

Stress Computation In Rigid Pavement

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Abstract — In a developing nation like India, the construction and improvement of high quality roads plays an important role. Highway pavement design plays an important role in the DPR projects. The satisfactory performance of the pavement will result in higher savings in terms of vehicle operating costs and travel time, which has a bearing on the overall economic feasibility of the project. Stresses in concrete pavement slabs are induced by wheel loads and by various temperature. The intensity and nature of stresses developed at different locations of the slab are dependent upon various factors, viz. magnitude of load, lateral placement of wheel loads, flexural strength of slab, temperature stresses, moisture, humidity and location of joints. Excessive stresses at the bottom of the concrete slab pavement produce crack. The design of rigid pavements have followed mechanistic-empirical (M-E) principles from the beginning. In M-E design methods, the slab thickness is determined by considering the fatigue failure of the slab due to damage cause by cumulative axel load passes. The input parameters related to sub-grade support, material characteristics, traffic load and climate. This paper discusses about the design methods that are traditionally being followed and examines the stresses. The stresses is calculated by three methods first - as per Westergaard and Bradbury approach, Second - using IIT-RIGID as per IRC guidelines which currently followed in field practice it is fatigue damage analysis approach. Third – By finite element method using software Ever Fe. It is observed that the stresses calculated from Westergaard is 4.85 MPa, Stress calculated using IIT-Rigid is 3.13 MPa and stresses available from finite element method using Software Ever FE is 2.396. The analysis results shows that stresses calculated form IIT-Rigid is more conservative.

Keywords- Finite element method (FEM), Ever Fe 2.24, Fatigue damage, Regression equations, Top down cracking, Bottom up cracking, IIT-Rigid, Westergaard's theory

I. INTRODUCTION

Rigid pavement are made-up of cement concrete to withstand heavy traffic loads in expressways, highways and runways. In early days, the thickness design of rigid pavement was based on empirical methods. With the advent of computers, researches many analytical methods have been widely used for rigid pavement design. The first guidelines published in 1974 for rigid pavements in India was based on only flexural stresses developed in concrete. Flexural strength of Pavement Quality Concrete (PQC) is one of the most important parameter in pavement design. After years passes the guidelines were revised to incorporate the cumulative fatigue damage criteria.

The cumulative fatigue damage of both bottom up cracking and top down cracking is influenced by various criteria. This paper studies, the thickness design of rigid pavement carried out by three methods. 1) As per Westergaard analysis which is based on load stress and warping stress. 2) As per IRC: 58- 2015 which is based on fatigue damage. 3) As per EVER Fe 2.24 software which is based on finite element method (FEM). A typical rigid pavement cross section is shown in Figure 1.

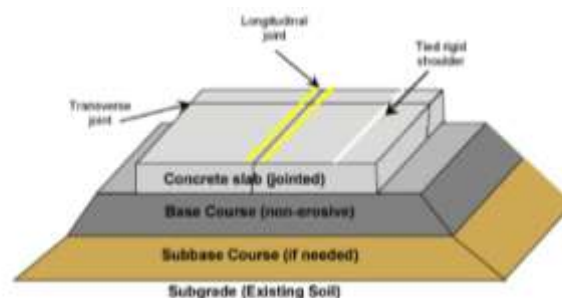


Figure 1 Rigid Pavement cross section

II. OBJECTIVES

1) The objective of this study is to understand the relative input parameters used in thickness design of plain jointed rigid pavement which is carried out as per Westergaard analysis 2) Stress calculation as IRC: 58-2015 3) Stress calculation as per EVER Fe 2.24 software. 4) Comparison of the stresses

III. THICKNESS DESIGN OF RIGID PAVEMENT

3.1 Design as per Westergaard approach

Westergaard theory consider three type of stresses first load stress due to wheel load, second warping stress due to daily temperature variation of top and bottom surface of slab and frictional stresses due to seasonal variations. Three typical locations considered in the analysis of rigid pavement i.e. interior, edge, and corner, critical combination of stresses is calculated. The design load is considered from data of traffic load survey-98th percentile load (which will be exceeded by only 2% of the maximum loads).

