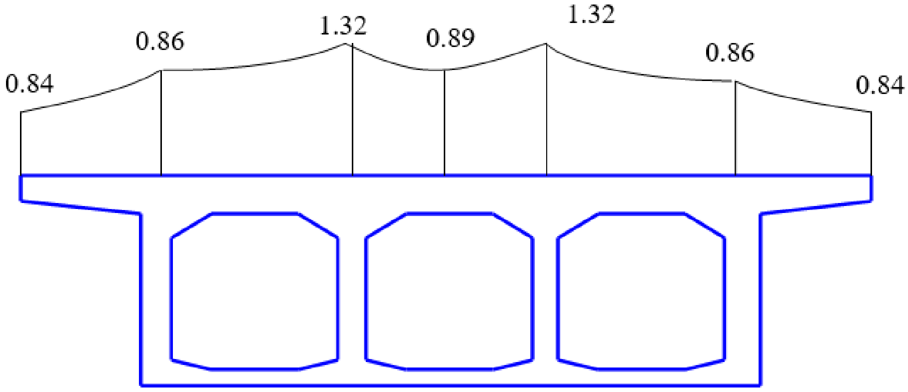



**Fig 3.** Normal Stress Distribution (MPa) at midspan for L=20m

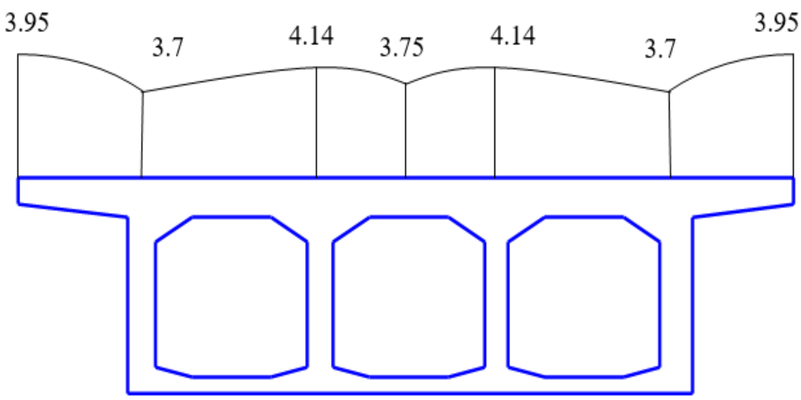



**Fig 4** Normal Stress Distribution (MPa) at midspan for L=40m

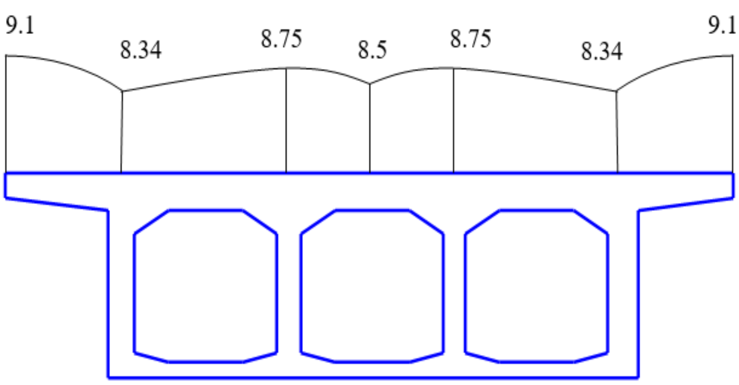



**Fig 5** Normal Stress Distribution (MPa) at midspan for L=60m

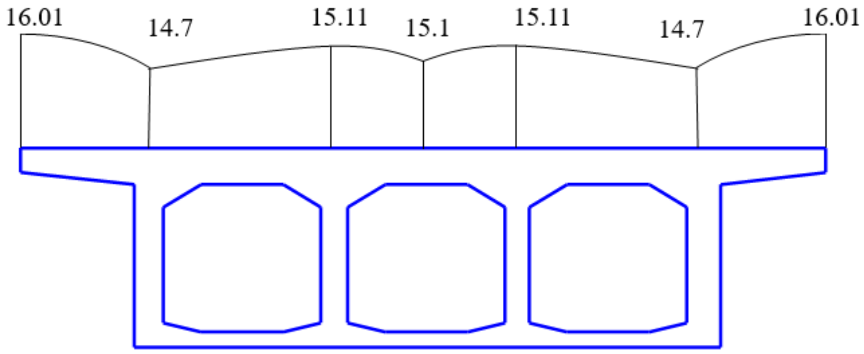



**Fig 6** Normal Stress Distribution (MPa) at midspan for L=80m

**3.2 EFFECT OF SHEAR LAG ON CANTILEVER LENGTH (a)**

**3.2.1 EFFECT ON SHORT SPAN**

The following Figures 7 & 8 modelled in SAP show the variation of longitudinal normal stress distribution induced in flanges when the box girder is subjected to self- weight for span of 20m and depth of 3.4 m

