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THE COMPARISON OF BONDAGE NUMBER AND MAXIMUM DEGREE OF AN INTERVAL GRAPH G USING AN ALGORITHM

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Abstract: *Interval Graphs are rich in combinatorial structures and have found applications in several disciplines such as Traffic Control, Computer Sciences and particularly useful in cyclic scheduling and computer storage allocation problems. The bondage number b(G) of a non empty graph G is the minimum cardinality among all sets of edges. In this paper we present the comparison of bondage number and maximum degree of an interval graph g using an algorithm.*

Keywords: *Interval family, Interval Graph, Bondage number, Domination number, Maximum vertex degree.*

I. INTRODUCTION

Graph theory is branch of mathematics, which has becomes quite rich and interesting for several regions. In last three decades thousand of research article have been published in graph theory there are several area of in graph theory which have reserved good attention from mathematician

Graphs are very convenient tool for representing the relationship among objects, which are represented by vertices. In there term relationships among vertices are represented by connections. In general, any mathematical objects involving points and connection among them can be called a graph are a hyper graph as well as an interval graph corresponding to an interval family I. For a grate diversity of problems such pictorial representations may lead to a solution .For example data basics map coloring web graph, physical net work, organic molecular as well as less tangible interactions occurring in social net works in a flow of a computer program.

Domination is a rapidly developing area of research in graph theory, and it various applications to ad-hock net work, distributed computing, social net works and web graph, partly explain the increased interest.

II. PRELIMINARIES

Dominations theory in particular various domination parameters in graph are the main objects of studying in this paper. The basic familiarity with the rotation of the graphs, the concept of domination number, bondage number and standard algorithms tools is assumed. Here we give introducing of basic concepts domination number and bondage number[7,8]. Let a subset S of V (G) is said to be dominating set if for every vertex v in V(G)-S, there is a vertex u in S, such that u is adjacent to v. A dominating set S of the graph G is said to be minimal dominating set if for every vertex v in S, S-{V} is not a dominating set[1,3,4]. That is no proper subset of s is a dominating set. The number of vertices in a minimum dominating set is called domination number of the graph G. It is denoted by $\gamma(G)$.

Next we define the bondage number of G the bondage number b (G) of a non empty graph G is the minimum cardinality among all sets of edges E_1 for which $\gamma(G - E_1) > \gamma(G)$ [2,6]. And also we will discuss the maximum vertex degree of G .We consider a graph G, we define the vertex degree of G. Suppose the vertex set $|V|$ = n. Let $d_1,d_2,\ldots\ldots d_n$ where $d_1 \le d_2 \le \ldots\ldots \le d_n$ be the sequence of degree of the vertices of an interval graph G corresponding into an interval family I. Sorted by size we refer to the sequence as the degree sequence of the graph G. We define the minimum degree and the maximum degree of G which are

$$\delta(G) = Min\{\deg v / v \in V(G)\} \text{ and } \Delta(G) = Max\{\deg v / v \in V(G)\}.$$

In general an undirected graph G=(V,E)is an interval graph (IG), if the vertex set V can be put into one-to-one correspondence with a set of interval I on the real line R, such that two vertices are adjacent in G if and only if their corresponding intervals have non-empty intersection the set I is called an interval representation of G and G is referred to as the intersection graph I. Let I = $\{I_1, I_2, I_3, \ldots\ldots I_n\}$ be any interval family where each I_i is an interval on the real line and $I_i= [a_i, bi]$ for i=1,2,…..n. Here a_i is called the left end point labeling and b_i is the right end point labeling of I_i, without loss of generality we assume that all end points of the intervals in I are distinct number between 1and 2n.

Two intervals i and j are said to be intersect each other if they have non empty intersection. Also we say that the intervals contain both its end points and that no two intervals share a common end point the intervals and vertices of an interval graph are one and the same thing the graph G is connected 1 and the list of sorted end point is given and the intervals in I are indexed by increasing right end point that is $d_1 \le d_2 \le \ldots\ldots \le d_n$.

III. MAIN THEOREMS

Theorem 1: For any finite interval graph G corresponding to an interval family I = {1, 2,…, n}. Let i, j $\in$ I and j is contained in i, i $\neq$ 1, then $b(G) \geq \frac{n \cdot \Delta(G)}{nc_d}$ where d is domination number.

Proof: Let I = {1, 2,… , n} be a finite interval family and G be a interval graph corresponding to I. Suppose we consider an interval graph corresponding to I. We define G be an interval graph corresponding to interval family I = {1, 2,… , n} .

Let i, j $\in$ I and suppose j is contained in i, i $\neq$ 1 and there is no other interval that intersects j, other than i then the bondage number b(G)=1. Now we have to show that the bondage number b(G)=1 of G corresponding to an interval family I. Let i, j be any two interval in I, which satisfy the hypothesis of the theorem. We know that the bondage number $b(G) = \gamma(G-e) > \gamma(G)$. In this consider first we will find the domination number $\gamma(G)$. Then clearly i $\in$ $\gamma(G)$, where $\gamma(G)$ is a minimum domination number of G because there is no other interval in I , other than i, that dominates j. Next we will find $\gamma(G-e)$ of G corresponding to i.