3.1.1 Design stipulation

Axle load survey data considered from illustrative example as calculated in IRC-58-2015, Appendix- VII. Panel size considered 3.5 m x 4.5 m

As per theory the responsible maximum stress for BUC is rear single axle load so the same is considered in the design in following example.

The maximum load of rear single axle is 190 kN. The design inputs is tabulated in the Table 1.

Table 1 Design Inputs for Westergaard

Axle load - P	19000 kg
	190 kN
Wheel load	9500 kg
Modulus of elasticity (E)	3.05 X 10 ⁵ Kg/cm ²
Thickness	28 cm
Poisson's ratio of concrete μ	0.15
CBR	8%
Radius of wheel load distribution	15 cm
Subgrade Reaction (K)	29.02 kg/m ³
	285 MPa/m
Thermal coefficient (ϵ)	10 X 10 ⁻⁶
Flexural strength of concrete	4.95 MPa
	48.56 Kg/cm ²
Density of concrete (γ)	24 kN/m ³

Radius of relative stiffness (l)

$$l = \left[\frac{Eh^3}{12K(1 - \mu^2)} \right]^{1/4}$$

$$l = \left[\frac{(3.05 \times 10^5) \cdot 28^3}{12 \times 29.02(1 - 0.15^2)} \right]^{1/4}$$

$$= 66.595 \text{ cm}$$

Radius of resisting section

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$= 14.92 \text{ cm}$$

3.1.2 Load stress calculation

Stress at interior

$$s_i = \frac{0.316p}{h^2} [4\log_{10}(l/b) + 1.069]$$

$$s_i = \frac{0.316 \times 9500}{28^2} [4\log_{10}(66.595/14.92) + 1.069]$$

$$= 14.04 \text{ kg/cm}^2$$

Stress at edge

$$s_e = 0.529 \frac{P}{h^2} (1 + 0.54\mu) \times [4\log_{10}(l/b) + \log_{10} b - 0.4048]$$

$$s_e = 0.529 \frac{9500}{28^2} (1 + 0.54(0.15)) \times [4\log_{10}(66.595/14.9) + \log_{10}(14.92) - 0.4048]$$

$$= 23.32 \text{ kg/cm}^2$$

Stress at corner

$$s_c = \frac{3P}{h^2} \left[1 - \left(\frac{a\sqrt{2}}{l} \right)^{1.2} \right]$$

$$s_c = \frac{3 \times 9500}{28^2} \left[1 - \left(\frac{15\sqrt{2}}{66.595} \right)^{1.2} \right]$$

