

**UNSTEADY MHD FREE CONVECTION FLOW THROUGH POROUS  
MEDIUM PAST AN ACCELERATED VERTICAL PLATE WITH  
CHEMICAL REACTION**

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**Abstract:** *In the present paper, an unsteady free convection flow of an electrically conducting fluid through a porous medium past an accelerated infinite vertical plate, with constant heat flux is precisely investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of chemical reaction with heat generation or absorption and Ohmic heating has been considered, when the magnetic lines of force are fixed to the fluid or to the plate. The governing coupled linear partial differential equations are solved numerically using the finite difference technique. The present problem finds typical applications in aeronautics, spacecraft design and the study of the thermal plumes into atmosphere which are responsible for atmospheric pollution. The numerical results of various parameters are presented through graphs and tables.*

**Keywords:** *Porous plate, Chemical Reaction, Heat Generation, Heat Absorption, MHD free convection.*

**1. Introduction**

The study of MHD boundary layer flows of electrically conducting fluids finds applications in several industrial and technological fields such as meteorology, electrical power generation, solar power technology, nuclear engineering, and geophysics. Debnath [1, 2] obtained an exact solution of the unsteady hydro magnetic boundary layer equation for a viscous incompressible and electrically conducting rotating fluid in the presence of an external magnetic field. Debnath [3] derived exact solutions of the unsteady hydrodynamic and hydro magnetic boundary layer flows including the effects of the pressure gradient and uniform suction or blowing. Georgantopoulos et al.[4] calculated the magnetohydrodynamic free convection flow past an impulsively started infinite vertical plate with constant temperature. Tokis and Pande [5] observed the unsteady two-dimensional flow of a viscous incompressible and electrically conducting fluid near a moving porous plate of an infinite extent in the presence of a transverse magnetic field. In recent years, hydro magnetic flows and heat transfer have essentially become more important because of numerous applications, for example, metallurgical processes in cooling of continuous strips through a quiescent fluid, thermonuclear fusion, aerodynamics, among others. Raptis and Singh [6] together studied the effect of a uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an accelerated vertical infinite plate when the magnetic lines of force are fixed relative to the fluid. They observed the fluid velocity attains to a non-zero steady state as the boundary layer increases. Tokis [7] analyzed a class of exact solutions of the unsteady free convection flow of an electrically conducting fluid near a moving vertical plate of an infinite extent in the presence of uniform transverse magnetic field fixed to the fluid or to the plate. Several studies have been continued on Magnetohydrodynamics free convection flows past a vertical surface under different physical situations [8 – 11].

However, it seems less attention was paid on hydro magnetic free convection flows near a vertical plate subjected to a constant heat flux boundary condition even though this situation involves in many engineering applications. Chandra et al. [12] obtained the effects of magnetic field and buoyancy force on the unsteady free convection flow of an electrically conducting fluid when the flow was generated by uniformly accelerated motion of an infinite vertical plate subjected to constant heat flux. They obtained an exact solution with the help of Laplace transform technique and the numerical results are computed with the approximated error functions appeared in the solution. Narahari and Debnath [13] considered the unsteady MHD free convection flow near an accelerated infinite vertical plate with constant heat flux and heat generation or absorption has been considered when the magnetic lines of force are fixed to the fluid or to the plate. The governing coupled linear partial differential equations are solved analytically using the Laplace transform technique without any restriction.

Rajaiah et al. [14] investigated an unsteady free convection flow of an electrically conducting fluid past an accelerated infinite vertical plate, with constant heat flux is precisely investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of chemical reaction with heat generation or absorption and Ohmic heating has been considered, when the magnetic lines of force are fixed to the fluid or to the plate.

In the present paper an unsteady free convection flow of an electrically conducting fluid through a porous medium past an accelerated infinite vertical plate, with constant heat flux is precisely investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of chemical reaction with heat generation or absorption and Ohmic heating has been considered, when the magnetic lines of force are fixed to the fluid

or to the plate. The governing coupled linear partial differential equations are solved numerically using the finite difference technique. The present problem finds typical applications in aeronautics, spacecraft design and the study of the thermal plumes into atmosphere which are responsible for atmospheric pollution. The numerical results are presented through graphs and tables

## 2. Formulation

The unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past an infinite non conducting vertical plate with Ohmic heating is considered under the following assumptions

1. The  $x'$ -axis is taken along the plate in the upward direction and the  $y'$ -axis is perpendicular to the plate into the fluid by choosing an arbitrary point on this plate as the origin.
2. A uniform magnetic field of strength  $B_0$  is applied in the horizontal direction that is in the  $y'$ -direction.
3. Initially, at time  $t' \leq 0$ , the plate and the fluid are at rest and at the same temperature  $T_\infty'$ . At time  $t' > 0$ , suddenly the plate accelerated with velocity  $u_0 f(t')$  in its own plane along the  $x'$ -axis against the gravitational field and heat is supplied to the plate at a constant rate in the presence of temperature dependent heat generation or absorption.
4. All the physical properties of the fluid are assumed to be constant, except the density variations with temperature in the body force term.
5. The magnetic Reynolds number of the flow is assumed to be small. So that, the induced magnetic field is neglected in comparison with applied magnetic field ( $B_0$ ).
6. The Ohmic heating is considered.
7. As the plate is infinite extent in  $x'$  direction, all the physical quantities are the functions of the space coordinate  $y'$  and time  $t'$  only and therefore the inertia terms are negligible.
8. Here the plate is subjected to exponential and uniform acceleration in the fluid, so that, there arise two cases:
  - a) Exponentially Accelerated Plate
  - b) Uniformly Accelerated Plate