Now consider the edge e = (i, j) in G if we remove this edges from G, then j becomes an isolated vertex in $\gamma(G-e)$, as there is no other vertex in G other than i, that is adjacent with j hence the domination number $\gamma(G^1) = \gamma(G) \cup \{j\}$ becomes a domination number of $\gamma(G-e)$ and since $\gamma(G)$ is a minimum domination number of G it follows that $\gamma(G^1)$ is also minimum domination number of $\gamma(G-e)$.

Therefore $|\gamma(G^1)| = \gamma(G-e) \quad = |\gamma(G^1)| + 1 > \gamma(G)$

Thus the bondage number b(G) = 1

Therefore $b(G) = \gamma(G-e) > \gamma(G) = 1$

Next we will prove that the maximum degree $\Delta(G)$ corresponding to an interval family I. Now we will prove that the maximum degree of G. We consider an interval graph G, we define the vertex degree of G. Suppose the vertex set $|V|$ = n. Let $d_1, d_2, \ldots\ldots d_n$ where $d_1 \leq d_2 \leq \ldots\ldots \leq d_n$ be the sequence of degree of the vertices of an interval graph G corresponding into an interval family I. Sorted by size we refer to the sequence as the degree sequence of the graph G. We define the minimum degree and the maximum degree of G which are

$$\delta(G) = Min\{\deg v / v \in V(G)\} \text{ and } \Delta(G) = Max\{\deg v / v \in V(G)\}$$

In this fact that the vertex degree { d_1, d_2,…… , d_n} be an interval graph sequence with $d_1 \leq d_2 \leq \ldots\ldots \leq d_n$. We have to show that there is an interval graph with vertex set {$v_1, v_2, \ldots\ldots v_n$} such that $\deg(v_k) = D_k$ for k = 1, 2,….,n with v_1 adjacent to vertices $v_1, v_2, \ldots\ldots v_{d+1}$. Let V = {v_1, v_2,……, v_n} and $\deg(v_k) = d_k$ where k = 1, 2, …., n. Let G be an interval graph for which the number L = $|N_G(v_1) \cap \{v_2, \ldots\ldots v_{d+1}\}|$ is maximum. If L = d_1, then the conclusion follows alternatively, if L< d_1, then there is a vertex $v_m = 2 \leq m \leq d_1 + 1$. Such that v_1 is not adjacent to v_m and there exists vertex $v_k : k > d_1+1$. Such that v_1 is adjacent to v_k, since $\deg(v_1) = d_1$

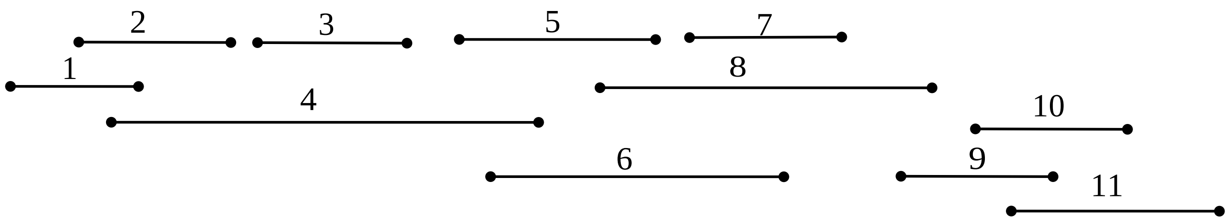
In this consistence we will find the maximum vertex degree.

$$\Delta(G) = Max\{\deg v / v \in V(G)\}$$

In this two cases we can verify easily $b(G) \geq \frac{n \cdot \Delta(G)}{nc_d}$

As follows the practical problem of an interval graph G corresponding to an interval family I.