$$= 27.12 \text{ kg/cm}^2$$

3.1.3 Warping stress calculation

Day time temperature = 16.8

Night time temperature = 18.4

Stress at interior

$$st_{(i)} = \frac{Eet}{2} \left[\frac{c_x + \mu c_y}{1 - \mu^2} \right]$$

Here, $L_x = 4.5\text{m}$, $L_y = 3.5\text{m}$ $e = 10 \times 10^{-6}$

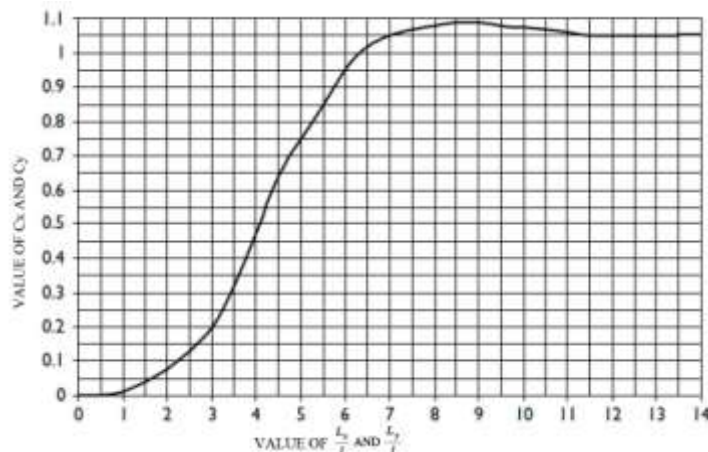


Figure 2 Warping stress coefficient chart

$$C_x = \frac{L_x}{l}$$

$$C_x = \frac{450}{66.595}$$

$$= 6.75$$

As per Figure 2,

$$C_x = 0.96$$

$$c_y = \frac{350}{66.595}$$

$$c_y = \frac{L_y}{1} = 5.25$$

As per Figure 2,

$$C_y = 0.96$$

$$st_{(i)} = \frac{Eet}{2} \left[\frac{c_x + \mu c_y}{1 - \mu^2} \right]$$

$$st_{(i)} = \frac{3.05 \times 10^5 \times 10 \times 10^{-6} \times 16.8}{2} \left[\frac{0.6 + (0.15 \times 0.25)}{1 - 0.15^2} \right]$$

$$= 28.65 \text{ kg/m}^3$$

Stress at corner

$$st_{(c)} = \frac{c_x Eet}{3(1 - \mu)} \sqrt{\frac{a}{l}}$$

$$st_{(c)} = \frac{3.05 \times 10^5 (10 \times 10^{-6}) \times (13.4)}{3(1 - 0.15)} \sqrt{\frac{15}{66.595}}$$

$$= 7.60 \text{ kg/cm}^2$$

3.1.4 Frictional Stress

$$s_f = \frac{(WL_c f)}{(2 \times 10^4)}$$

$$s_f = \frac{(2400 \times 4.51 \times 1.1)}{(2 \times 10^4)}$$

$$= 0.594 \text{ kg/cm}^2$$

Table 2 Stresses as per location

Stress location in slab panel	Load stress in Kg/cm ²	Warping in Kg/cm ²	Frictional stress in Kg/cm ²
Edge	23.32 (S _e)	24.59 (St _e)	0.594 (S _f)
Corner	27.12 (S _e)	7.60 (St _e)	0.594 (S _f)
Interior	14.04 (S _i)	28.65 (St _i)	0.594 (S _f)

3.1.5 Critical combination of stress

During summer mid-day

$$\begin{aligned} &= S_e + S_{te} - S_f \\ &= 23.32 + 24.59 - 0.594 \\ &= 47.32 \text{ kg/cm}^2 \end{aligned}$$

During winter mid-day

$$\begin{aligned} &= S_e + S_{te} + S_f \\ &= 23.32 + 24.59 + 0.594 \\ &= 48.50 \text{ kg/cm}^2 \end{aligned}$$

During summer mid-day

$$\begin{aligned} &= S_e + S_{te} \\ &= 23.32 + 24.59 \\ &= 47.91 \text{ kg/cm}^2 \end{aligned}$$

As shown in above calculation the stress available during winter mid-day is highest among all the three stresses. It is considered as critical.

$$\begin{aligned} \text{Critical Stress} &= 48.50 \text{ kg/cm}^2 \\ &= 4.85 \text{ MPa} \end{aligned}$$

3.2 Design as Per IRC 58-2015 without concrete shoulders

The pavement slab thickness has two mode of failures-1).Bottom-Up fatigue Cracking (BUC) and 2). Top-Down fatigue Cracking (TDC).

3.2.1 Bottom up fatigue cracking

During the day hours, the top surface of slab is hotter than the bottom surface so the slab tend to warp in a convex shape. Thus, when the slab is warped as per **Figure 3**, higher tensile stresses are repeated due to rolling wheel loads causing higher fatigue damage.