### 3. Exponentially Accelerated Plate (EAP)

Using the finite difference scheme, the governing equations along with the initial and boundary conditions for the case of an exponentially accelerated plate turn into

$$\frac{u(i, j+1) - u(i, j)}{\Delta t} = \frac{u(i+1, j) - 2u(i, j) + u(i-1, j)}{\Delta y^2} - \left(M + \frac{1}{k}\right) [u(i, j) - K_1 \exp(a_0 t)] + Gr \theta(i, j) + Gc \phi(i, j) \quad (1)$$

$$\frac{\theta(i, j+1) - \theta(i, j)}{\Delta t} = \frac{1}{Pr} \frac{\theta(i+1, j) - 2\theta(i, j) + \theta(i-1, j)}{\Delta y^2} - \frac{Q}{Pr} \theta(i, j) - Mc [u(i, j)]^2 \quad (2)$$

$$\frac{\phi(i, j+1) - \phi(i, j)}{\Delta t} = \frac{1}{Sc} \frac{\phi(i+1, j) - 2\phi(i, j) + \phi(i-1, j)}{\Delta y^2} - \gamma \phi \quad (3)$$

The initial and boundary conditions are represented as

$$u(i, 0) = 0, \quad \theta(i, 0) = 0, \quad \phi(i, 0) = 0 \quad \text{for all } i \quad (4)$$

$$u(0, j) = \exp(a_0 t), \quad \theta(0, j) = 1, \quad \phi(0, j) = 1 \quad \text{for all } i$$

$$u(i, j) \rightarrow 0, \quad \theta(i, j) \rightarrow 0, \quad \phi(i, j) \rightarrow 0 \quad \text{for all } j$$

### 4. Uniformly Accelerated Plate (UAP) Case

$$\frac{u(i, j+1) - u(i, j)}{\Delta t} = \frac{u(i+1, j) - 2u(i, j) + u(i-1, j)}{\Delta y^2} - \left(M + \frac{1}{k}\right) [u(i, j) - K_1 t] + Gr \theta(i, j) + Gc \phi(i, j) \quad (5)$$

$$\frac{\theta(i, j+1) - \theta(i, j)}{\Delta t} = \frac{1}{Pr} \frac{\theta(i+1, j) - 2\theta(i, j) + \theta(i-1, j)}{\Delta y^2} - \frac{Q}{Pr} \theta(i, j) - M Ec [u(i, j)]^2 \quad (6)$$

$$\frac{\phi(i, j+1) - \phi(i, j)}{\Delta t} = \frac{1}{Sc} \frac{\phi(i+1, j) - 2\phi(i, j) + \phi(i-1, j)}{\Delta y^2} - \gamma \phi \quad (7)$$

The initial and boundary conditions are represented as

$$u(i, 0) = 0, \quad \theta(i, 0) = 0, \quad \phi(i, 0) = 0 \quad \text{for all } i \quad (8)$$

$$u(0, j) = K_1 t, \quad \theta(0, j) = 1, \quad \phi(0, j) = 1 \quad \text{for all } i$$

$$u(i, j) \rightarrow 0, \quad \theta(i, j) \rightarrow 0, \quad \phi(i, j) \rightarrow 0 \quad \text{for all } j$$

### 6. Stability Analysis

These computations are carried out for  $Pr = 0.71, 1, 7$  and  $11$ ,  $K_1 = 0$  and  $1$  and for various values of  $K$ . To judge the accuracy of the convergence of the finite difference scheme, the same program was run with the  $\Delta t = 0.0009$  and  $0.00125$  and no significant change was observed. Hence, we conclude the finite difference scheme is stable and convergent

### 8. Analysis of Results through Graphs

(a) Exponentially Accelerated Plate (EAP)

The velocity profiles for Exponentially Accelerated Plate (EAP) are discussed through the graphs 1 to 4, when the magnetic field is being fixed to the fluid ( $K_1 = 0$ ) and to the moving plate ( $K_1 = 1$ ), for various physical parameters such as Hartmann number ( $M$ ), Permeability Parameter ( $K$ ), the heat generation or absorption parameter ( $Q$ ), thermal Grashof number ( $Gr$ ) and the Solutal Grashof number ( $Gc$ ) etc.