IV. ILLUSTRATION-I



Interval Family I

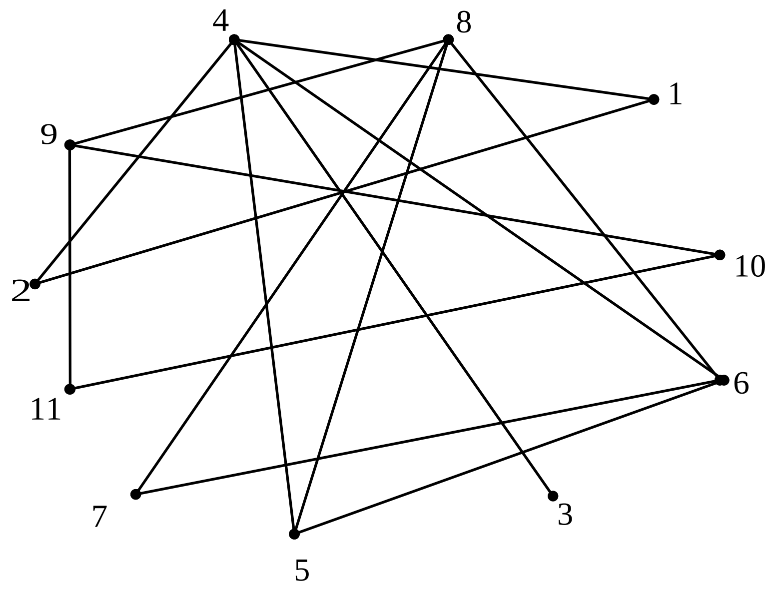


Fig.2: Interval graph G

Dominating set $\gamma(G) = \{4, 8, 10\} = 3$

$$|\gamma(G)| = |3|$$
$$= 3$$

Remove the edge e = (3, 4) from G

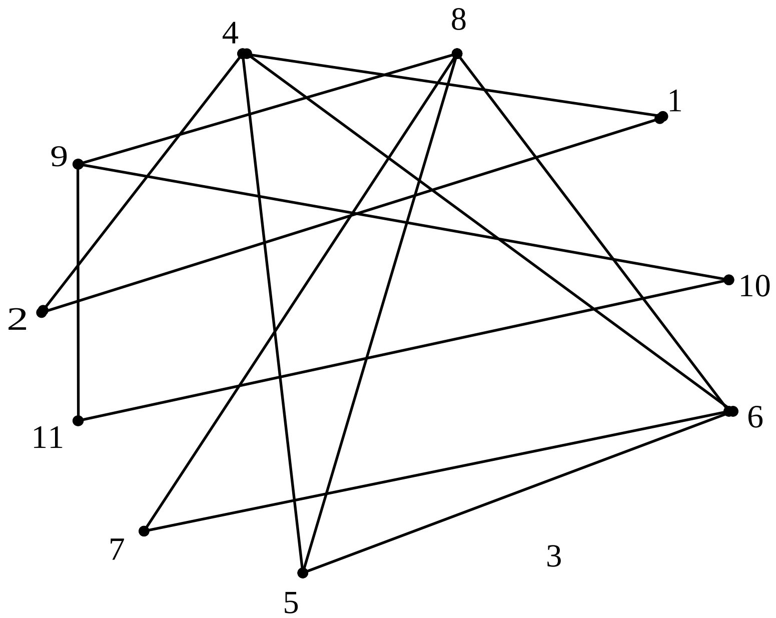


Fig.3: Interval graph G-e

Dominating set $\gamma(G-e) = \{3, 4, 8, 10\} = 4$

$$|\gamma(G-e)| = |4|$$
$$= 4$$

Therefore $\gamma(G-e) > \gamma(G)$

b(G) = 1

V. TO FIND MAXIMUM VERTEX DEGREE $\Delta(G)$

Deg(1) = 2	Deg(2) = 2	Deg(3) = 1	Deg(4) = 5
Deg(5) = 3	Deg(6) = 4	Deg(7) = 2	Deg(8) = 4
Deg(9) = 3	Deg(10) = 2	Deg(11) = 2	

$$\Delta(G) = 5$$

$$b(G) \geq \frac{n \cdot \Delta(G)}{nc_d}$$

$$1 \geq \frac{11 \cdot 5}{11c_3}$$

$$1 \geq \frac{55}{165}$$

$$1 \geq 0.333$$

VI. PROCEDURE FOR MINIMUM DOMINATING NUMBER $\gamma(G)$ TOWARDS AN ALGORITHM

Input: Interval family I = { 1, 2, …., n}
Output: Minimum dominating set of an interval graph G(I)
Step 1: Set MDS {max(i) }
Step 2: LI = The largest interval in MDS
Step 3: Compute NI(LI)
Step 4: If NI(LI) = null then go to Step 8
Step 5 : Find max(NI(LI))
Step 6: If max (NI(LI)) does not exist then
  Step 6.1: max (NI(LI)) = NI(LI)

Step 7: MDS = {MDS}U{max(NI(LI)} go to step 2
Step 8 : end.

VII. FOR FINDING MINIMUM DOMINATING NUMBER $\gamma(G)$ FROM THE ALGORITHM

nbd [1] = { 1, 2, 4 }	max(1) = 4	NI(1) = 3
nbd [2] = { 1, 2, 4 }	max(2) = 4	NI(2) = 3
nbd [3] = { 3, 4 }	max(3) = 4	NI(3) = 5
nbd [4] = { 1, 2, 3, 4, 5, 6 }	max(4) = 6	NI(4) = 7
nbd [5] = { 4, 5, 6, 8 }	max(5) = 8	NI(5) =7
nbd [6] = { 4, 5, 6, 7, 8 }	max(6) = 8	NI(6) = 9
nbd [7] = { 6, 7, 8 }	max(7) = 8	NI(7) = 9
nbd [8] = { 5, 6, 7, 8, 9 }	max(8) = 9	NI(8) = 10
nbd [9] = { 8, 9, 10, 11 }	max(9) = 11	NI(9) = null
nbd [10] = { 9, 10, 11 }	max(10) = 11	NI(10) = null
nbd [11] = { 9, 10, 11 }	max(11) = 11	NI(11) = null

VIII. PROCEDURE FOR A MINIMUM DOMINATING NUMBER OF $\gamma(G)$ WITH PRACTICAL PROBLEM

Input: Interval family I = { 1, 2,… 11}
Step 1: MDS = 4
Step 2: LI = The largest interval in MDS = 4
Step 3: NI(LI) = NI(4) = 7
Step 4: max (NI (LI)) = max (7) = 8
Step 5: MDS = {4} U {8} = {4, 8} go to step 2
Step 6: LI = The largest interval in MDS = 8
Step7: NI(8) = 10
Step 8: max (NI(LI)) = max (10) = 11
Step 9: MDS = {4, 8} U {11} = {4, 8, 11} go to step 2

