

**Prediction Of Maximum Crack Width For Moderate Deep Beam**

Influence On Crack Width and Crack Pattern Due To Size Effect in Reinforced Concrete Moderate Deep Beam

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Abstract — A new formula for predicting maximum crack width in reinforced concrete moderate deep beams have been developed incorporating size effect parameters. Ratio of effective length to overall depth of beam, diameter of bar, longitudinal reinforcement ratio have been taken as modifying factor for crack width prediction formula. The performance of proposed formula is checked using the author's test results which includes maximum crack width measurement, crack pattern, modes of failure, propagation load, ultimate and first crack load etc. from 10 reinforced concrete beams without shear reinforcement. The results of proposed formula well correlates with the experimental results.

Keywords- specimens, Beams, Reinforced cement concrete, maximum crack width, moderate deep beams, size effect, first crack load

I. INTRODUCTION

The recent tendency towards utilizing performance based concept in design more efficiently is a driving force of the investigation on cracking characteristics. Cracking analysis constitutes a major step in the serviceability design of concrete structures. Cracks of excessive widths contribute to corrosion of the reinforcement, surface deterioration and consequently damage the structure's long-term wellbeing. The occurrence of cracks in RC structures is unavoidable because of the low tensile strength of concrete. Cracks form when the tensile stress in concrete exceeds its tensile strength[22]. Cracks in a RC member will always be a threat for the satisfactory performance and serviceability of the structure; it has significant influence on serviceability, durability, aesthetic and force transfer. Hence, prediction and control of cracking and crack widths is essential for serviceability considerations of reinforced concrete beams.

Prediction of crack widths has been studied by many researchers (Base et. al., 1966; Broms [23], 1965; Gergely and Lutz, 1968[19]; Meier and Gergely, 1981[24]; Suri and Dilger, 1986). Which are related to crack control in concrete structures provide some design formulae for crack width prediction, most of them were originally developed for tensile and flexural crack width. They were experimentally obtained and cannot be applied directly to shear crack width prediction, because shear cracking is caused by a different mechanism.

In reinforced concrete beams subjected to shear forces, shear cracks form diagonally with an inclination towards the axis of the beam. These inclined shear cracks can begin as flexural cracks or inside the web area. According to ASCE-ACI joint Committee 426 (ASCE-ACI 1973), the shear failure mechanism in RC beams is characterized by the occurrence of inclined shear cracks either before or after a flexural crack forms nearby. A number of investigations in last decades (Adebar and Leeuwen 1999; Adebar 2001; De Silva *et al.* 2005, 2008; Hassan *et al.* 1985, 1987, 1991; Witchukreangkrai *et al.* 2004, 2006; Piyamahant 2002; Collins *et al.* 2007; Sherwood *et al.* 2007; Zararis 2003) were focused on shear cracking mechanism and diagonal shear failure in RC members. In spite of these studies, the factors affecting the spacing between shear cracks and shear cracks width are still not known for all conditions.

Maximum crack width can be affected by various parameters such as longitudinal reinforcement ratio, clear cover, side cover, steel stress and strain, diameter of bar, size of beam (L_e/D), effective area of concrete around steel bar, bond between reinforcement and concrete etc. Hassan *et al.* (1985, 1987 and 1991) [25-27] carried out one of the most significant studies concerning shear cracking mechanism in RC beams. In those studies, the factors to affect shear crack width which were shear reinforcement characteristics (bond characteristics, spacing, angle with member axis and its configuration) and ratio of shear span to depth were well investigated. Furthermore, Witchukreangkrai *et al.* (2004, 2006) reported that the stirrup ratio has an important effect on shear crack width in RC beams and prestressed concrete beams. While Piyamahant (2002) showed that shear crack width depends on the compressive strength of concrete and diameter of the shear reinforcement.

Based on the previous literature survey as shown above, it can be concluded that the understanding of shear cracking behavior in RC moderate deep beams has not been well clarified. This paper tries to throw the light on crack width prediction factor responsible for shear cracking of moderate deep beam. The objectives of the present study are to clarify the effect of beam size, effective length to overall depth ratio, stress in longitudinal reinforcement, longitudinal reinforcement ratio and diameter of bar on the diagonal crack width in reinforced concrete beams by conducting the experiment of 10 simply supported beam specimens. In the experiment shear crack widths were measured by digital Vernier gauge with a precision of 0.01mm at various loading stages. A new formula developed considering size effect

parameter using statistical analysis in SPSS software provides satisfactory results when compared with experimental values. Effect of fibers on crack width shows better results as it is intended to be used.