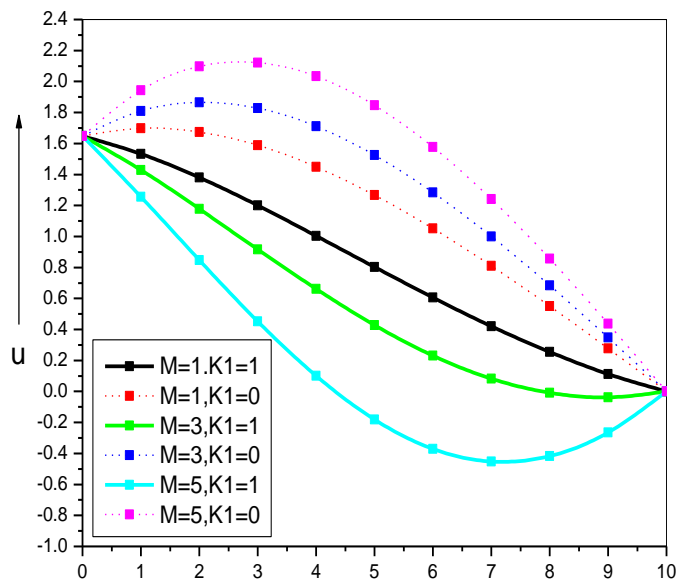


Fig. 1  
 Variation of Velocity profiles for different values of M

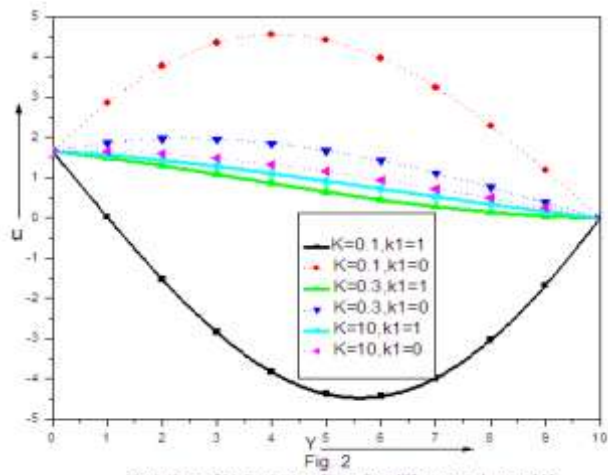


Fig. 2  
 Variation of Velocity profiles for different values of K.

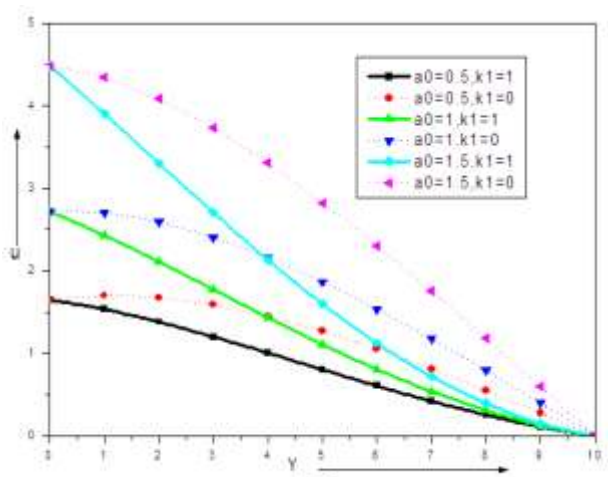