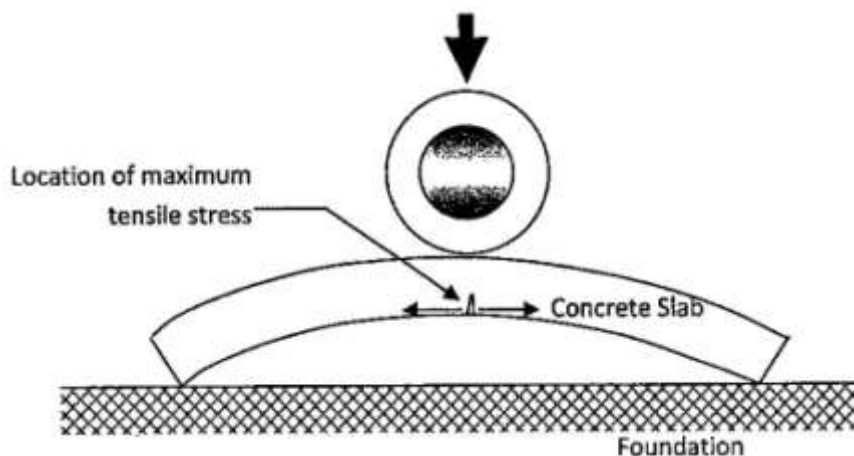


Figure 3 Axle load placed in the middle of the slab during day time

3.2.2 Top down fatigue cracking

During the night hours, the top surface is cooler than the bottom surface and the ends of the slab curl up in a concave shape resulting in loss of support for the slab. **Figure 4** shows the placement of axle loads close to transverse joints during night period causing high flexural stresses in the top layer leading to top-down cracking.

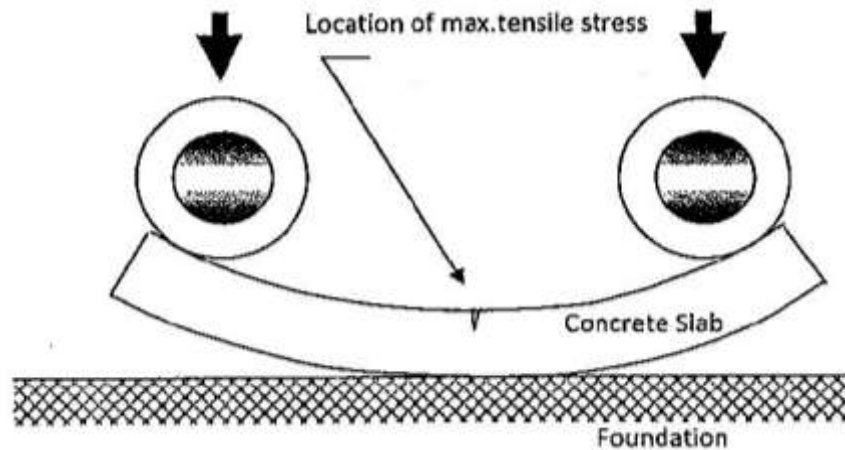


Figure 4 Placement of two axles of a commercial vehicle on a slab curled during night hours

Table 3 Design inputs

Parameter	Input
Design life (years)	30
Effective CBR of subgrade soil (%)	8
Design traffic (CVPD)	4500
Grade of concrete	M40
Traffic growth rate (%)	7.5
Traffic during night (%)	60
Temperature zone	4
Type of sub-base	Untreated Granular Subbase
Use of dowel bars	Yes
Average number of axles per commercial vehicle	2.35

Table 4 Axle load spectrum

Load Case	Single axle	
	Axle load class kN	Frequency (% of single axles)
1	185-195	18.15

- a) Selection of modulus of subgrade reaction
 - Effective CBR of compacted subgrade = 10 %
 - Modulus of subgrade reaction = 50.3 MPa/m (Table 2, IRC:58-2015)
 - Assumed thickness of untreated granular sub base = 150mm - Provide DLC thickness = 150mm
 - Effective modulus of subgrade reaction of combined foundation (Subgrade + Subbase), $k = 285 \text{ MPa/m}$ (From Table 3, IRC:58-2015)
 - 28 day cube compressive strength of cement concrete = 40 MPa
- b) Selection of flexural strength of Concrete
 - 28 day flexural strength of concrete = 4.5 MPa
 - 90 day flexural strength of concrete = $1.1 \times 4.5 = 4.95 \text{ MPa}$
- c) Selection of Design traffic for Fatigue analysis
 - Annual rate of growth of commercial traffic = 0.075 (assumed)
 - Two-way commercial traffic volume per day = 6000 commercial vehicles/day
 - Percentage of traffic in predominant direction = 50% (3000 CVs in each direction)
 - Total two way commercial vehicles during design period
 - $C = 365 \times 6000 \times ((1 + 0.075)^{30}) / 0.075 = 22,64,44,692 \text{ CVs}$
 - Average number of axles (steering/single/tandem/tridem) per commercial vehicle = 2.35
 - Total two-way axle load repetitions during the design period = $22,64,44,692 \times 2.35 = 53,21,45,025 \text{ axles}$