II. EXPERIMENTAL INVESTIGATION

A test program was carried out to study the cracking behavior of reinforced concrete moderate deep beam. Total of 10 beams were experimentally investigated for crack width development and widening.

A. Specimens

Moderate deep beams of 10 different sizes were taken to identify the size effect on crack width. Each of the beam have rectangular cross section and constant breadth (b) of 75mm. Shear span to depth ratio was kept 1 for specimen A2 to E and for remaining five specimens (F to J) it was kept 2. Specimens' details are given in Table 1.

Table 1 Details of investigated specimens.

BEAM NAME	Over All Depth (mm)	Effective Depth (mm)	Shear Span (mm)	Effective Length (mm)	Concrete Cover (mm)	a/d	Le/D	Diameter of bar(Φ) (mm)	Longitudinal reinforcement ratio(ρ)
A2	225	200	200	800	25	1	3.56	2#8 Φ	0.006698667
B	275	250	250	900	25	1	3.27	2#10 Φ	0.008373333
C	325	300	300	1000	25	1	3.08	2#10 Φ	0.006977778
D	375	350	350	1100	25	1	2.93	2#10 Φ	0.005980952
E	425	400	400	1200	25	1	2.82	2#12 Φ	0.007536
F	175	150	300	600	25	2	3.43	2#8 Φ	0.008931556
A1	225	200	400	800	25	2	3.56	2#8 Φ	0.006698667
H	275	250	500	1000	25	2	3.64	2#10 Φ	0.008373333
I	325	300	600	1200	25	2	3.69	2#10 Φ	0.006977778
J	375	350	700	1400	25	2	3.73	2#10 Φ	0.005980952

B. Material

All 10 specimens were cast in adjustable molds made of cast iron. Grade of concrete used for the investigation was M25. Mix proportioning of the material was obtained according to IS: 10262-2009. Maximum size of aggregate utilized was 20mm though the blending of 65:35 for 20:10mm aggregates respectively was adopted to achieve proper workability of 100mm slump value. In the mix proportions of concrete, ordinary Portland cement was used and water-cement ratio was kept at 0.45 with the addition of an admixture (refer Table 2). In all test specimens, high yielding strength deformed reinforced bar (HYSD) of grade Fe500 with diameter as shown in Table 1 were used as longitudinal reinforcement.

Table 2 Mix proportions of concrete.

Characteristic cube strength	25MPa
Cement type	OPC-53 grade
Maximum aggregate size	20mm
Slump for concrete	100mm
Free-water content	152 kg/m ³
Cement content	338 kg/m ³
Coarse aggregate content	1132 kg/m ³
Fine aggregate content	837 kg/m ³
Water Cement Ratio	0.45
Admixture (Superplasticizer)	2.36 kg/m ³

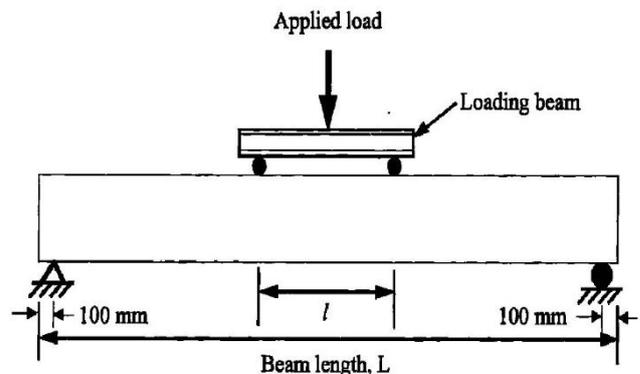


Figure 1 Loading configuration

C. Instrumentation and test procedure

Tests were carried out using a universal testing machine having 200 tones direct compression capacity and 100 tones bending capacity. All the beams were simply supported with different spans varying from 700mm to 1500mm. beams were centered on platform and levelled horizontally and vertically to achieve uniform distribution of load. Bubble tube was used to check levelling of specimens. Surface of loading and support plates were cleaned with wire brush to avoid instability of

system during loading which could be caused by sudden braking of layer of concrete slurry. To easily identify the difference between cracked and uncracked specimens, photographs were taken prior to load application of each beam. The load was applied through the two loading points for beams A2 to E and through central loading points for beams F to J using a series of load stages. Interval of loading varies for each of the specimen. At each interval, the loading was halted and all cracks were marked with a black marker on white painted surface and the crack width was measured using Digital Vernier Gauge. Crack propagation load and points at which crack width was to be measured, were marked by a dash line directly on the surface of beams. To record the reading of width of cracks, an observation sheet was prepared in Auto CAD. Sheet contains drawing of beam scaled to 1:100. Grid lines were created to accurately mark the points.