Fig. 3  
 Variation of Velocity profiles for different values of  $a_0$

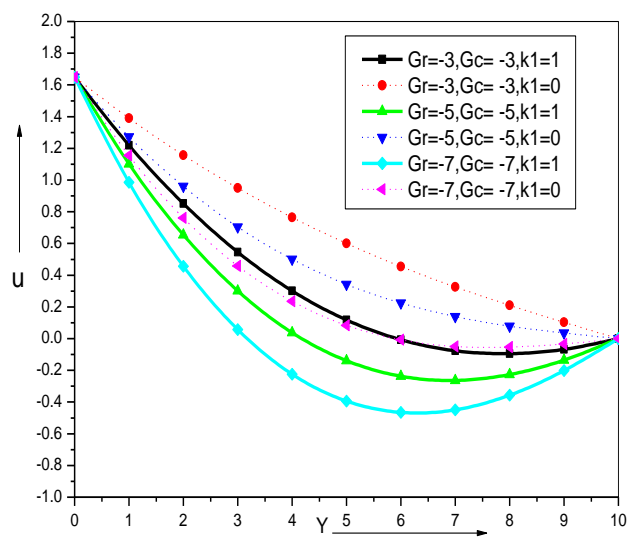
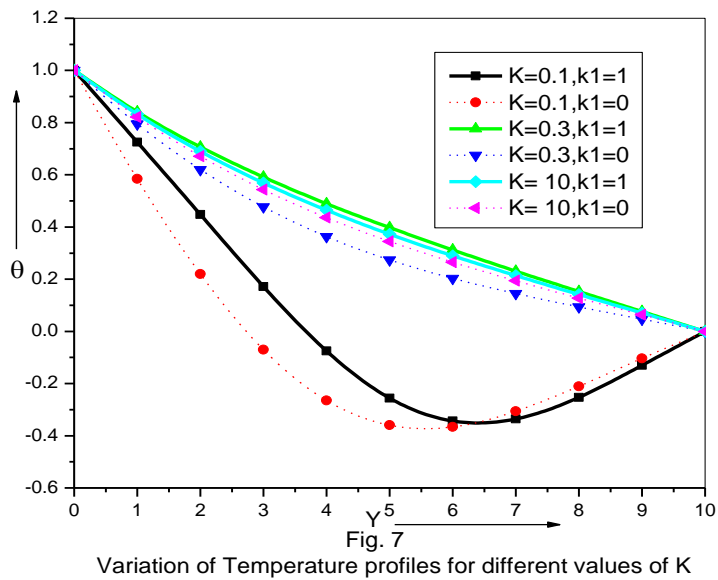
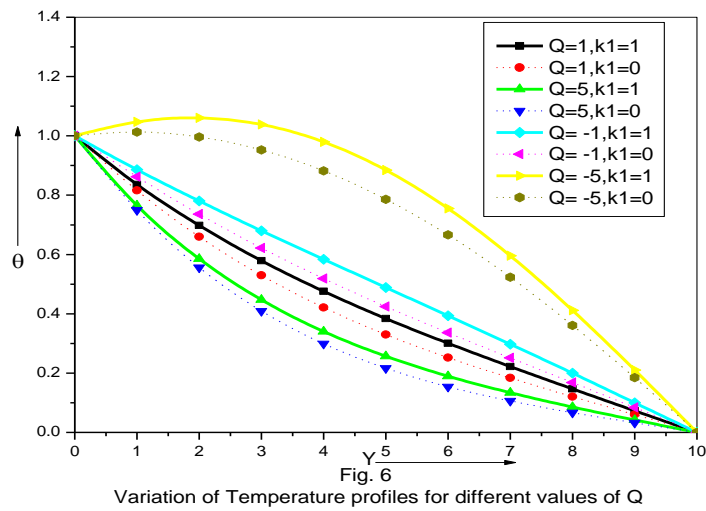
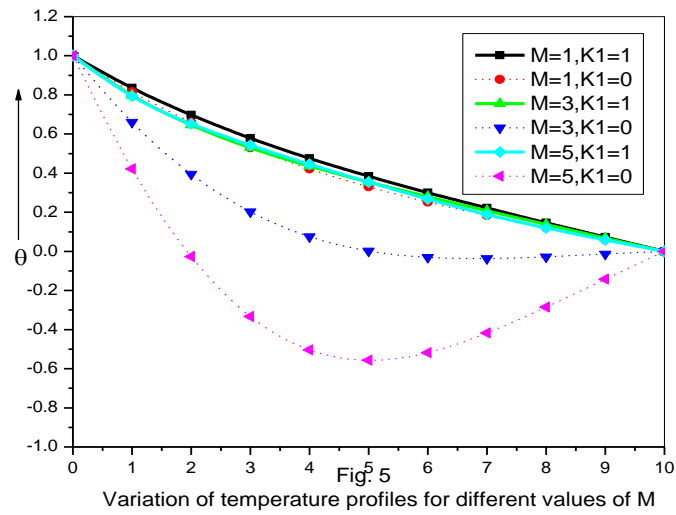