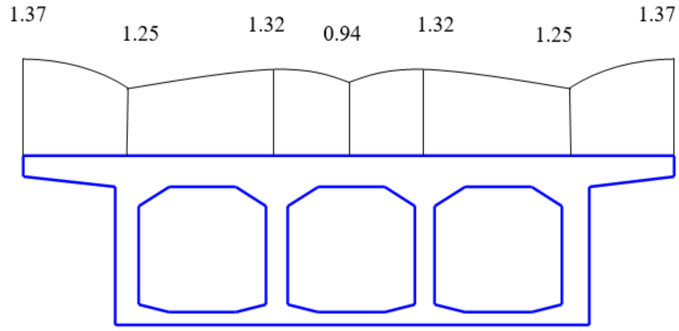



**Fig 7** Normal Stress Distribution (MPa) at midspan for a=1.5 m

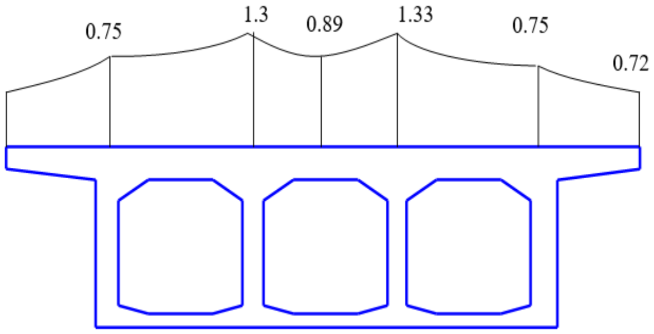



**Fig 8** Normal Stress Distribution (MPa) at midspan for a=4 m

**3.2.2 EFFECT ON LONG SPAN**

The following figure 9 & 10 modelled in CSI BRIDGE show the variation of longitudinal normal stress distribution induced in flanges when the box girder is subjected to self-weight for span of 80m and depth of 3.4 m

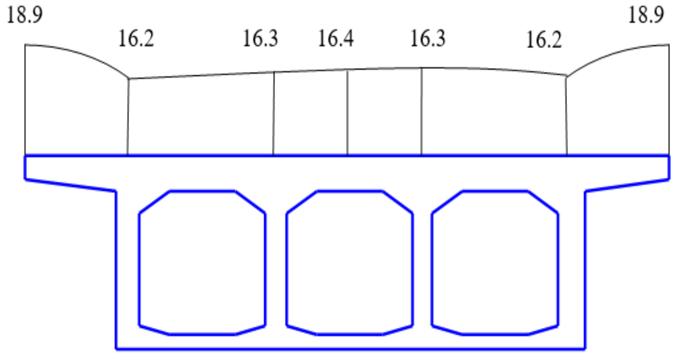



**Fig.9** Normal Stress Distribution (MPa) at midspan for L=80m & a=1.5m

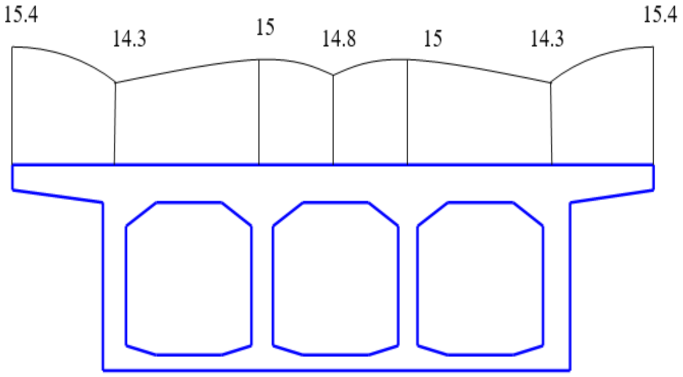



**Fig.10** Normal Stress Distribution (MPa) at midspan for L=80m & a=4m

For short span with large cantilever the full width is not effective in taking compression as seen in Figure 3&8 where cantilever tip stress is low than at other locations which indicates that that cantilever is not fully stressed since at tip of cantilever the section is reduced and hence should give higher stress value. For large span with large as well as short cantilever the full width is effective in taking compression as seen in Figures 4, 5, 6, 9 and 10 where cantilever is fully stressed thus showing high stress value due to reduced section at tip. Similar trend is observed for short span with short cantilever length as observed in Figure 7