Step 10: LI = 11
Step 11: NI(11) = null
Step 12 = end

IX. TO FIND THE MINIMUM DOMINATING NUMBER $\gamma(G-e)$ TOWARDS AN ALGORITHM

Input: Interval family I = { 1, 2, …., n}
Output: Minimum domination number of an interval graph G (I) since $\gamma(G-e)$
Suppose there is a disconnected graph or an isolated vertex then we
have to add {MDS} U {removal edge }

Step 1: Set MDS = {max (i) }
Step 2: LI = the largest interval in MDS
Step 3: Compute NI (LI)
Step 4: If NI (LI) = null then go to Step 8
Step 5 : Find max(NI(LI))
Step 6: If max (NI(LI)) does not exist then
Step 6.1: max (NI(LI)) = NI(LI)
Step 7: MDS = {MDS} U {max (NI (LI)} go to step 2
Step 8 : At $\gamma(G-e)$, we get MDS = MDS U { max(NI(LI)) + Removal edges}
Step 9: end

From G¹ = $\gamma(G-e)$

nbd [1] = { 1, 2, 4 }	max(1) = 4	NI(1) = 3
nbd [2] = { 1, 2, 4 }	max(2) = 4	NI(2) = 3
nbd [3] = null that is isolated vertex , we should take separate vertex		
nbd [4] = { 1, 2, 4, 5, 6 }	max(4) = 6	NI(4) = 7
nbd [5] = { 4, 5, 6, 8 }	max(5) = 8	NI(5) =7
nbd [6] = { 4, 5, 6, 7, 8 }	max(6) = 8	NI(6) = 9
nbd [7] = { 6, 7, 8 }	max(7) = 8	NI(7) = 9
nbd [8] = { 5, 6, 7, 8, 9 }	max(8) = 9	NI(8) = 10
nbd [9] = { 8, 9, 10, 11 }	max(9) = 11	NI(9) = null
nbd [10] = { 9, 10, 11 }	max(10) = 11	NI(10) = null
nbd [11] = { 9, 10, 11 }	max(11) = 11	NI(11) = null

X. PROCEDURE FOR A MINIMUM DOMINATING NUMBER $\gamma(G-e)$ WITH PRACTICAL PROBLEM

Input: Interval family I = { 1, 2, …., 11}
Step 1: MDS = 4
Step 2: LI = The largest interval in MDS = 4
Step 3: NI(LI) = NI(4) = 7
Step 4: max (NI(LI)) = max(7) = 8
Step 5 : MDS = {4} U {8} = {4, 8} go to step 2
Step 6: LI = The largest interval in MDS = 8
Step 7: NI(8) = 10
Step 8: max (NI(LI)) = max(10) = 11
Step 9 : MDS = {4, 8} U {11} = {4, 8, 11} go to step 2
Step 10: LI = 11
Step 11: NI(11) = null
Step 12: At $\gamma(G-e)$, we get MDS = {MDS} U {removed edge}
{ 4, 8, 11} U { 3} = { 3, 4, 8, 11 }
Step 13: end

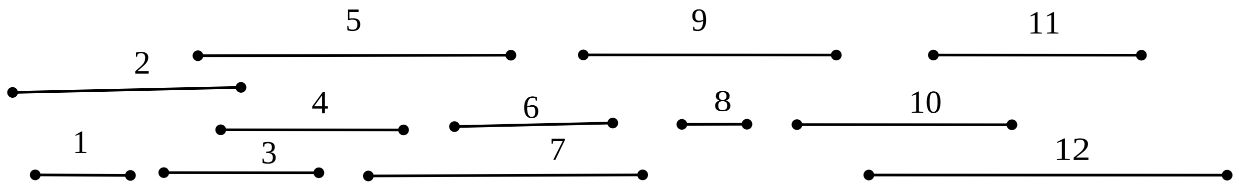
Therefore $\gamma(G-e) > \gamma(G)$

Theorem 2: For any finite interval graph G corresponding to an interval family I Let i = 2 , j = 1 and j is contained in i, then $b(G) \geq \dfrac{n \cdot \Delta(G)}{nc_d}$ where d is domination number.

Proof: Let I = {1, 2,….,n} be an interval family and G is a finite interval graph corresponding to an interval family I. We consider an interval family I , let i = 2, j = 1 and j is contained in i. suppose there is no interval other than i that intersect j then bondage number b(G) = 1. Let e = (1, 2) and $\gamma(G)$ be a minimum domination number of G corresponding to I. The vertex 1 becomes an isolated vertex in $\gamma(G-e)$ as there is no vertex other than 2 that is adjacent with 1 then $\gamma(G_1) = \{\gamma(G)\} U \{1\}$ becomes a minimum dominating number of $\gamma(G-e)$ so that $\gamma(G-e) > \gamma(G)$ which implies b(G) = 1.Therefore the bondage number b(G) = 1. Next we have to prove that the maximum vertex degree of an interval graph G corresponding to I = { 1, 2, ….., n} already we proved the procedure of the maximum vertex degree in theorem 1. Therefore $b(G) \geq \frac{n \cdot \Delta(G)}{nc_d}$.