- Number of axles in predominant direction = $53,21,45,025 \times 0.5 = 26,60,72,513$
- Design traffic after adjusting for lateral placement of axles (25 percent of the total two-way commercial traffic for two-lane two-way carriageway) = $26,60,72,513 \times 0.25 = 6,65,18,128$ (Clause no. 5.5.2.3, IRC-58:2015)
- Day time (12-hour) design axle repetitions = $66,518,128 \times (1-0.6) = 2,66,07,251$ (40% traffic during day time)
- Day time Six-Hour axle load repetitions = $2,66,07,251 / 2 = 1,33,03,626$
- Hence, design number of axle load repetitions for bottom-up cracking analysis = $1,33,03,626$
- Night time (12-hour) design axle repetitions = $66,518,128 \times 0.6$ (60% traffic during night time) = $3,99,10,877$
- Night time Six-Hour axle load repetitions $3,99,10,877 / 2 = 1,99,55,439$
- % of commercial vehicles having the spacing between the front (steering) axle and the first axle of the rear axle unit less than 4.50 m = 55%
- Hence, design number of axle load repetitions for Top-down cracking analysis = $1,99,55,439 \times 0.55 = 1,09,75,491$

The design axle load repetitions for bottom-up and top-down fatigue cracking analysis are given in **Table 5**.

Table 5 Design axle load repetitions

Axle Category	Proportion of the axle category	Number of axles Responsible for	
		Top down cracking	Bottom up cracking
Single axle	15%	1995544	1646324

- d) Cumulative Fatigue Damage (CFD) analysis for Bottom-Up Cracking (BUC) and Top-Down Cracking (TDC) and Selection of Slab Thickness

- Concrete pavement with tied concrete shoulder with dowel bars across transverse joint.
- Max. Day-time Temperature Differential in slab = 16.8°C
- Night-time Temperature Differential in slab = $\text{day-time diff} / 2 + 5 = 13.4^{\circ}\text{C}$
- Elastic modulus of concrete (E) 30000MPa
- Radius of relative stiffness, $I = (Eh^3 / (12k(1-\mu^2)))^{0.25} = 0.66621\text{ m}$

- e) Flexure stress calculation for bottom up crack using regression equation for single axle without concrete shoulders.

- (Considering $k > 150\text{ MPa/m}$ as per IRC 58-2015 Appendix-V Eqn. no V.6) considering radius of relative stiffness 0.66 m (as calculated in Westergaard approach). The beta factor B is 0.66 for transverse joint with dowel bar

$$S = -0.219 + 1.686 \frac{BPh}{kl^4} + 168.48 \frac{h^2}{kl^2} + 0.1089 \Delta T$$

$$S = -0.219 + 1.686 \frac{0.66 \times 19080.28}{285 \times 0.66^4} + 168.48 \frac{0.28^2}{285 \times 0.66^2} + 0.1089 \Delta 13.4$$

$$S = 2.399\text{ MPa}$$

The summarized result of flexural stress and fatigue damage for bottom up and top down cracking are computed in **Table 6** and **Table 7**.

Table 6 Analysis for bottom-down cracking

Single axles				
Expected rep. (Ni)	Flex. Stress MPa	Stress Ratio (SR)	Allowable Rep. (Ni)	Fatigue Damage
362191	3.13	0.506	588331	0.616

Table 7 Analysis for Top-Down Cracking

Single axles				
Expected rep. (Ni)	Flex. Stress MPa	Stress Ratio (SR)	Allowable Rep. (Ni)	Fatigue Damage
298808	2.399	0.485	1768731	0.169

- The flexural stress for max. load for Bottom up Cracking is 3.13 MPa and for Top down cracking is 2.399 MPa.