### 3.3 DEAD LOAD ANALYSIS

***3.3.1 Effect of Span (L) on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder.***

Figure 11 shows the variation of stress ($\sigma_{wf}$) with span for different lengths of cantilever. It can be observed that with depth and cantilever length (a) constant, the longitudinal bending normal stress at web flange junction ($\sigma_{wf}$) increases with increase in span. Figures 11a to 11d predict the similar trend.

***3.3.2 Effect of cantilever length on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder.***

Figure 12 shows the variation of stress ($\sigma_{wf}$) with cantilever length for different spans. The stress ($\sigma_{wf}$) *decreases* with increase in cantilever length as the moment of inertia increases because of cantilever flanges. Similar trend is observed for spans in range 20 m to 80m from Figures 12a to 12d

***3.3.3 Effect of Depth of box girder on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder.***

Figure 13 shows the variation of stress ($\sigma_{wf}$) with depth of box girder for different spans. With cantilever length and span constant , the stress ($\sigma_{wf}$) *increases* with decreases in depth of box girder which contributes to the moment of inertia as seen in Figures 13a to 13f

### 3.4 LIVE LOAD ANALYSIS (with eccentricity of 70R wheeled loading)

***3.4.1 Effect of Span (L) on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder***

Figure 14 shows how the stress ($\sigma_{wf}$) varies with span as Class 70R wheeled vehicles moves along the bridge maintaining the edge distance of 1.2m from crash barrier. With depth constant, the stress ($\sigma_{wf}$) increases with increase in span for different cantilever lengths and is compressive in nature. But for larger cantilever length of 4m where wheel comes on outer web flange junction the stress ($\sigma_{wf}$) becomes tensile in nature (for span 20m to 60m) and is compressive for larger span of 70m and 80m as seen in Figures 14a to 14d

*3.4.2 Effect of cantilever length on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder.*

Figure 15 shows the variation of stress ($\sigma_{wf}$) with cantilever length for different spans. The stress ($\sigma_{wf}$) decreases with increase in cantilever as the vehicle eccentricity changes keeping depth and span constant and is compressive in nature. For larger cantilever length of 3.5m and 4m the stress changes its sign developing tension at the outer web flange junction as seen in Figures 15a to 15d

*3.4.3 Effect of Depth of box girder on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder.*

Figure 16 shows the variation of stress ($\sigma_{wf}$) with depth of box girder for different spans. With cantilever length and span constant the stress ($\sigma_{wf}$) increases with decreases in depth of box girder as seen in Figures 16a to 16f.