As follows the practical problem with algorithm

XI. ILLUSTRATION-II



Interval Family I

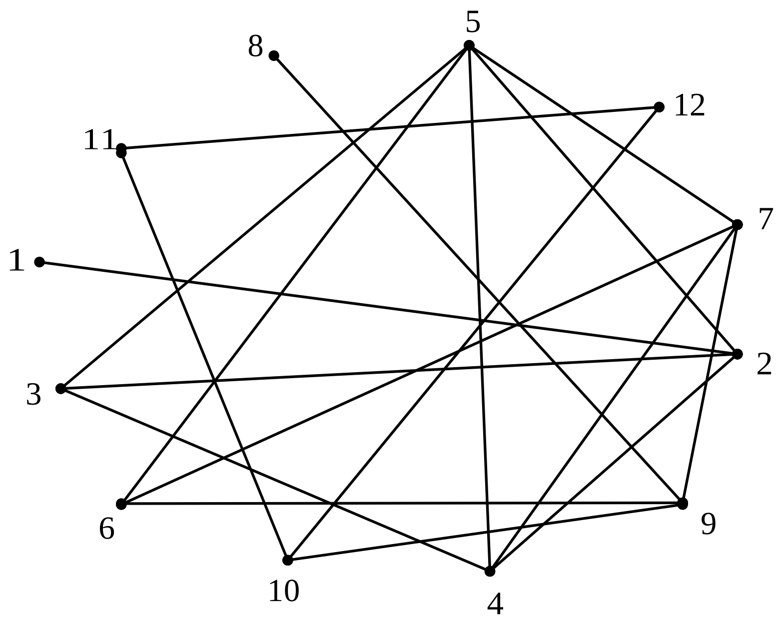


Fig.2: Interval graph G

Dominating set $\gamma(G) = \{2, 9, 11\} = 3$

$$|\gamma(G)| = |3|$$

$$= 3$$

Remove the edge e = (1, 2) from G

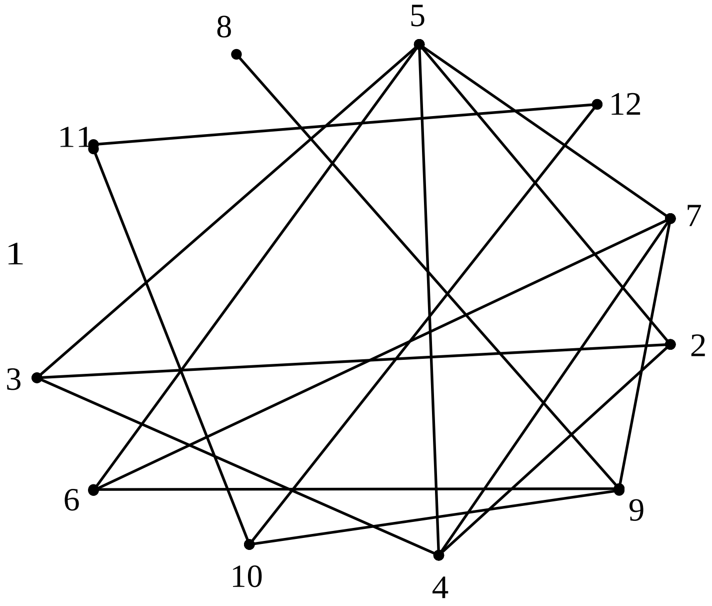


Fig.2: Interval graph G - e

Dominating set $\gamma(G-e) = \{1, 2, 9, 11\} = 4$

$$|\gamma(G-e)| = |4|$$
$$= 4$$

Therefore $\gamma(G-e) > \gamma(G)$

b(G) = 1

XII. TO FIND MAXIMUM VERTEX DEGREE $\Delta(G)$

Deg(1) = 1	Deg(2) = 4	Deg(3) = 3	Deg(4) = 4
Deg(5) = 5	Deg(6) = 3	Deg(7) = 4	Deg(8) = 1
Deg(9) = 4	Deg(10) = 3	Deg(11) = 2	Deg(12) = 2

$$\Delta(G) = 5$$

$$b(G) \geq \frac{n \cdot \Delta(G)}{nc_d}$$

$$1 \geq \frac{12 \cdot 5}{12c_3}$$

$$1 \geq \frac{60}{240}$$

$$1 \geq 0.25$$

XIII. PROCEDURE FOR MINIMUM DOMINATING NUMBER $\gamma(G)$ TOWARDS AN ALGORITHM

Input: Interval family I = { 1, 2, …., n}

Output: Minimum dominating set of an interval graph G(I)

nbd [1] = { 1, 2 }	max(1) = 2	NI(1) = 3
nbd [2] = { 1, 2, 3, 4, 5 }	max(2) = 5	NI(2) = 6
nbd [3] = { 2, 3, 4, 5 }	max(3) = 5	NI(3) = 6
nbd [4] = { 2, 3, 4, 5, 7 }	max(4) = 7	NI(4) = 6
nbd [5] = { 2, 3, 4, 5, 6, 7 }	max(5) = 7	NI(5) = 8
nbd [6] = { 5, 6, 7, 9 }	max(6) = 9	NI(6) = 8

nbd [7] = { 5, 6, 7, 9 }	max(7) = 9	NI(7) = 8
nbd [8] = { 8, 9 }	max(8) = 9	NI(8) = 10
nbd [9] = { 6, 7, 8, 9, 10 }	max(9) = 10	NI(9) = 11
nbd [10] = { 9, 10, 11, 12 }	max(10) = 12	NI(10) = null
nbd [11] = { 10, 11, 12 }	max(11) = 12	NI(11) = null
nbd [12] = { 10, 11, 12 }	max(12) = 12	NI(12) = null

