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PID Control of High-Order Non-minimum Phase Systems using Advantages of Model Reduction

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Abstract — *In this paper, a simple controller design method is presented for design of a higher order non minimum phase (NMP) systems. First the original higher order NMP system in reduced to a lower order NMP model using a model reduction method. Then a Proportional Integral Derivative (PID) controller is designed for the obtained lower order model. The Controller thus obtained is attributed to the original NMP system for its control. The model reduction method employed is based on Singular Perturbation Approximation (SPA) method and the controller design is on Magnitude Optimum Multiple Integration (MOMI) method. The validity of the proposed design method is supported through a numerical example.*

Keywords- *controller design, model reduction, PID controller, singular perturbation approximation, magnitude optimum.*

I. INTRODUCTION

Many stable industrial process control systems, hydraulic turbines, torpedoes, airplanes are characterized by NMP characteristics having right-half s-plane zeros. It is a well known fact that such NMP systems are difficult to control. Earlier work in the control of NMP systems, include control using adaptive control, predictive control and evolutionary soft computing techniques [1 – 4]. Most of these practical systems when mathematical modeled result in high order transfer functions. Controller design of such high order NMP systems throws out an open challenge for continuous research in this domain as designing controllers for higher order systems is a more complicated task. There is a dire need to carry out further work in this direction as control of higher order systems with NMP characteristics poses a much larger problem. The problem of controlling high order systems can be partially solved by designing controllers using model reduction techniques. For the last few decades Model Order Reduction (MOR) and application of MOR techniques to real time systems has been an important subject of research. A lot of such methods were earlier suggested both in the time domain as well as in the frequency domain [5 – 14] each of them having its own domain of applicability. The controller is first designed from the reduced order model which is constructed from the original high order system or plant via a model reduction procedure and then the so obtained controller is attributed for the original system. Presently, most industrial processes and practical systems are controlled by (PID) controllers. The popularity of PID controllers is due to their simplicity in design and parameter tuning. Very intense work in PID tuning was reported in the literature. Here, the original high order NPM system to be controlled is first approximated using SPA based model reduction method. A PID controller with a derivative filter is designed with the help of reduced order model (ROM) using MOMI method which is then used to control original system by cascading it in its forward path under unity feedback. Section I deals with introduction, section II with internal balancing of systems, overview of SPA theory of model reduction and MOMI method. In Section III, the actual design procedure is mentioned, Section IV deals with a numerical example followed by Section V Conclusions and the remainder with references.

II. MODEL REDUCTION

2.1) Internal Balancing of Continuous-Time Systems

If (A,B,C,D) is an n-th order stable system that is both controllable and observable, then it is known that there exists a transformation such that the transformed controllability and Observability Grammians are equal to a diagonal matrix Σ. Such a realization is generally referred to as *balanced realization* or *internally balanced realization*. Obtaining an internally balanced realization is a preliminary step to SPA model reduction method. Stepwise procedure to obtain balanced realization is as follows.

Let (A,B,C) be a state space representation of a strictly proper continuous –time system with an assumption that (A,B) is controllable,(A,C) is observable, and A is stable, T, a non singular balancing transforming matrix. All the matrices carry proper dimensions.

Step1: Controllability and Observability Grammians (C_G, O_G) are computed by solving, respectively, the Lyapunov equations:

$$AC_G + C_G A^T + BB^T = 0 \qquad (2.1a)$$

and

$$A^T O_G + O_G A + C^T C = 0, \tag{2.1b}$$

Step 2: Cholesky factors L_c and L_o of C_G and O_G are found out:

$$C_G = L_c L_c^T \tag{2.2a}$$

and

$$O_G = L_o L_o^T \tag{2.2b}$$

Step 3: SVD of the matrix $L_o^T L_c$ is found :

$$L_o^T L_c = U\Sigma \ V^T \tag{2.3}$$

Step 4:

$$\Sigma^{-1/2} = \mathrm{diag}\left(\frac{1}{\sqrt{\sigma_1}}, \frac{1}{\sqrt{\sigma_2}}, \ldots, \frac{1}{\sqrt{\sigma_n}}\right) \tag{2.4}$$

is computed where $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$ with positive entries.