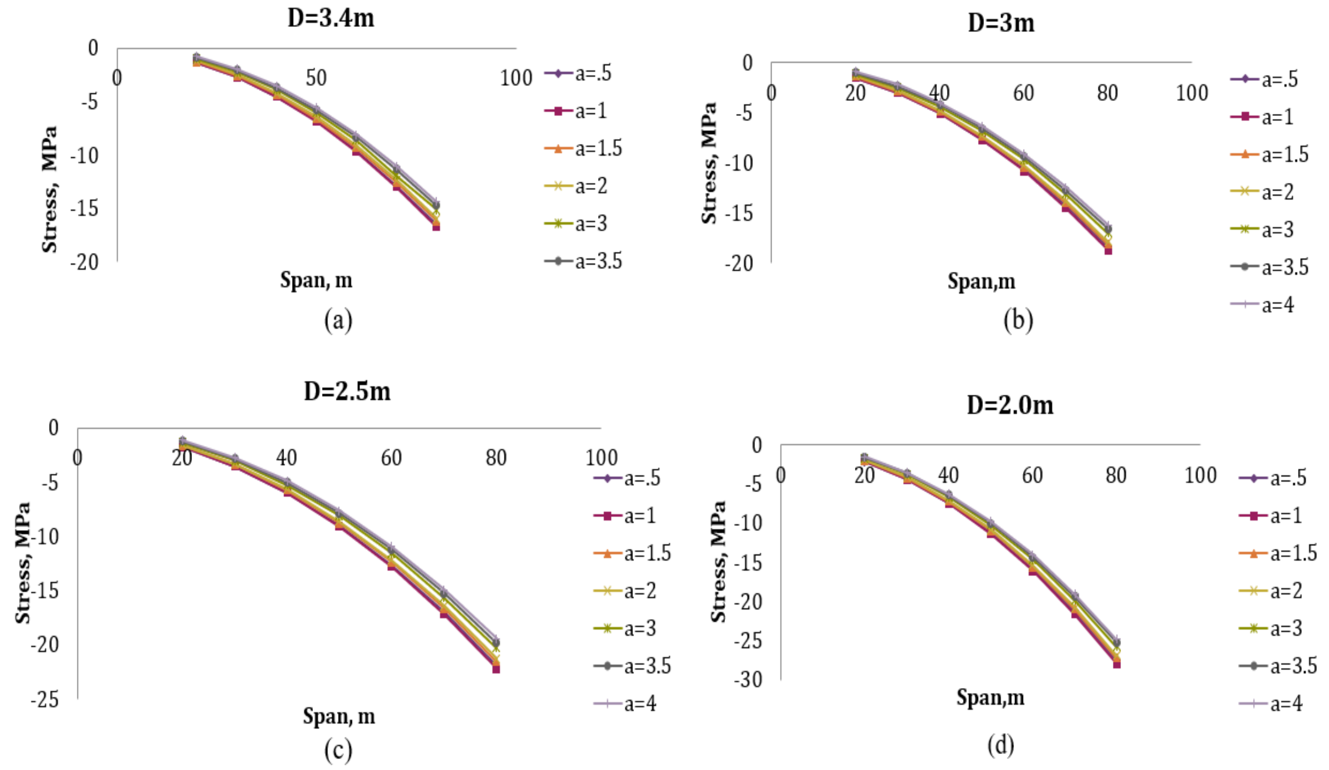



**Fig 11** (a) to (d) Effect of Span (L) on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder for dead load analysis