3.3 Design as per EVER Fe 2.24 software

Ever FE (current version 2.24) is a user friendly 3D finite-element analysis tool for simulating the response of jointed plain concrete pavement (JPCP) systems to axle loads and environmental effects. Ever FE is useful for both concrete pavement researchers and designers who must perform either complex nonlinear or simple linear stress analyses of JPCP. The Universities of Maine and Washington jointly developed EverFE with funding from the Washington and California State Departments of Transportation. EverFE is available free from this website to any interested users. The software (open source software) is based on finite element method and in the analysis it is used to calculate the stresses for Top down and bottom up cracking.

3.3.1 Data inputs for Ever Fe-stresses calculation

Table 8 Geometry considered

Number of slab panels	1
Length of Panel	4.50 m
Width of Panel	3.50 m
Thickness of PQC Slab	280 mm
Thickness of DLC	150 mm
Base & Subgrade	1 layer option is selected.

Table 9 Material properties

Modulus of Elasticity, E	30000MPa
Poisson's Ratio, μ	0.15
Co-efficient of Linear Expansion, alpha	10e-006
Density of concrete	2400 kg/m ³
Modulus of Elasticity of DLC	13600 MPa
Poisson's Ratio	0.20

Table 10 Slab base interface

Bound Base	In-click check-box
Initial Stiffness	0.0mpa/mm for frictionless interface
Slip Displacement	0 mm

Table 11 Dense liquid subgrade

Tensionless	Un-click check-box
K value for Subgrade	0.0515mpa/mm

Table 12 Axle loading for BUC

Parameters	Inputs
Load (kN)	95
X (mm)	2250
Y (mm)	-700
Z (mm)	430
W (mm)	296
A (mm)	1800
Day time Temperature Change in Slab-1 & 2	16.8 ⁰ c & 0 ⁰ c

Table 13 Axle loading for TDC

Parameters	Inputs
Load (kN)	47.5 (FA*) 95 (TA**)
X (mm)	215 (FA) 4261 (TA)
Y (mm)	-689 (FA) -770 (TA)
Z (mm)	308(FA) 436(TA)
W (mm)	212(FA) 300(TA)
A (mm)	1800(FA) 1800(TA)
Day time Temperature Change in Slab-1 & 2	16.8 ⁰ c & 0 ⁰ c

* Front axle ** Tandem axle

Table 14 Finite Element meshing

Number of Elements along X in column	3
Number of Elements along Y in row	3
Number of Elements along Z in slab	2
Number of Elements along Z in subgrade	1