Step 5: Transforming matrix is computed as

$$T = L_c V \Sigma^{-1/2} \tag{2.5}$$

Step 6:matrices of the balanced realization are computed as

$$\tilde{A} = T^{-1}AT, \tilde{B} = T^{-1}B, \tilde{C} = CT \tag{2.6}$$

2.2) Singular Perturbation Approximation

Singular Perturbation Approximation is a kind of *state truncation* and *residualization* method which has the advantage of matching steady state value of reduced order model with that of the original, when compared with the simple *balanced truncation* technique which introduces steady state errors between original and the model. The overview of the method can be found in [13] and [14].

Suppose $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ is the balanced realization of any linear time invariant system, and an r-th order approximant is required, then the matrices are to be partitioned as

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{2.7}$$

with

A_{11} as r x r matrix (r < n).

The state space realization of r-th order stable approximant (A_r, B_r, C_r, D_r) is given by the following equations

$$A_r = A_{11} - A_{12}A_{22}^{-1}A_{21} \tag{2.8a}$$

$$B_r = B_1 - A_{12}A_{22}^{-1}B_2 \tag{2.8b}$$

$$C_r = C_1 - C_2 A_{22}^{-1}A_{21} \tag{2.8c}$$

$$D_r = D - C_2 A_{22}^{-1}B_2 \tag{2.8d}$$

The corresponding transfer function of the reduced model is given by

$$\tilde{R}(s) = C_r(sI - A_r)^{-1}B_r + D_r \tag{2.9}$$

The inherent property of the SPA method is that, in general, it results in proper and non minimum phase reduced models. The latter property seems to be an advantage for considering this method for reduction of NMP systems. The former property is a disadvantageous one in the present context of controller design. Most of the practical systems mathematically modeled result in strictly proper transfer function i.e. the pole- zero difference is not zero. In order to obtain strictly proper transfer reduced order models, $\tilde{R}(s)$ is modified to $R(s)$ by discarding some of the non-dominant zeros and retaining only dominant zeros. If the pole–zero difference of the reduced model is greater than one, then the approximation tends to be poorer. It is suggested that the difference be equal to one. In case of complex conjugate pair of zeros , then their real parts are only considered and the most dominant ones are retained with only their real parts.

2.3) Magnitude Optimum Multiple Integration Method

It is a modified version of the original magnitude optimum(MO) method originally suggested for process control[15]. Recently the MO method is improved using the concept of 'moments' originated from the identification theory. This is further enhanced to the so called MOMI method [16] which multiple integrals to calculate the PID parameters. Let the transfer function of the stable LTI system to be controlled be defined as

$$G(s) = \frac{a_0 + a_1 s + ... + a_m s^m}{b_0 + b_1 s + ... + b_n s^n}, \ (m<n) \tag{2.10}$$

The above transfer function can be expanded around s=0 as an infinite series in positive powers of 's' as

$$G(s) = A_0 - A_1 s + A_2 s^2 - ...... \tag{2.11}$$

The parameters $A_i (i = 0,1,2,3,....)$ are called time-weighted integrals of the impulse response of the system. Finding the parameters from the impulse response by applying an exact impulse input to the system is rather difficult. Therefore the above defined 'moments' can be obtained by repetitive (multiple) integrals of the systems input and output during the change of steady state [16]. It is also suggested in literature that the moments can be directly calculated from the coefficients of the system transfer function [19,20].In MOMI method, only the first six coefficients are required to realize the PID parameters. The moments $A_i (i = 0,1,2,3,4,5)$ are computed as

$$A_i = (-1)^{i+2} c_i \quad i= 0,1,2,3,4,5 \tag{2.12}$$

where c_i s are calculated using the following recursive algorithm

$$c_0 = \frac{a_0}{b_0}, c_k = \left[a_k - \sum_{j=1}^{k} b_j \cdot c_{k-j} \right], \tag{2.13}$$

k=1,2,3,4,5

III. CONTROLLER DESIGN

PID controller design entails the following two main steps:Finding a reduced order model of the original high order system to be controlled; Realizing a controller with the help of the reduced model and attributing it to the original high order system. The above steps are elaborated below.