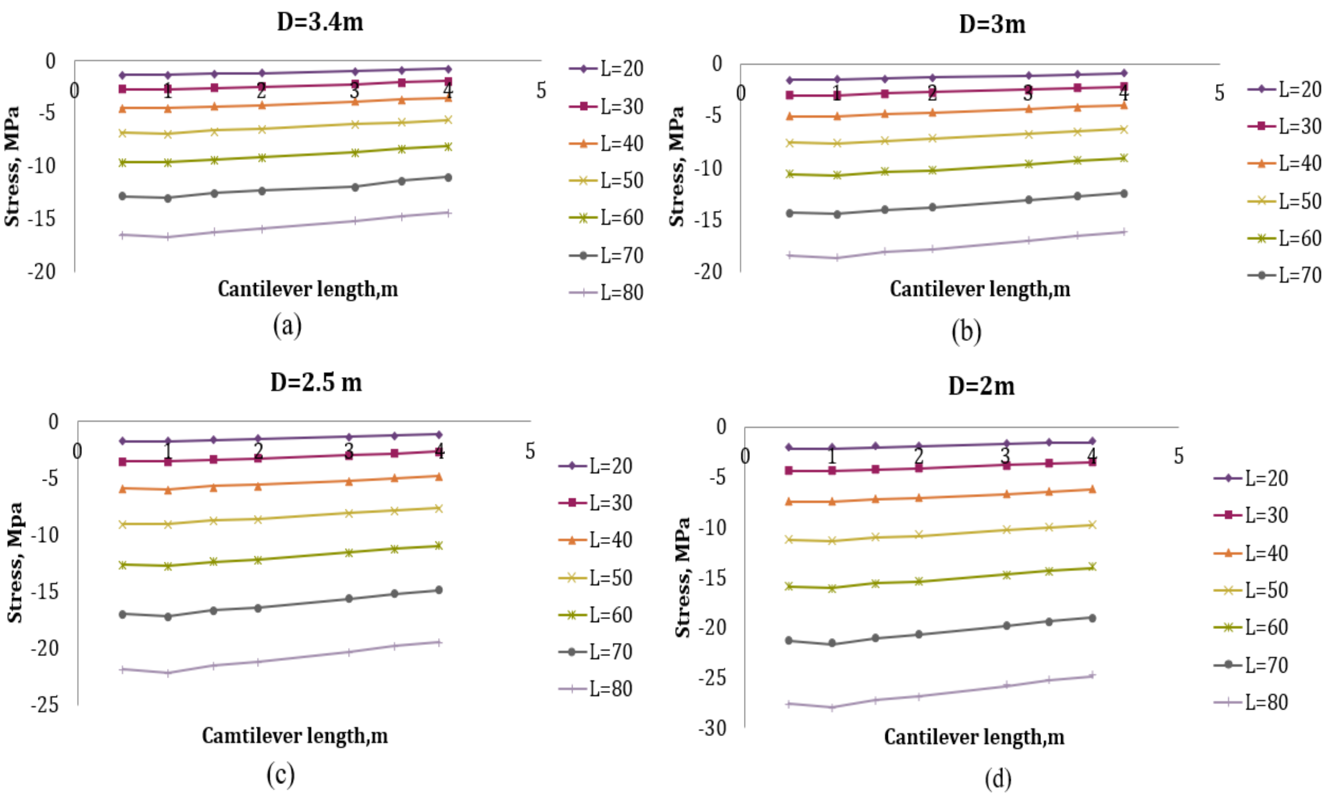



**Fig 12** (a) to (d) Effect of Cantilever length on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder for dead load analysis.

