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Frequency Distribution of a Resistance formed by Series combination through Convolution

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Abstract:Methods based on the Fourier transform are used in virtually all areas of engineering and science. Fourier transform is a pervasive and versatile tool. Fourier transform, especially convolution and some special functions, is a very useful tool to solve problems of probability distribution.Numerical practice on serial products confirms the feeling for convolution and incidentally draws attention to practical character of numerical evaluation. Here in this paper, the authors used Unit rectangle function, convolution and central limit theorem to solve the problem from Probability distribution of serial product.*

Keywords: *Fourier transforms, Convolution, Central limit theorem, Unit rectangle function, Probability distribution.*

I. INTRODUCTION

Fourier transform is a vital in so many fields. It plays an important role in the theory of many branches of science. It is a great advantage to be able to more from one physical field to another and to carry over the experience already gained, but it is necessary to have the key which interprets the terminology of the new field. Fourier transform arises in Statistics and in physical subjects involving random phenomena in several ways, many of them traceable to a certain convolution relation that plays a basic role. In this paper we used Fourier transform to solve the problem of Probability distribution.

II.THE FOURIER TRANSFORM

A. Definition

The function$F(s)$defined by$F(S) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi xs}\,dx$is called the Fourier Transform of $f(x)$and$f(x) = \int_{-\infty}^{\infty} F(S)e^{i2\pi xs}\,dS$is called the inverse Fourier transform ofF(s).

Existence of the Fourier Transform

1. $f(x)$is piecewise continuous on every finite interval.
2. $f(x)$ is absolutely integrable on the x-axis.

B. Convolution

Convolution is a composition of products. Not only is convolution widely significant as a physical concept, it also offers an advantageous starting point for theoretical developments. Conversely, because of its adaptability to computing, it is an advantageous terminal point for theory before one starts numerical work.

The convolution of two functions $f(x)$and $g(x)$is defined as

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du$$

The convolution itself is also a function of x.The operation of convolution is commutative, associative and also distributive over addition.

C.Central limit theorem

If a large number of functions are convolved together, the resultant may be very smooth, and as the number increases indefinitely, the resultant may approach Gaussian form. The rigorous statement of this tendency of protracted convolution is the central-limit theorem.

The Central-limit theorem in its general form states that under applicable conditions the convolution of $n$functions is equal to a Gaussian function whose variance is the sum of the variances of the different functions, plus a remainder which diminishes with increasing n in a certain way.

III.PROBABILITY DISTRIBUTION

To each valuex_i, we associate a number, $p_i = P(X = x_i)$then p_i is called the Probability distribution of random variable X, provided $p_i, i = 1,2, \dots$ satisfies the following conditions:

i) $p_i \geq 0$, for all i

ii) $\sum_{i=1}^{\infty} p_i = 1$; for a discrete random variable X which takes values$x_1, x_2, \dots$
and $\int_1^{\infty} p_i = 1$; for a continuous random variable X which takes values$x_1, x_2, \dots$

iii) $P(A \cup B) = P(A) + P(B)$.

If we have a large number of $n_1$resistances with $a\%$ tolerance, the resistance range is from $n_1 - a$ to $n_1 +$ aand there are roughly as many in 1-ohm interval as in any other. The variations are compatible with those to be expected if the resistors were drawn from an infinite number of supply containing equal numbers of resistors in any 1-ohm interval between $n_1 -$ a to $n_1 + a$ and none outside that range. This is a consequence of the method of manufacture and sorting by which the resistors are produced. Let us suppose, therefore, that our stock of resistors has been drawn from a supply in which the frequency of occurrence of resistances between R and $R + dR$is $P_1(R)dR$, where

$$P_1(R) = \frac{1}{2a} \prod \left(\frac{R - n_1}{2a}\right) \dots \dots \dots \dots (1)$$

where$\prod(\mathrm{R})$ is the unit rectangle function, which is defined by

$$\prod(x) = \begin{cases} 0 & |x| > 1/2 \\ (1/2 & |x| = 1/2) \\ 1 & |x| < 1/2 \,. \end{cases}$$

and$f(x) = h\prod\left(\frac{x-c}{b}\right)$ is a displaced rectangle function of height h and base b, centered on $x = \mathrm{c}$.Properties of the unit rectangle function are,