3.1) Algorithm for Finding Reduced Order Model

The procedure to obtain reduced order model suitable for controller design is given in the following stepwise algorithm.

Step 1: Define the transfer function of high order NMP system to be controlled $G(s)$ as in eqn.(2.10)

Step 2: Obtain its balanced realization $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ using eqns. (2.1) – (2.6)

Step 3: Obtains its reduced order model (A_r, B_r, C_r, D_r)applying SPA method using eqns.(2.7- (2.8)

Step 4: Obtain the corresponding transfer function of the reduced model $\tilde{R}(s)$ and its modified version $R(s)$.

3.2) Realizing Controller Transfer function

The PID controller structure to be realized is considered as

$$G_{PID}(s) = \frac{K_I + K_P s + K_D s^2}{s(1 + sT_F)} \tag{2.14}$$

where K_I, K_P, K_D are the integral, proportional and derivative gains respectively and T_F, the filter time constant.

The transfer function of the reduced order model is expanded in positive powers of s (eqn.2.11) and the moments ($A_0, A_1, A_2, A_3, A_4, A_5$) are calculated using eqns. (2.12 and 2.13).The controller parameters are now calculated as

$$\begin{bmatrix} K_I \\ K_P \\ K_D \end{bmatrix} = \begin{bmatrix} -\hat{A}_1 & \hat{A}_0 & 0 \\ -\hat{A}_3 & \hat{A}_2 & -\hat{A}_1 \\ -\hat{A}_5 & \hat{A}_4 & -\hat{A}_3 \end{bmatrix}^{-1} \begin{bmatrix} -0.5 \\ 0 \\ 0 \end{bmatrix} \tag{2.15}$$

where

$\hat{A}_0 = A_0$

$\hat{A}_1 = A_1 + A_0 T_F$

$\hat{A}_2 = A_2 + A_1 T_F + A_0 T_F^2$

......

The controller parameters can be calculated iteratively by first choosing $T_F = 0$ and then calculating controller parameters using eqn.(2.15). In the second iteration filter time constant T_F is chosen as

$$T_F = \frac{K_D}{K_P N} \tag{2.16}$$

with typical values of N ranging from 8 to 20 [18]. The value of ' N' need not be strictly within the above range but can be suitably chosen such that the realized controller stabilizes the closed loop and produces a response as per the required specifications.

The PID controller transfer function can thus be realized as
as in eqn.(2.14) using eqn.(2.15) and eqn. (2.16).

3.3) Obtaining Closed loop System with Controller

The PID controller thus designed is now attributed to the original high order NMP system in the forward path in cascade. The resultant closed loop system as shown in figure Figure.1.

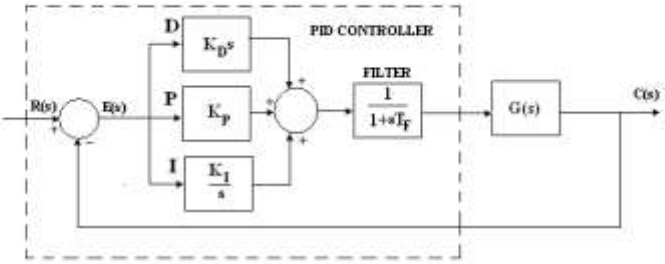

Figure.1 Closed loop PID configuration

3.4) Algorithm for Controller Design

The following is a stepwise algorithm for the proposed PID controller design

Step 1: Obtain the transfer function of the original high order NMP system $G(s)$ to be controlled

Step 2: Obtain its reduced order transfer function $R(s)$ using the proposed model reduction method.

Step 3: Realize the proposed PID controller $G_{PID}(s)$ using the reduced order transfer function obtained

Step 4: Control the original system with the obtained controller and check for performance

Step 5: If the closed loop response does not meet the specifications, then the controller is reconfigured with a different value of 'N' and tried until satisfactory operation is achieved.