XIV. PROCEDURE FOR A MINIMUM DOMINATING NUMBER OF $\gamma(G)$ WITH PRACTICAL PROBLEM

Input: Interval family I = { 1, 2, …., 12}
Step 1: MDS = 2
Step 2: LI = the largest interval in MDS =2
Step 3: NI (LI) = NI (2) = 6
Step 4: max (NI (LI)) = max (6) = 9
Step 5 : MDS = {2} U {9} = {2, 9} go to step 2
Step 6: LI = the largest interval in MDS = 9
Step 7: NI (9) = 11
Step 8: max (NI (LI)) = max (11) = 12
Step 9: MDS = {2, 9} U {12} = {2, 9, 12} go to step 2
Step 10: LI = 12
Step 11: NI (12) = null
Step 12 = end

XV. TO FIND THE MINIMUM DOMINATING NUMBER OF $\gamma(G-e)$ TOWARDS AN ALGORITHM

Input: Interval family I = { 1, 2, …., n}

Output: Minimum domination number of an interval graph G(I) since $\gamma(G-e)$

From G^1 = $\gamma(G-e)$

nbd [1] = null that is isolated vertex , we should take separate vertex

nbd [2] = { 2, 3, 4, 5 }	max(2) = 5	NI(2) = 6
nbd [3] = { 2, 3, 4, 5 }	max(3) = 5	NI(3) = 6
nbd [4] = { 2, 3, 4, 5, 7 }	max(4) = 7	NI(4) = 6
nbd [5] = { 2, 3, 4, 5, 6, 7 }	max(5) = 7	NI(5) = 8
nbd [6] = { 5, 6, 7, 9 }	max(6) = 9	NI(6) = 8
nbd [7] = { 5, 6, 7, 9 }	max(7) = 9	NI(7) = 8
nbd [8] = { 8, 9 }	max(8) = 9	NI(8) = 10
nbd [9] = { 6, 7, 8, 9, 10 }	max(9) = 10	NI(9) = 11
nbd [10] = { 9, 10, 11, 12 }	max(10) = 12	NI(10) = null
nbd [11] = { 10, 11, 12 }	max(11) = 12	NI(11) = null
nbd [12] = { 10, 11, 12 }	max(12) = 12	NI(12) = null

XVI. PROCEDURE FOR A MINIMUM DOMINATING NUMBER $\gamma(G-e)$ WITH PRACTICAL PROBLEM

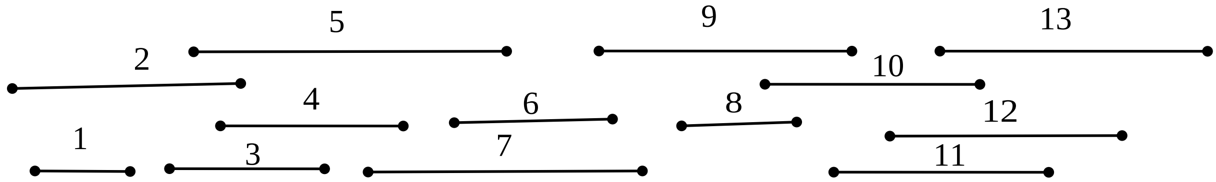
Input: Interval family I = { 1, 2, …., 12}
Step 1: MDS = 5
Step 2: LI = The largest interval in MDS = 5
Step 3: NI(LI) = NI(5) = 8
Step 4: max (NI(LI)) = max(8) = 9
Step 5 : MDS = {5} U {9} = {5, 9} go to step 2
Step 6: LI = The largest interval in MDS = 9
Step 7: NI(9) = 11
Step 8: max (NI(LI)) = max(11) = 12
Step 9 : MDS = {5, 9} U {12} = {5, 9, 12} go to step 2
Step 10: LI = 12
Step 11: NI(12) = null
Step 12: At $\gamma(G-e)$, we get MDS = {MDS} U {removed edge}
{ 5, 9, 12} U { 1} = { 1, 5, 9, 12 }
Step 13: end

Therefore $\gamma(G-e) > \gamma(G)$

Theorem 3: Let G be an interval graph G corresponding to I. Let i = n , j = n-1 and j is contained in i, then $b(G) \geq \dfrac{n \cdot \Delta(G)}{nc_d}$ where d is domination number.