1) $\int_{-\infty}^{\infty} \prod(x) dx = 1$

2) $\prod^n(x) = \prod(x)$

3) $\prod(x)\prod\left(x/b\right) = \prod(x), b > 1$

4) More general: multiplication with $\prod\left(x/b\right)$

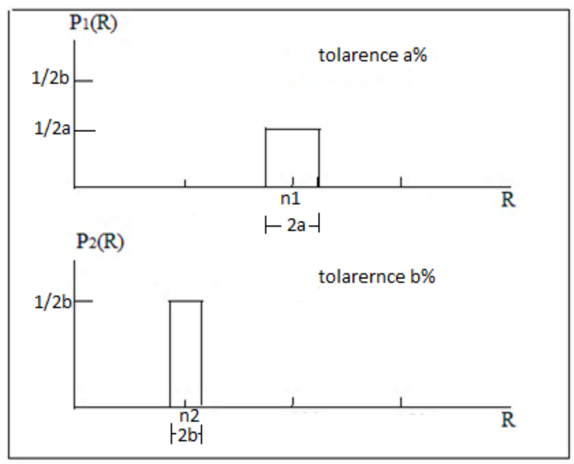

Fig.1 Frequency Distribution of resistance R of two stocks $\boldsymbol{n_1}$ ***and*** $\boldsymbol{n_2}$.

changes only functions, for which $f\left(|x| > \frac{b}{2}\right) \neq 0$.

The factor $1/2a$in equation-1 allows for the requirement that

$$\int_0^{\infty} P_1(R) dR = 1 \dots \quad \dots \quad \dots \quad (2)$$

The frequency distribution $P_1(R)$ is shown in Figure 1.

If we also have a stock of n_2-ohm resistors (we take $n_1 > n_2$), with a b% $\left(= a/_2\%\right)$ tolerance, we have a second frequency distribution $P_2(R)$ given by

$$P_2(R) = \frac{1}{2b} \prod \left(\frac{R - n_2}{2b}\right) \dots \quad \dots \quad \dots \quad (3)$$

Now we build an electric circuit of $n_1 + n_2$ resistors by using$n_1$and n_2 resistors in series, and we require to know what tolerance should be ascribed to the composite element. In other words, we want to find the frequency distribution $P(R)$of resistance lying between the minimum possible value of $\left((n_1 + n_2) - (a + b)\right)$-ohms and the maximum of $\left((n_1 + n2+a+b\right.$-ohms.

It is seen that both $P_1(R)$ and $P_2(R)$ enter into the determination of $P(R)$. The problem is solved as follows.

Let the resistor picked from the n_1-ohm stock have a resistance R'; then if the series combination is to have total resistance R, the resistor from the n_2-ohm stock will have to have resistance $R - R'$. The frequency of occurrence of a total resistance R will be the product of the frequencies of occurrence of R′ in the first component and R − R′ in the second, integrated over the range of possibilities of R′; that is

$$P(R) = \int_{-\infty}^{\infty} P_1(R')P_2(R - R')dR' \dots \quad \dots \quad \dots \quad (4)$$

In this integral the limits of integration could be put $(n_1 - a)$to $(n_1 + a)$, since that is the range of possible values of R′ in the particular circumstances under consideration. Since, outside the range of $(n_1 - a)$to $(n_1 + a)$,$P_1(R)$ has been defined to be zero, the value of the integral is not affected by the use of infinite limits, and there is a gain in generality of the formula.

By inspection we say that equation-4 is a convolution of $P_1(R)$and $P_2(R)$; in fact in the asterisk notation,

$$P = P_1 * P_2.$$

This is instance of the basic convolution relation between the frequency distribution describing the sum of two quantities and the frequency distributions of the given quantities. We show this result as follows,

Here resistance R remains between $\big((n_1 + n_2) - (a + b)\big)$ ohms to $\big((n_1 + n_2) + (a + b)\big)$ ohms.