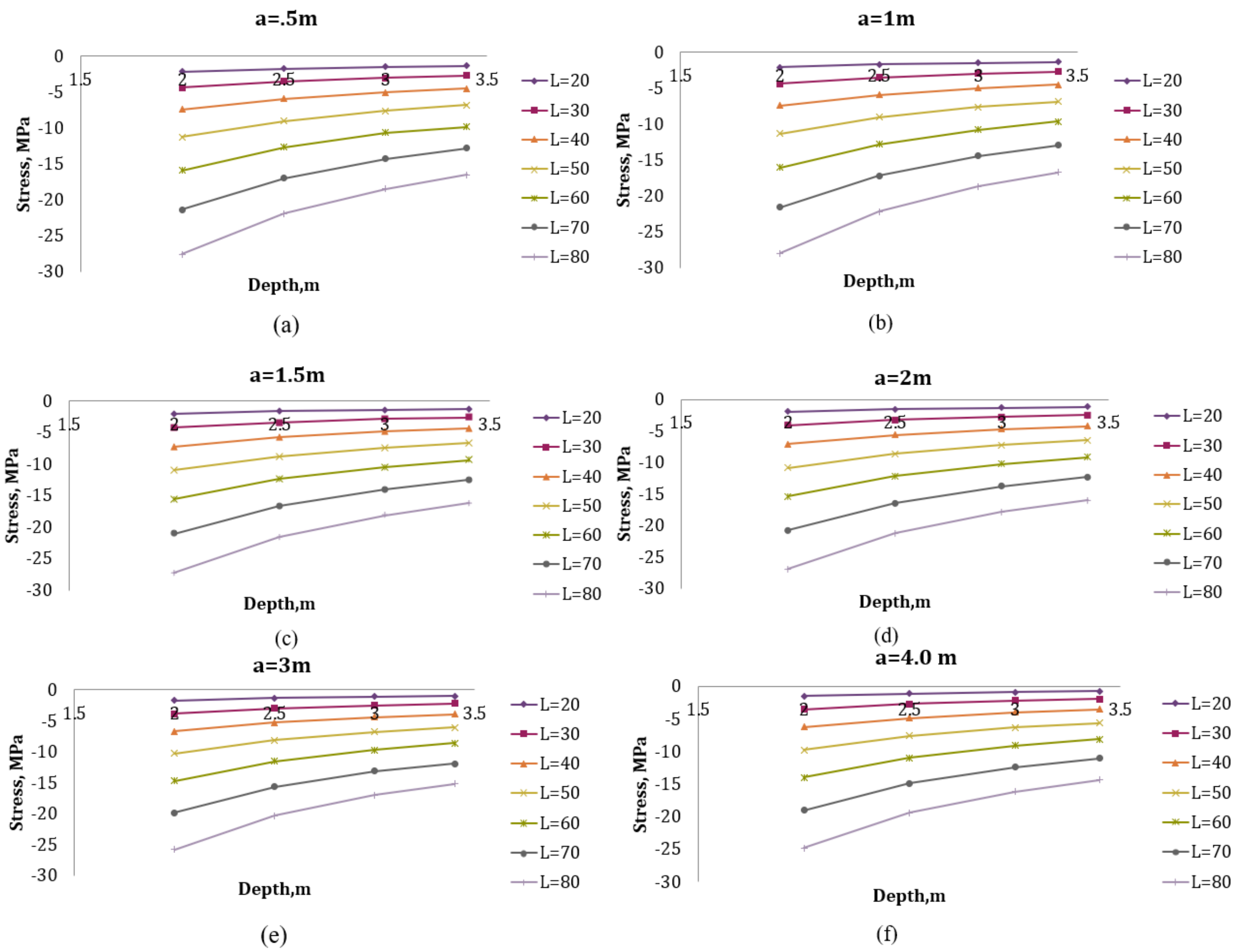



**Fig 13** (a) to (f) Effect of Depth on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder for dead load analysis

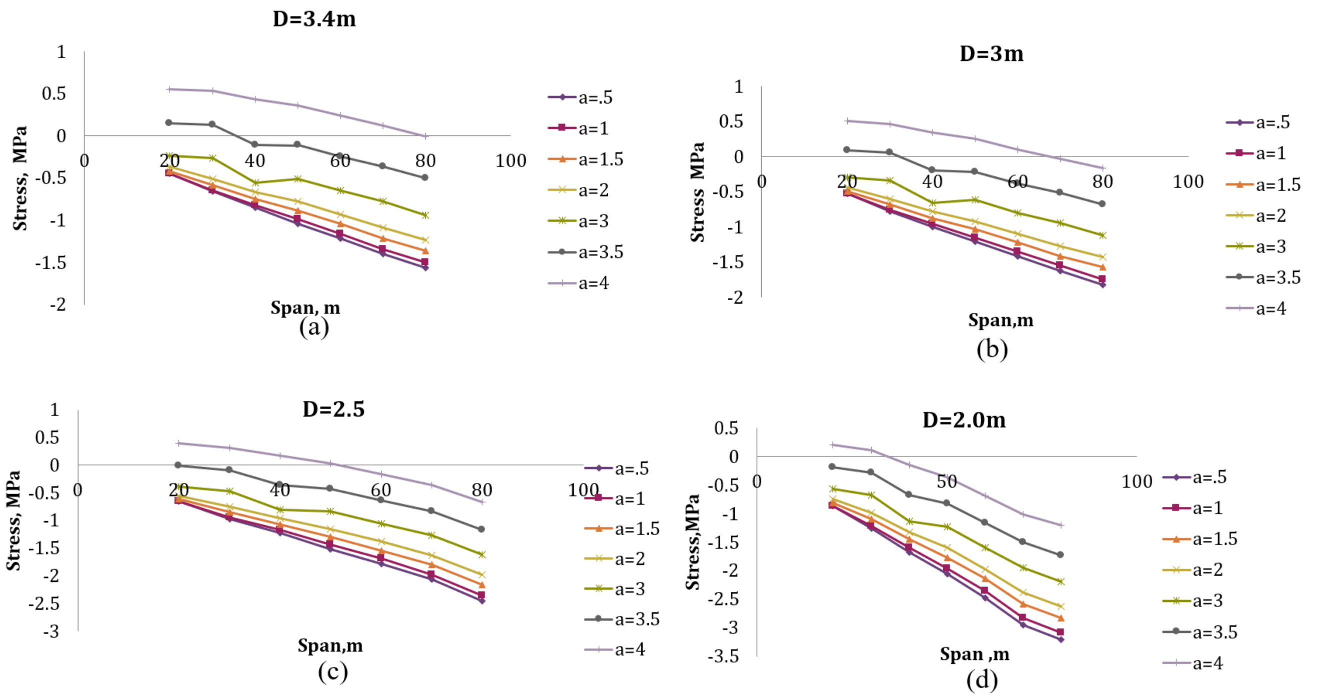



**Fig 14** (a) to (d) Effect of Span (L) on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder for live load analysis