Fig. 4  
 Variation of Velocity profiles for different values of GR, Gc



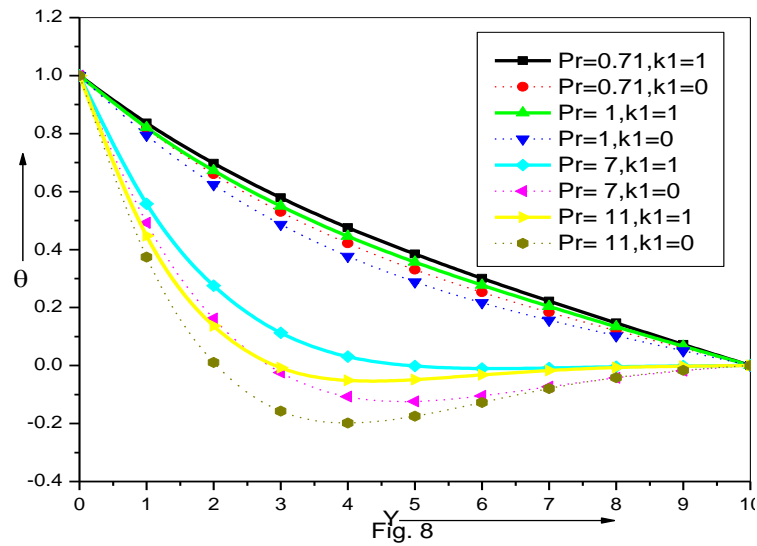


Fig. 8  
 Variation of Temperature profiles for different values of Pr

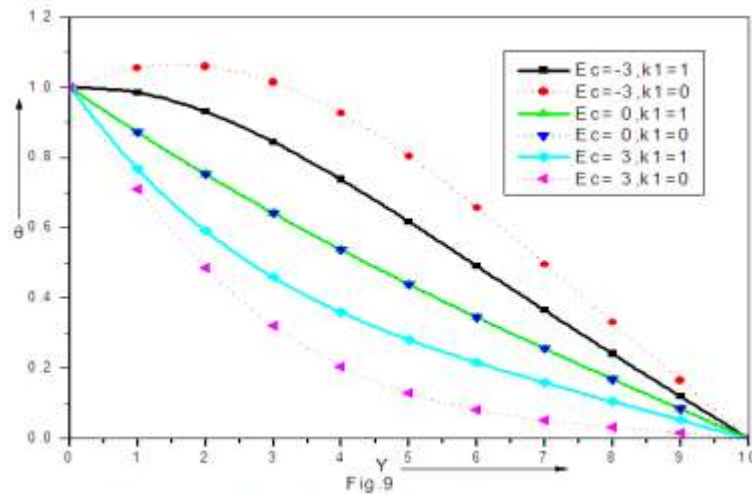


Fig. 9  
 Variation of Temperature profiles for different values of Ec

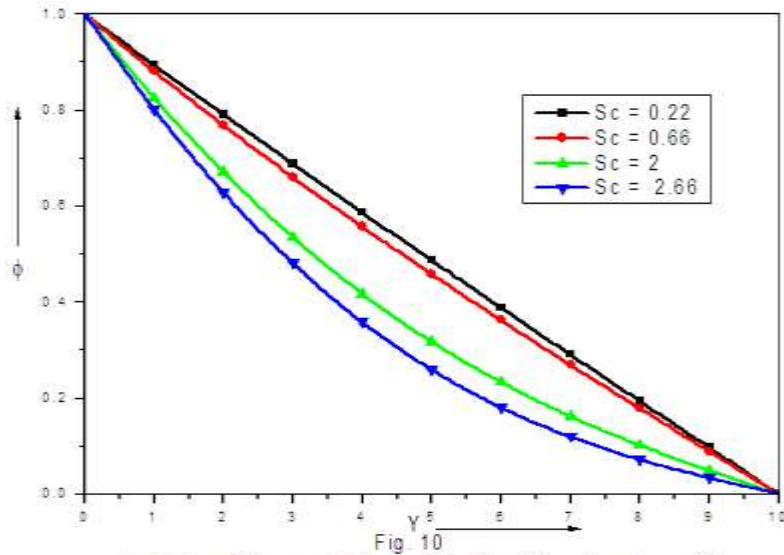


Fig. 10  
 Variation of Concentration profiles for different values of Sc

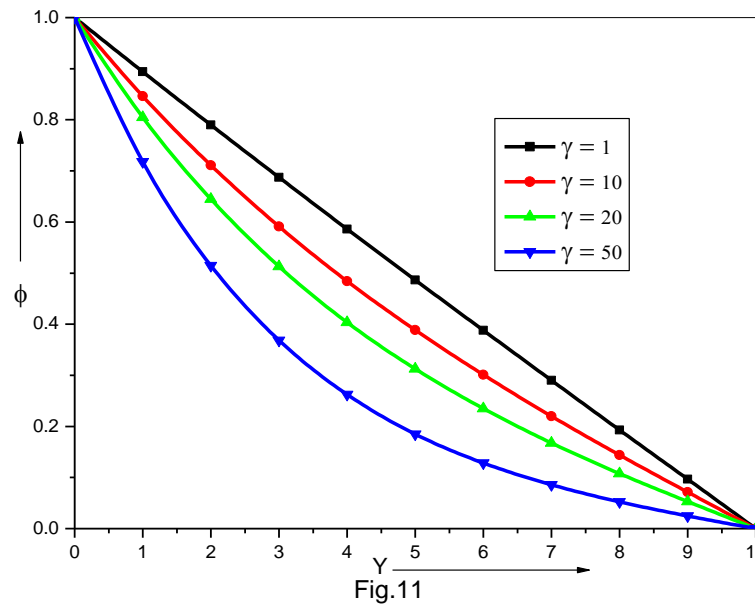


Fig.11  
 Variation of Concentration profiles for different values of  $\gamma$

The velocity profiles, for an Exponentially Accelerated Plate (EAP), are discussed through the graphs 1 to 8, when the magnetic field is being fixed to the fluid ( $K_1 = 0$ ) and to the moving plate ( $K_1 = 1$ ), for various physical parameters such as Hartmann number (M), Eckert number (Ec), thermal Grashof number (Gr), the Solutal Grashof number (Gc), the acceleration parameter ( $a_0$ ) and the heat generation or absorption parameter (Q) etc.

The presence of a magnetic field in an electrically conducting fluid introduces the Lorentz force, which acts against the flow results in the decrease of the velocity with the increase of magnetic field (M). The effect of the Hartmann number (M), when the magnetic field is being fixed to the fluid ( $K_1 = 0$ ) and to the moving plate ( $K_1 = 1$ ), is shown in the figure 1. From the figure, the decrease of velocity is observed with the increase of the magnetic field for the moving plate and the velocity increases in the boundary layer.

The presence of porous medium is to increase or decrease the velocity according as the size of the pores. In the figure 2 the variation of velocity with permeability parameter (K) is shown. It is evident that as permeability of the pore size increases the velocity increases. But it can be reduced with decrease of the size of the pores. But velocity observes an opposite behavior relative to fixed ( $K_1 = 0$ ) as well as moving plate ( $K_1 = 1$ ).

The variation of velocity for  $a_0$  and  $K_1$  is shown in the figure 3. From the figure it is clear that as  $a_0$  increases the velocity increases but it decreases rapidly in the boundary layer. The velocity decreases in the case of the moving plate i.e. for  $K_1 = 1$ . The variation of velocity distribution to the thermal and Solutal Grashof numbers is discussed in the figure 4. The thermal Grashof number is the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. The positive values of Grashof number indicates the cooling of the plate. The rise in the velocity is observed due to the enhancement of the thermal buoyancy force. As thermal buoyancy increases, the velocity increases rapidly near the plate and gradually decreases to free stream velocity. Similar to the thermal Grashof number, the Solutal Grashof number effect is also to increase in the velocity. The rise in the velocity distribution is observed from the figure.