Now let$R = \big((n_1 + n_2) - (a + b)\big)$ then

$$P\big((n_1 + n_2) - (a + b)\big) = \int_{n_1-a}^{n_1+a} P_1(R')P_2\Big(\big((n_1 + n_2) - (a + b)\big) - R'\Big)\, dR'$$

$$= \frac{1}{4ab}\int_{n_1-a}^{n_1+a} \prod\left(\frac{R' - n_1}{2a}\right)\prod\left(\frac{\Big(\big((n_1 + n_2) - (a + b)\big) - R'\Big) - n_2}{2b}\right) dR'$$

As$n_1 - a \le R' \le n_1 + a \;\Rightarrow\; -(n_1 + a) \le R' \le -(n_1 - a)$

$$\Rightarrow\; \big((n_1 + n_2) - (a + b)\big) - (n_1 + a) \le \big((n_1 + n_2) - (a + b)\big) - R' \le \big((n_1 + n_2) - (a + b)\big) - (n_1 - a)$$

$\Rightarrow n_2 - (2a + b) \le R - R' \le n_2 - b$

But $R - R'$ is defined for $n_2 - b$to $n_2 + b$.

$$\text{So}P\big((n_1 + n_2) - (a + b)\big) = 0$$

Similarly, if $R < (n_1 + n_2) - (a + b), P(R) = 0.$

Now, if $R = (n_1 + n_2) - a$ then

$$P\big((n_1 + n_2) - a\big) = \int_{n_1-a}^{n_1+a} P_1(R')P_2\Big(\big((n_1 + n_2) - a\big) - R'\Big)\, dR'$$

$$= \frac{1}{4ab}\int_{n_1-a}^{n_1+a} \prod\left(\frac{R' - n_1}{2a}\right)\prod\left(\frac{\Big(\big((n_1 + n_2) - a\big) - R'\Big) - n_2}{2b}\right) dR'$$

As$n_1 - a \le R' \le n_1 + a \;\Rightarrow\; -(n_1 + a) \le R' \le -(n_1 - a)$

$$\Rightarrow\; \big((n_1 + n_2) - a\big) - (n_1 + a) \le \big((n_1 + n_2) - a\big) - R' \le \big((n_1 + n_2) - a\big) - (n_1 - a)$$

$$\Rightarrow\; n_2 - 2a \le R - R' \le n_2$$

But $R - R'$ is defined for $n_2 - b$to $n_2 + b$. So up to $n_2 - 2a$to $n_2 - b$, $P_2\big((n_1 + n_2) - a - R'\big) = 0$

So, for $n_2 - 2a$ to $n_2 - b$

$$P\big((n_1 + n_2) - a\big) = 0$$

Since $a > b \;\Rightarrow\; n_2 - 2a < n_2 - 2b$ and we know that $n_2 - 2b < n_2 - b$

$$\Rightarrow n_2 - 2a < n_2 - 2b < n_2 - b < n_2$$

And from $n_2 - b$to n_2, $P\big((n_1 + n_2) - a\big) = \frac{1}{4ab}.b$ $\quad$ ($\because$ length of $n_2 - b$ to n_2 is b)

$$\Rightarrow P\big((n_1 + n_2) - a\big) = \frac{1}{4a}.$$

For $R = (n_1 + n_2) - b$,

$$P\big((n_1 + n_2) - b\big) = \int_{n_1-a}^{n_1+a} P_1(R')P_2\Big(\big((n_1 + n_2) - b\big) - R'\Big)\, dR'$$

As $n_1 - a \le R' \le n_1 + a$

$$\Rightarrow\; (n_1 + n_2) - b - (n_1 + a) \le (n_1 + n_2) - b - R' \le (n_1 + n_2) - b - (n_1 - a)$$

$$\Rightarrow\; n_2 - (a + b) \le R - R' \le n_2 + (a - b)$$

For $n_2-(a+b)$to $\mathrm{n}_2-\mathrm{b}$; $P_2\left(\left((n_1+n_2)-b\right)-R'\right)=0 \Rightarrow P\left((n_1+n_2)-b\right)=0$
Since $n_2-(a+b)<n_2-b$
For $n_2-b$to $n_2+(a-b)$, length of the interval is a.
So, $P\left((n_1+n_2)-b\right)=\frac{1}{4ab}.a \Rightarrow P\left((n_1+n_2)-b\right)=\frac{1}{4b}$.
Similarly,
For $R=n_1+n_2, P(n_1+n_2)=\frac{1}{4ab}a=\frac{1}{4b}$.
And for $R=(n_1+n_2)+b, P\left((n_1+n_2)+b\right)=\frac{1}{4ab}a=\frac{1}{4b}$.
that is, between $(n_1+n_2)-b$ to $(n_1+n_2)+b$, $P(R)=\frac{1}{4b}$.
Nowfor $R=(n_1+n_2)+a, P\left((n_1+n_2)+a\right)=\frac{1}{4ab}b=\frac{1}{4a}$.
For $R=(n_1+n_2)+(a+b), P\left((n_1+n_2)+(a+b)\right)=0$.
Now the equation for $R=(n_1+n_2)-(a+b)$to $R=(n_1+n_2)-b$ is, that is, for $\left((n_1+n_2)-(a+b),0\right),\left((n_1+n_2)-b,14b\right.$ is