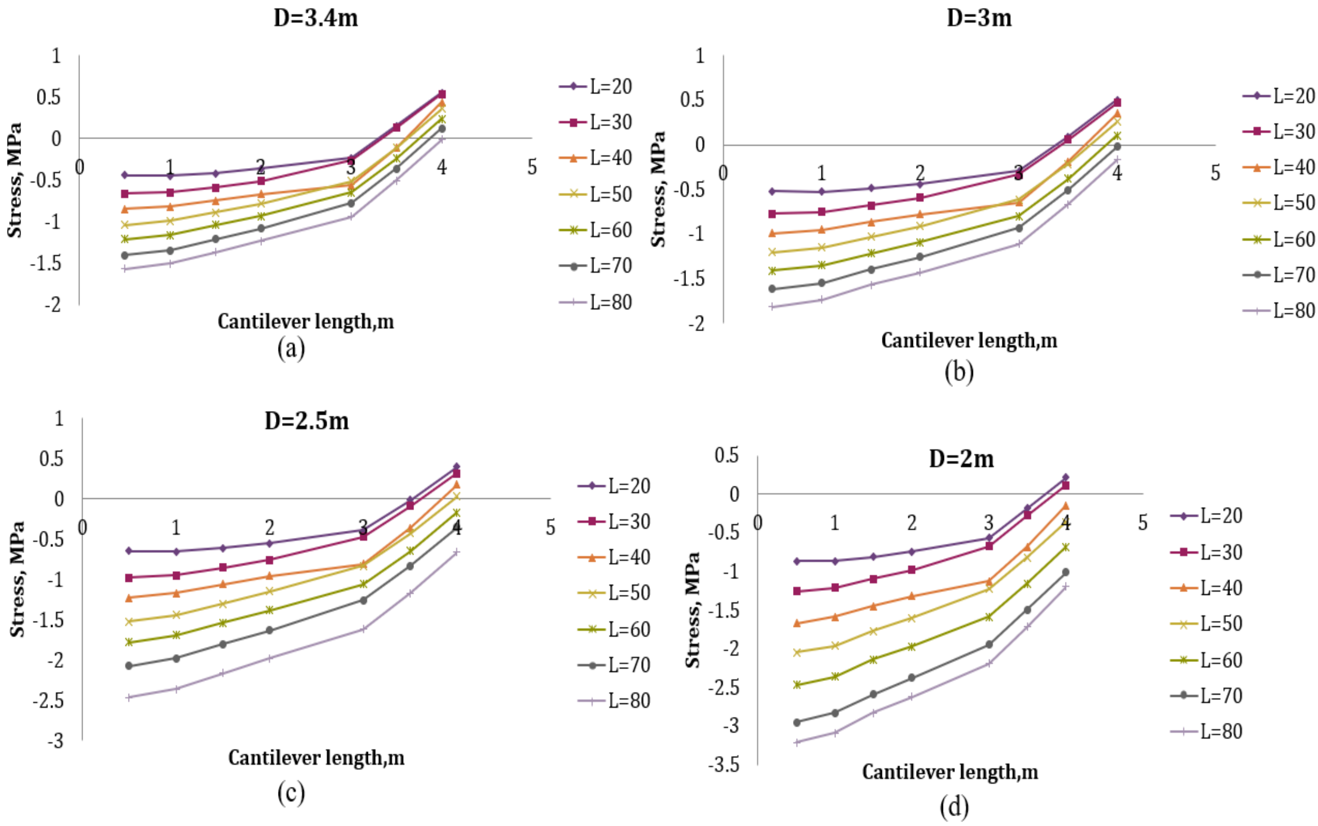
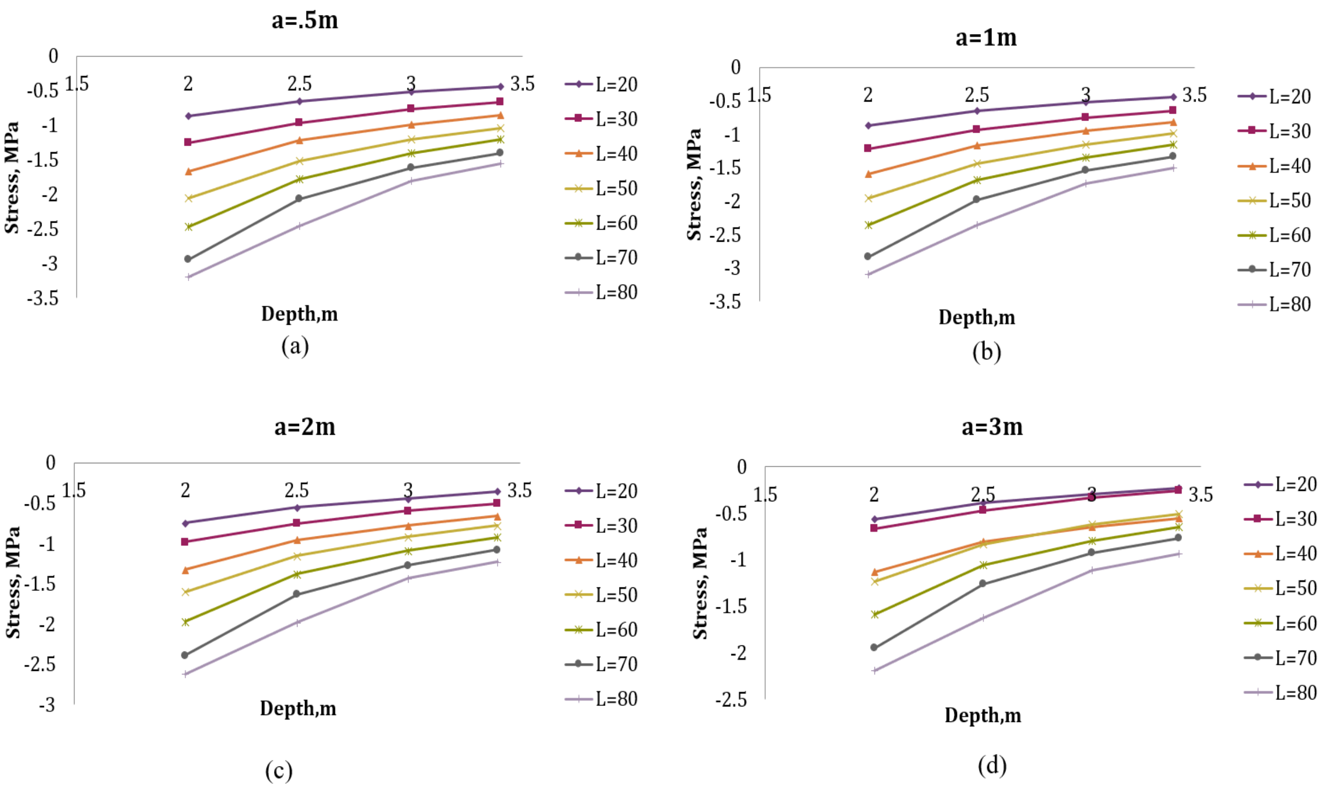
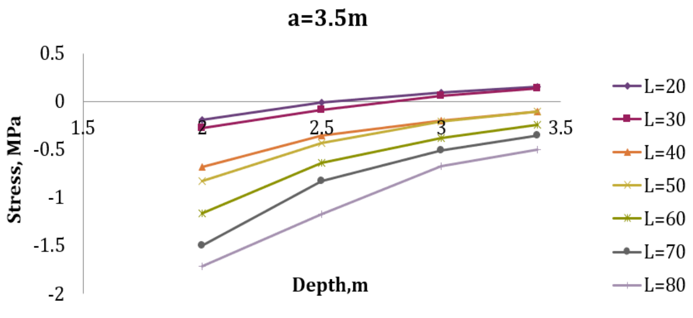
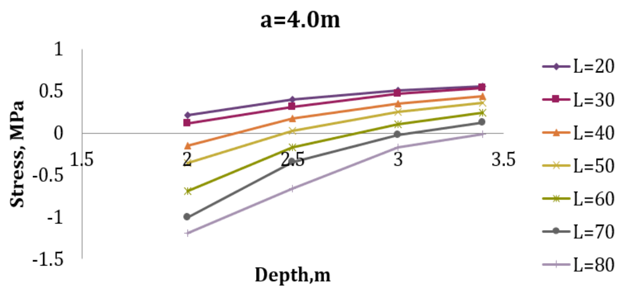



**Fig 15**(a) to (d) Effect of Cantilever length on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder for live load analysis






(e)




(f)

**Fig 16** (a) to (f) Effect of Depth on Longitudinal Bending normal stress at outer web flange junction ($\sigma_{wf}$) across the cross section of box girder for live load analysis

## IV CONCLUSIONS

1. The effect of shear lag is critical for short span and large width of box girder as the full width of box girder is not effective in taking compression.
2. For long span with short as well as large cantilevers negative shear lag is observed as full width is effective since stress at cantilever tip is high compared to outer web flange junction due to reduced thickness at cantilever tip.

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