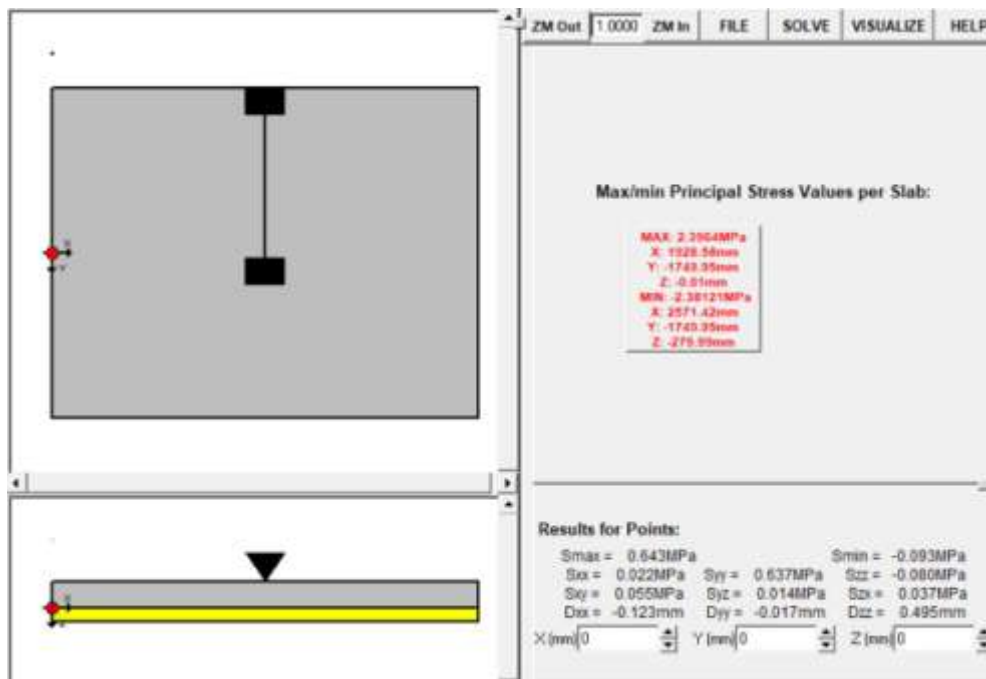


Figure 5 Stresses for bottom up cracking

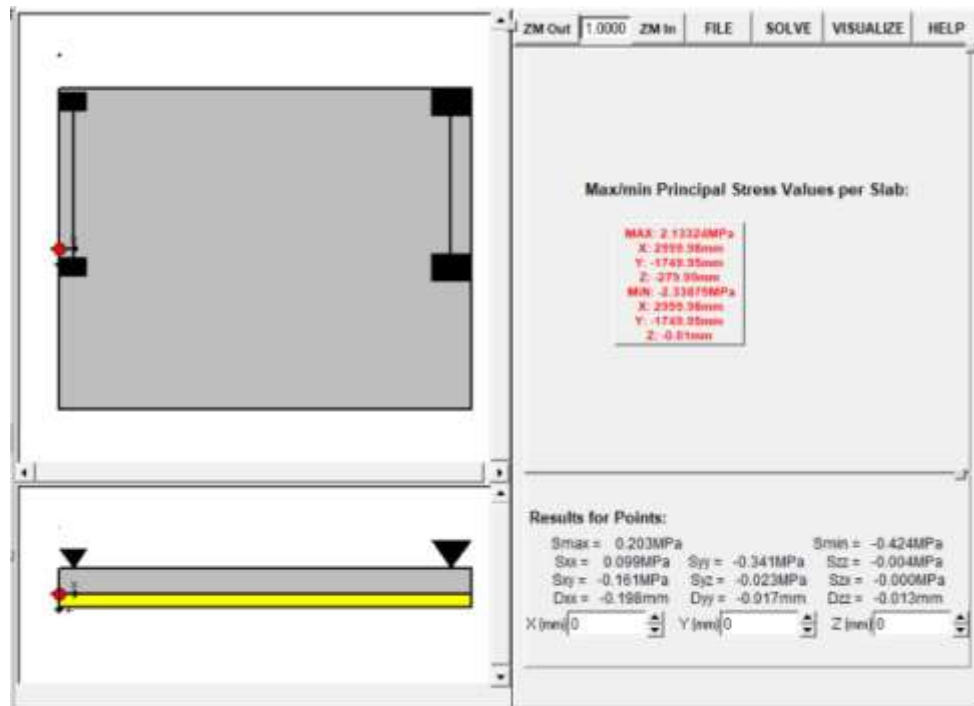


Figure 6 Stresses for top down cracking

- The maximum flexural stress of 2.3964 MPa at the edge of longitudinal joint and bottom of slab
- The maximum flexural stress of 2.1332 MPa at the edge of longitudinal joint and at top level of slab

IV. RESULTS

Table 15 Comparison of stress result

Software Programs	Flexural stress in MPa	
	BUC	TDC
Westward's Theory (Panel 3.5m x 4.5m x 0.28m)	4.85	-
IIT RIGID – 2015 (Panel 3.5m x 4.5m x 0.28m)	3.13	2.399
Ever - Fe - 2.24 (Panel 3.5m x 4.5m x 0.28m)	2.396	2.133

V. CONCLUSION

1. Thickness Design of rigid pavement carried out by three methods and stresses are calculated and observed that, stress calculated by Westward's Theory is 4.85 MPa, by IIT Rigid – 2015 the responsible stress for BUC is 3.13 MPa and stresses for TDC is 2.399 MPa and stresses by Ever - Fe is 2.24 responsible stress for BUC is 2.396 MPa and stresses for TDC is 2.133 MPa
2. The stress analysis shows that stress calculated by fatigue damage approach using IIT-Rigid is 3.13 MPa which is maximum compared to other methods
3. The maximum stress calculated by IIT-Rigid is about 30 % higher than EVER-Fe which is finite element base method, which shows that IIT-Rigid is more conservative than other methods

VI. REFERENCE

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