IV. NUMERICAL EXAMPLE

Consider a fourth order system which is supposed to be controlled by the proposed PID controller. The required specifications are : % overshoot < 25,settling time < 20 sec. and steady state error zero.

$$G(s) = \frac{10s^2 - 30s + 20}{s^4 + 7.20s^3 + 13.66s^2 + 4.22s + 3.12} \tag{2.17}$$

The second order reduced model obtained by the proposed method is

$$R(s) = \frac{-2.1230s + 1.6240}{s^2 + 0.2065s + 0.2533} \tag{2.18}$$

The PID controller realized for the above system meeting the required specifications is

$$G_{PID}(s) = \frac{0.1929s^2 + 0.03778s + 0.04896}{s(1 + 0.2422s)} \tag{2.19}$$

with $K_P = 0.03778, K_I = 0.04896, K_D = 0.1929$ and $T_F = 0.2422$ for $N = 20$.

The unit step responses of the original full order system G(s) and its reduced second order model R(s) are compared in Figure.2.The unit step response simulations of the uncontrolled and PID controlled systems are shown in Figure.3.

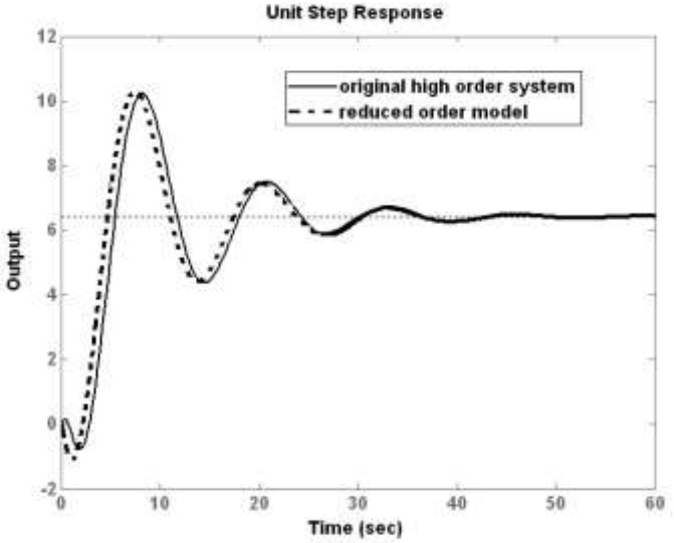

Figure.2 Unit step response of G(s) and R(s)

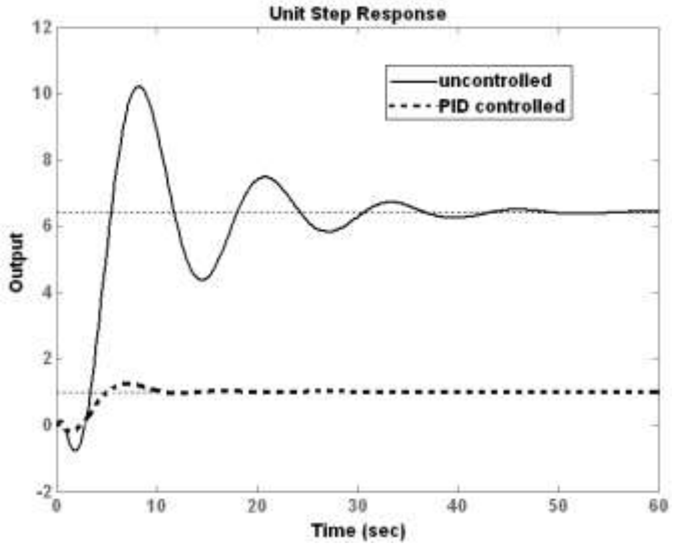

Figure.3 Unit step response of uncontrolled and PID controlled systems

The time domain characteristics of the closed loop system are given in Table1. They clearly suggest that the PID controlled system has met the required specifications and has a better performance. If the controlled system does not meet the specifications then either the specifications are to be slightly relaxed or the PID parameters need to be fine-tuned about their values.

Table1. Time Domain characteristics

characteristic	uncontrolled	PID controlled
% peak overshoot	59.6	24.8
settling time	40.7	13.7
steady state error	5.41	0

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