Proof: Let G be finite interval graph G corresponding to an interval family
I = { 1, 2,……, n} . Let i = n, j = n-1 and j is contained in i. Suppose there is no interval other than i that intersect j then the bondage number b(G) = $\gamma(G-e) > \gamma(G)$. In this we have to prove that $b(G) \geq \dfrac{n \cdot \Delta(G)}{nc_d}$ where d is domination number. Already we have to proved in Theorem 1 of the bondage number b(G) as well as the maximum degree of G corresponding to the interval family I ={1, 2,....,n-1, n}. As follow the practical problem

XVII. ILLUSTRATION-III



Interval Family I

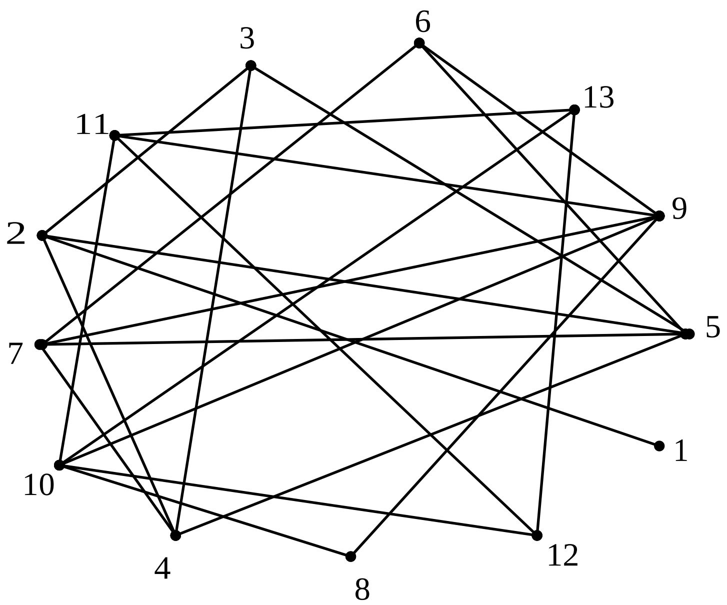


Fig.2: Interval graph G

Dominating set $\gamma(G) = \{2, 9, 12\} = 3$

$|\gamma(G)| = |3|$

$= 3$

Remove the edge e = (8, 9) from G

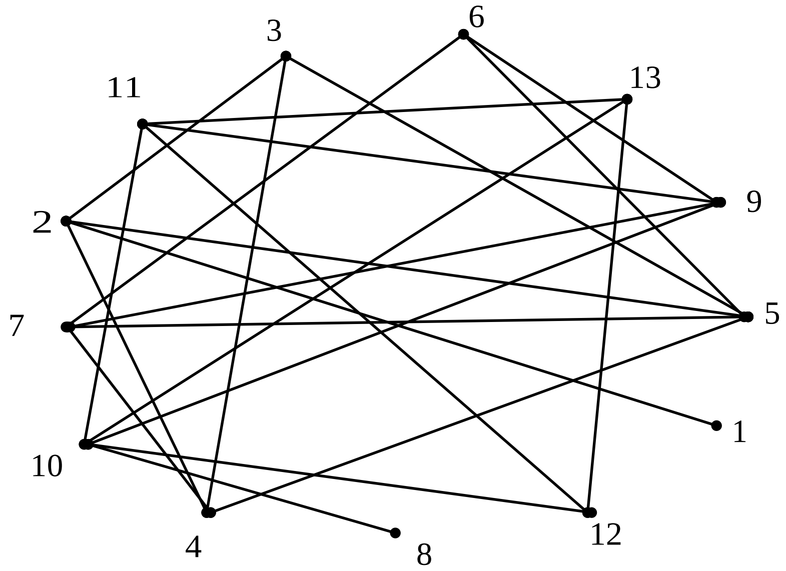


Fig.2: Interval graph G - e

Dominating set $\gamma(G-e) = \{2, 8, 9, 12\} = 4$

$$|\gamma(G)| = |4|$$

$$= 4$$

Therefore $\gamma(G-e) > \gamma(G)$

b(G) = 1

XVIII. TO FIND MAXIMUM VERTEX DEGREE $\Delta(G)$

Deg (1) = 1 Deg (2) = 4 Deg (3) = 3 Deg (4) = 4
Deg (5) = 5 Deg (6) = 3 Deg (7) = 4 Deg (8) = 2
Deg (9) = 5 Deg (10) = 5 Deg (11) = 4 Deg (12) = 3
Deg (13) = 3

$$\Delta(G) = 5$$

$$b(G) \geq \frac{n \cdot \Delta(G)}{nc_d}$$

$$1 \geq \frac{13 \cdot 5}{13c_3}$$

$$1 \geq \frac{65}{286}$$

$$1 \geq 0.227$$

XIX. FOR FINDING MINIMUM DOMINATING NUMBER $\gamma(G)$ FROM THE ALGORITHM

nbd [1] = { 1, 2 }	max(1) = 2	NI(1) = 3
nbd [2] = { 1, 2, 3, 4, 5 }	max(2) = 5	NI(2) = 6
nbd [3] = { 2, 3, 4, 5 }	max(3) = 5	NI(3) = 6
nbd [4] = { 2, 3, 4, 5, 7 }	max(4) = 7	NI(4) = 6
nbd [5] = { 2, 3, 4, 5, 6, 7 }	max(5) = 7	NI(5) = 8
nbd [6] = { 5, 6, 7, 9 }	max(6) = 9	NI(6) = 8
nbd [7] = { 4, 5, 6, 7, 9 }	max(7) = 9	NI(7) = 8
nbd [8] = { 8, 9, 10 }	max(8) = 10	NI(8) = 11

nbd [9] = { 6, 7, 8, 9, 10, 11 }	max(9) = 11	NI(9) = 12
nbd [10] = {8, 9, 10, 11, 12, 13 }	max(10) = 13	NI(10) = null
nbd [11] = {9, 10, 11, 12, 13 }	max(11) = 13	NI(11) = null
nbd [12] = { 10, 11, 12, 13 }	max(12) = 13	NI(12) = null
nbd [13] = { 10, 11, 12, 13 }	max(13) = 13	NI(13) = null