Temperature profile decreases with increase in Hartman number presented in figure5. The effect of the heat generation ( $Q < 0$ ) or absorption ( $Q > 0$ ) on the temperature is shown in the figure 6. From the figure it is precisely observed that near the plate, the temperature increases with the heat generation while with the heat absorption within the boundary layer produces opposite effect. The temperature increases 106.89% with the heat generation i.e., as Q varies from -5 to -1 and temperature increases 111.19% as Q varies from 1 to 5.

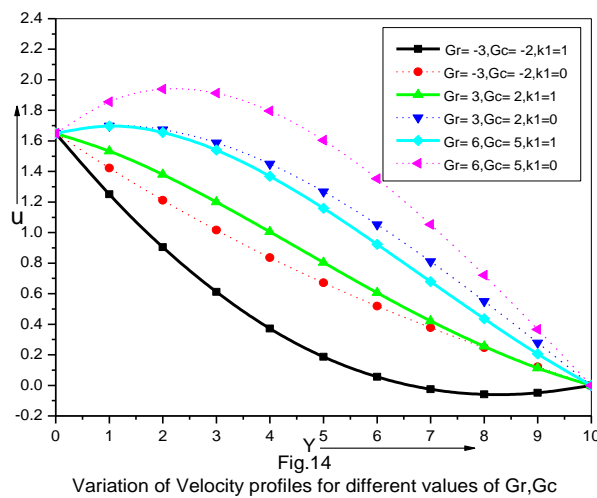
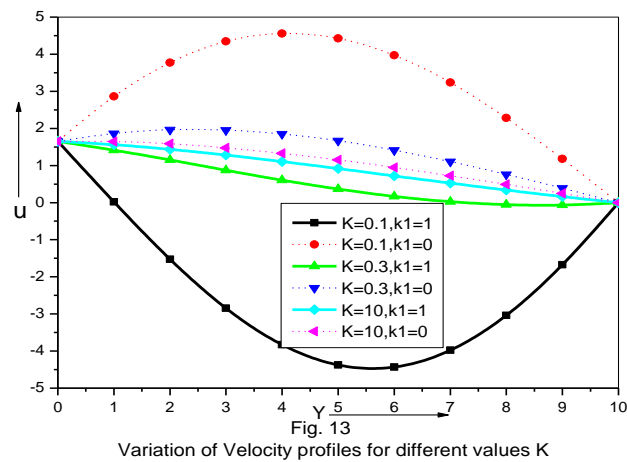
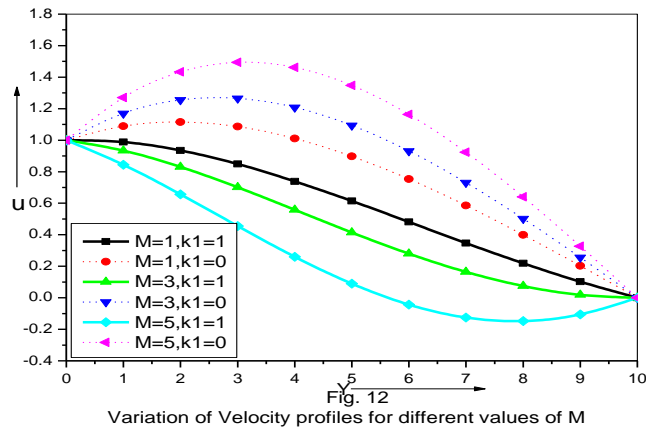
The temperature distribution decreases for increase of permeability of the medium with the moving plate. This effect is seen from the figure 7. For large value of K i.e.  $K = 10$  the temperature distribution remains almost same. The temperature distribution decreases with the increase of permeability in the boundary layer rapidly. With the increase of Prandtl number (Pr) the temperature distribution decreases. It also decreases in the boundary layer. This observation is shown in the figure 8. The temperature decreases with the increase of the viscosity i.e. with the increase of Eckert number