$$\frac{P(R)-0}{{}^{1}\!/_{4b}-0}=\frac{R-(n_1+n_2)-(a+b)}{(n_1+n_2)-b-(n_1+n_2)-(a+b)}$$

$$\Rightarrow \quad P(R)=\frac{R-(n_1+n_2)-(a+b)}{4ab}.$$

Equation for $R=(n_1+n_2)-b$to $=(n_1+n_2)+b$, that is,
for$\left((n_1+n_2)-b,1/4b\right),\left((n_1+n_2),\frac{1}{4b}\right),\left((n_1+n_2+b),\frac{1}{4b}\right)$is

$$P(R)={}^{1}\!/_{4b}.$$

Equation for $R=(n_1+n_2+b)\ to\ R=(n_1+n_2+a+b)$, that is, for $\left((n_1+n_2+b),1/4b\right),\left((n_1+n_2+a),1/2a\right)$, $\left((n_1+n_2+a+b),0\right)$is

$$\frac{P(R)-0}{{}^{1}\!/_{4b}-0}=\frac{R-(n_1+n_2+a+b)}{(n_1+n_2+b)-(n_1+n_2+a+b)} \quad\Rightarrow\quad P(R)=\frac{\left((n_1+n_2)+(a+b)\right)-R}{4ab}.$$

Summary:

$$P(R)=\begin{cases} 0 & ; \quad R\leqq(n_1+n_2)-(a+b) \\ \dfrac{R-(n_1+n_2)-(a+b)}{4ab} & ; \quad (n_1+n_2)-(a+b)<R\leqq(n_1+n_2)-b \\ {}^{1}\!/_{4b} & ; \quad (n_1+n_2)-b\leqq R<(n_1+n_2)+b \\ \dfrac{(n_1+n_2)+(a+b)-R}{4ab} & ; \quad (n_1+n_2)+b\leqq R<(n_1+n_2)+(a+b) \\ 0 & ; \quad R\geqq(n_1+n_2)+(a+b) \end{cases}$$

Total Area under $P(R)=$ Area of ΔAFB + Area of $\blacksquare$BCEF + Area of ΔCED

$$=\frac{1}{2}(a)\left(\frac{1}{4b}\right)+(2b)\left(\frac{1}{4b}\right)+\frac{1}{2}(a)\left(\frac{1}{4b}\right)=\frac{a}{4b}+\frac{1}{2}$$

$$=1(\because a=2b)$$

that is, The area under the convolution is the product of the areas under the convolved functions; since both $\mathrm{P}_1(\mathrm{R})$ and $\mathrm{P}_2(\mathrm{R})$ had unit area so the same should be true of $\mathrm{P(R)}$.

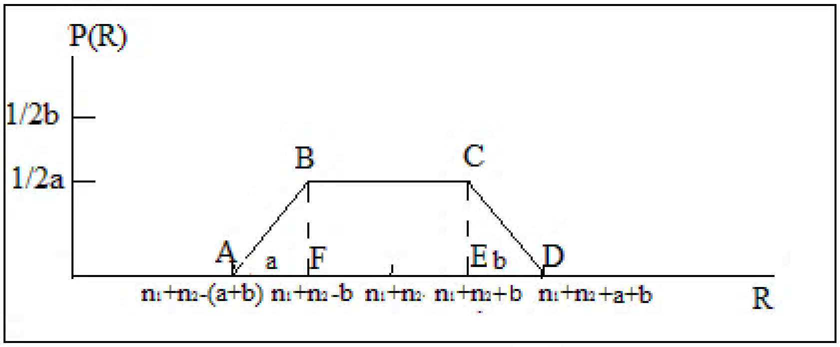

Fig 2Frequncy distribution of a Resistance R formed by series combination of two stocks, having tolerance $a\%$ and $b\%$ with $a = 2b$.

IV.CONCLUSION

The composite resistance has a mean value of $(n_1 + n_2)$-ohms and is distributed trapezoidally as shown in figure-2. If more elements were connected in series, then the tendency toward a Gaussian result in accordance with the central limit theorem would be more advanced, and the use of the standard deviation instead of the extreme range as a measure of spread would be quite appropriate.

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