XX.PROCEDURE FOR A MINIMUM DOMINATING NUMBER OF $\gamma(G)$ WITH PRACTICAL PROBLEM

Input: Interval family I = { 1, 2, …., 13}
Step 1: MDS = 2
Step 2: LI = the largest interval in MDS =2
Step 3: NI (LI) = NI (2) = 6
Step 4: max (NI (LI)) = max (6) = 9
Step 5: MDS = {2} U {9} = {2, 9} go to step 2
Step 6: LI = the largest interval in MDS = 9
Step 7: NI (9) = 12
Step 8: max (NI (LI)) = max (12) = 13
Step 9: MDS = {2, 9} U {13} = {2, 9, 13} go to step 2
Step 10: LI = 13
Step 11: NI (13) = null
Step 12 = end

XXI. TO FIND THE MINIMUM DOMINATING NUMBER $\gamma(G-e)$ TOWARDS AN ALGORITHM

From G^1 = $\gamma(G-e)$

nbd [1] = { 1, 2 }	max(1) = 2	NI(1) = 3
nbd [2] = { 1, 2, 3, 4, 5 }	max(2) = 5	NI(2) = 6
nbd [3] = { 2, 3, 4, 5 }	max(3) = 5	NI(3) = 6
nbd [4] = { 2, 3, 4, 5, 7 }	max(4) = 7	NI(4) = 6
nbd [5] = { 2, 3, 4, 5, 6, 7 }	max(5) = 7	NI(5) = 8
nbd [6] = { 5, 6, 7, 9 }	max(6) = 9	NI(6) = 8
nbd [7] = { 4, 5, 6, 7, 9 }	max(7) = 9	NI(7) = 8
nbd [8] = { 8, 10 }	max(8) = 10	NI(8) = 11
nbd [9] = { 6, 7, 9, 10, 11 }	max(9) = 11	NI(9) = 12
nbd [10] = {8, 9, 10, 11, 12, 13 }	max(10) = 13	NI(10) = null
nbd [11] = { 9, 10, 11, 12, 13 }	max(11) = 13	NI(11) = null
nbd [12] = { 10, 11, 12, 13 }	max(12) = 13	NI(12) = null
nbd [13] = { 10, 11, 12, 13 }	max(13) = 13	NI(13) = null

XXII. PROCEDURE FOR A MINIMUM DOMINATING NUMBER $\gamma(G-e)$ WITH PRACTICAL PROBLEM

Input: Interval family i = { 1, 2, …., 13}
Step 1: MDS = 2
Step 2: LI = the largest interval in MDS =2
Step 3: NI (LI) = NI (2) = 6
Step 4: max (NI (LI)) = max (6) = 9
Step 5: MDS = {2} U {9} = {2, 9} go to step 2
Step 6: LI = the largest interval in MDS = 9
Step 7: NI (9) = 12
Step 8: max (NI (LI)) = max (12) = 13
Step 9: MDS = {2, 9} U {13} = {2, 9, 13} go to step 2
Step 10: LI = 13
Step 11: NI (13) = null
Step 12: At $\gamma(G-e)$ we get MDS = {MDS} U {removed edge}
= {2, 9, 13} U {8}
= {2, 8, 9, 13}
Step 13: end

Therefore $\gamma(G-e) > \gamma(G)$

XXIII. REFERENCES

[1]. E.J.Cockayne, S.T.Hedetniemi, Towards a theory of domination in graphs, Net works,vol7(1977), 247-261.
[2]. Dr.A.Sudhakaraiah, K.Ramakrishna, T.venkateswarlu, The comparison of bondage number and the average distance of interval graphs – IJSER –vol.6 ,Issue-10,oct-2015.
[3]. T.W.Haynes, S.T.Hedetniemi and P.J.Slater, Domination in graphs, Marcel Dekker, Inc., New York(1998).
[4]. T.W.Haynes, S.T.Hedetniemi and P.J.Slater, Domination in graphs, advanced topics Marcel Dekker, Inc., New York(1998).
[5]. J.E.Dunbar, T.W.Haynes, U.Teschner and L.Volkmann, Bondage, insensitivity and reinforcement, in: T.W.Haynes, S.T.Hedetniemi, P.J.Slter(Eds.), Domination in graphs: Advanced topics (Marcel Dekker, New York, 1998) 471-489.
[6]. J.F.Fink, M.S.Jacobson, L.F.Kinch and J.Roberts, The bondage number of a graph, Discrete math. 86 (1990)47-57.
[7]. Hailong Liu and Liang Sun, The bondage and connectivity of graph, Discrete math. 263 (2003) 289-293.
[8]. U.Teschner, A new upper bound for the bondage number of a graph with small domination number, Aus. J.Combin 12 (1995)27-35.