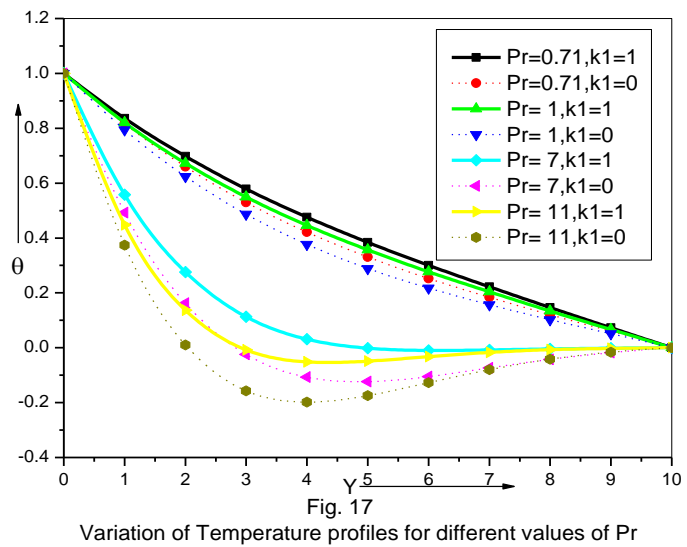
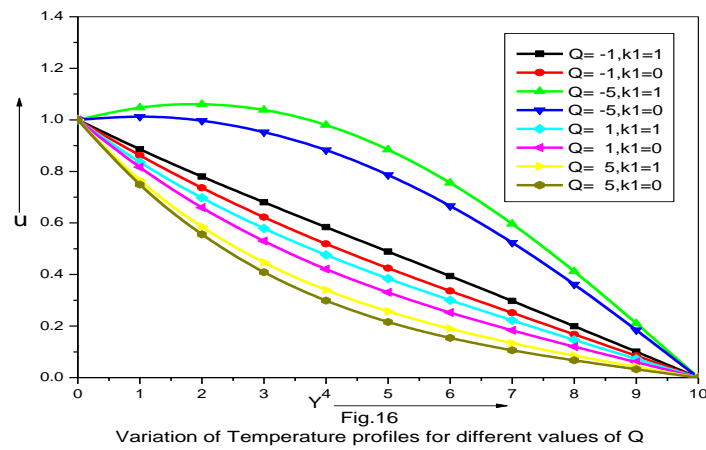
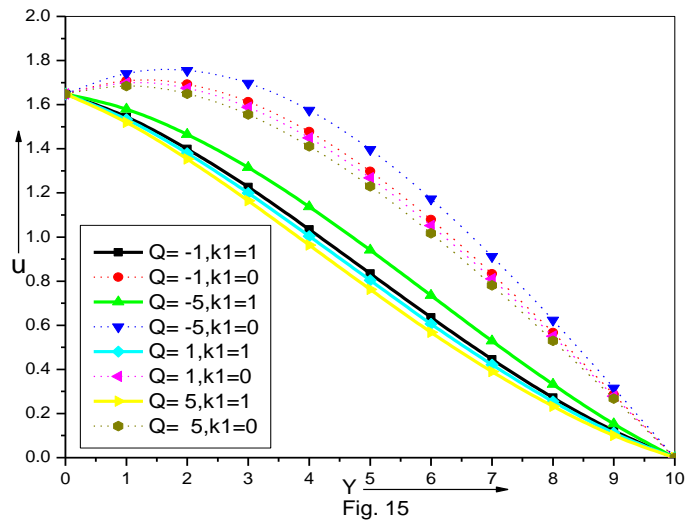
(Ec). But it decreases in the boundary layer rapidly. The temperature increases with the moving plate. This effect is observed in the figure 9.

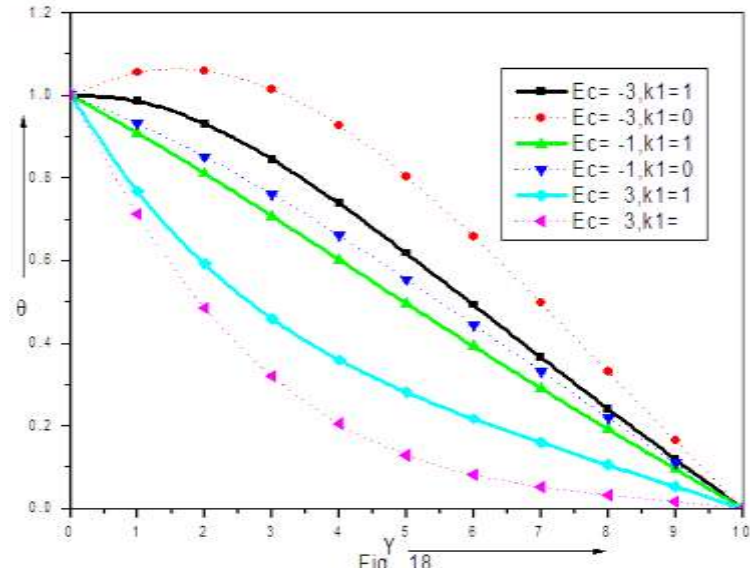
The concentration distribution is significantly affected by the presence of foreign species such as Hydrogen ( $Sc=0.22$ ), Oxygen ( $Sc=0.66$ ), Pentane ( $Sc=2.0$ ) and Octane ( $Sc=2.66$ ) which are given in the figure 10. The concentration is decreased with the increase of the Schmidt number i.e. with the presence of heavy foreign species. For the increase of chemical reaction parameter ( $\gamma$ ) also the concentration decreases. It is shown in the figure 11.

**(b) Uniformly Accelerated Plate (UAP)**

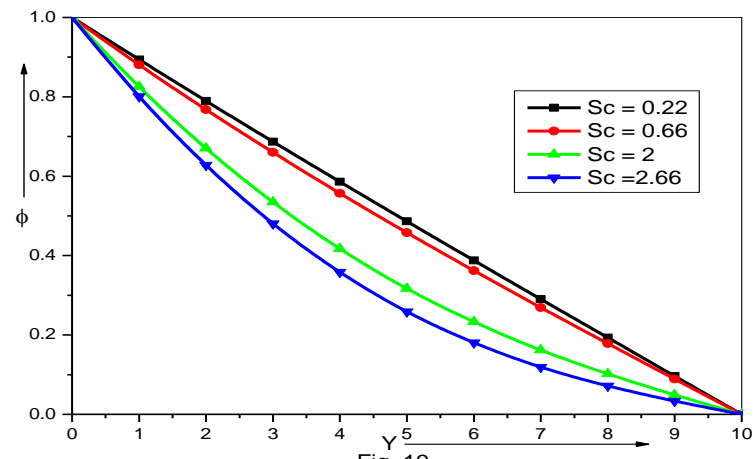




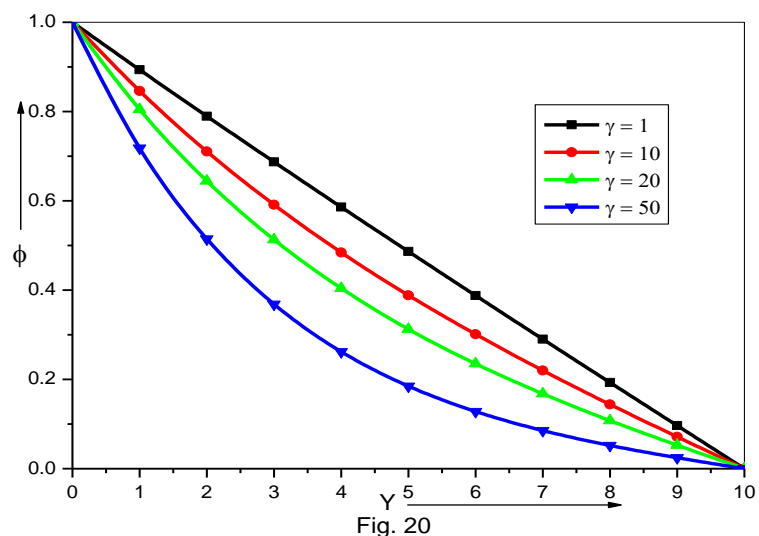




Variation of Temperature profiles for different values of  $E_c$



Variation of Concentration profiles for different values of  $Sc$



Variation of Concentration profiles for different values of  $\gamma$

The velocity profiles for Uniformly Accelerated Plate (UAP) are discussed through the graphs 12 to 15, when the magnetic field is being fixed to the fluid ( $K_1 = 0$ ) and to the moving plate ( $K_1 = 1$ ), for various physical parameters such as Hartmann number (M), Permeability Parameter (K), the heat generation or absorption parameter (Q), thermal Grashof number (Gr) and the Solutal Grashof number (Gc) etc.

The velocity variation with the magnetic field (M) is shown in the figure 12 near a Uniformly Accelerated Plate (UAP). The velocity decreases with the increasing transverse magnetic field when  $K_1 = 0$  where as it exhibits the opposite effect away from the plate when  $K_1 = 1$ .

The gradual decrease of velocity with the increasing permeability (K) is observed for  $K_1 = 0$  where as it exhibits the opposite effect away from the plate when  $K_1 = 1$  which is shown in the figure 13.

The variation of velocity distribution to the thermal and Solutal Grashof numbers is meticulously discussed in the figure 14. The rise in the velocity is carefully observed due to the enhancement of the thermal buoyancy force. As thermal buoyancy increases, the velocity increases rapidly near the plate and gradually decreases to free stream velocity. Similar to the thermal Grashof number, the modified Grashof number effect is also to increase in the velocity.

The velocity variation with heat generation ( $Q < 0$ ) and absorption ( $Q > 0$ ) is studied through the figure 15. On absorption of heat ( $Q > 0$ ) the velocity increases but with heat generation it showed an opposite behavior. The figure shows a decrease of velocity from near the plate to the moving plate.

The effect of the heat generation ( $Q < 0$ ) or absorption ( $Q > 0$ ) on the temperature is shown in the figure 16. From the figure, it is observed that near the plate the temperature decreases with the heat generation while with the heat absorption within the boundary layer produces opposite effect. The temperature decreases 106.89% with the heat generation i.e., as Q varies from -5 to -1 and temperature decreases 111.19% as Q varies from 1 to 5. The temperature decreases 5.30% and 7.53% in the case of heat absorption i.e., as Q varies from -5 to -1 and 1 to 5 respectively.

The temperature of the fluid with the increase of Prandtl number is given in the figure 17 when  $K_1 = 0$  and  $K_1 = 1$ . The temperature distribution rapidly decreases in the boundary layer with the increase of Prandtl number. The temperature decreases with the increase of the viscosity i.e. with the increase of Eckert number (Ec). But it decreases in the boundary layer rapidly. The temperature increases with the moving plate. This effect is observed in the figure 18.

The concentration distribution is significantly affected by the presence of foreign species such as Hydrogen ( $Sc=0.22$ ), Oxygen ( $Sc=0.66$ ), Pentane ( $Sc=2.0$ ) and Octane ( $Sc=2.66$ ) which are given in the figure 19. The concentration is decreased with the increase of the Schmidt number i.e. with the presence of heavy foreign species. For the increase of chemical reaction parameter ( $\gamma$ ) also the concentration decreases. It is shown in the figure 20.

## 9. Conclusions

The non linear coupled equations are solved by using numerical technique and presented the results as

1. Velocity profiles decrease with increase of M when magnetic field fixed to the moving plate and opposite nature when magnetic field fixed to the fluid in both EAP and UAP cases.
2. Increase in Gr & Gc causes the decrease in velocity profiles.
3. Temperature profiles decrease for heat generation and increase for heat absorption in both EAP and UAP cases.
4. Temperature profiles decrease for increase in Pr, Ec and increase for porosity parameter.
5. The increase in Schmidt number and Chemical Reaction parameter causes the decrease in Concentration profiles.

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