III. DERIVATION OF CRACK WIDTH FORMULA

A. General Remarks

Widening of the crack depends on many variables. Steel stress (f_s) is the most important variable to be considered in evaluating the crack width. Since the width of a crack is usually desired at working loads, the elastic cracked section theory was used to evaluate the steel stress. This was done by Hognestad, Rusch-Rehm, Clark and Broms in their investigation. Gergely and Lutz have shown that crack width is in direct relation with $\beta f_s^3 \sqrt{d_c A_0}$. Many researchers have worked for estimating the effect of Φ/ρ ratio on crack width. Size of the beam also have a significant effect on crack widening, propagation and pattern.

Based on the above findings, two variables were incorporated in the statistical analysis used herein to derive the maximum shear crack width formula for moderate deep beam. They are:

- a) the Φ/ρ ratio;
- b) L_e/D ratio.

The regression equation takes the form:

$$W_{max} = \frac{C \beta f_s^3 \sqrt{d_c A_0} \left(\frac{\Phi}{\rho}\right)^{(K * L_e/D)}}{1 + \left(\frac{L_e}{D}\right)^B} \quad (1)$$

Where C, K and B are the regression coefficients to be determined from the statistical analysis.

B. Maximum Crack Width Formula

The relevant data from ten reinforced concrete beams were used in the proposed regression analysis, the relevant data for which are presented in Table 1. Regression analysis was carried out using SPSS software with the help of 58 beams crack width data. Ratio of L_e/D varies from 3 to 4 which comes under moderate deep beam. The size of beams were so selected that its effect on crack width were clearly noticeable. Percentage of longitudinal reinforcement was varied for all of the specimens.

The solutions for the regression coefficients (after appropriate rounding) led to the following equation for predicting the maximum shear crack width:

$$W_{max} = \frac{0.03217 \beta f_s^3 \sqrt{d_c A_0} \left(\frac{\Phi}{\rho}\right)^{(0.052 * L_e/D)}}{1 + \left(\frac{L_e}{D}\right)^{-1.245}} \times 10^{-3} \quad (2)$$

Where,

- W_{max} = Maximum crack width in mm
- β = $\frac{h - x}{d - x}$
- Φ/ρ = Diameter of bar/Longitudinal reinforcement ratio
- L_e = Effective length of beam in mm
- D = Overall depth of beam in mm
- f_s = Steel stress = $m \frac{M}{I_{cr}} (d - x)$ in N/mm^2
- m = Modular ratio (E_s/E_c)
- I_{cr} = $b \frac{x^3}{3} + m A_{st} (d - x)^2$ in mm^4
- x = Depth of neutral axis of a cracked section in mm
- M = Bending moment in $N \cdot mm$
- A_{st} = Area of steel reinforcement in mm^2
- d = Effective depth in mm
- d_c = Distance from centroid of reinforcement to bottom of beam in mm
- A_0 = A_e/n effective area of concrete around steel bars ($2 * d_c * b/n$) in mm^2

Which is the new formula for predicting the maximum shear crack widths in reinforced concrete moderate deep beam.

IV. COMPARISION OF PROPOSED EQUATION WITH EXPERIMENTAL RESULTS

To check the relative performance of the proposed equation, crack widths obtained from equation (2) was again compared with experimental results and its graphical representation is shown in Figure 2

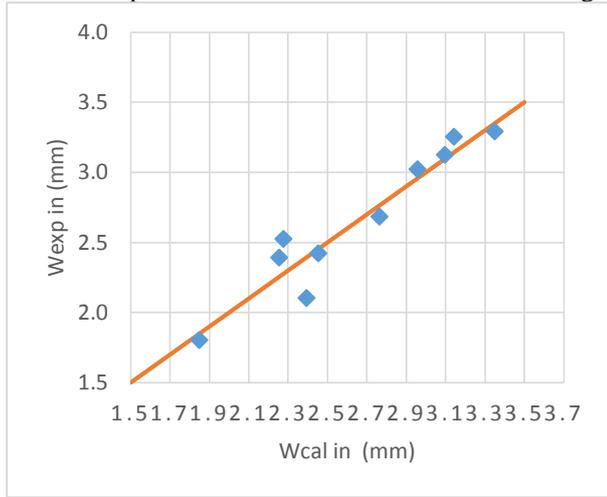


Table 3 Comparison Of Proposed Equation With Experimental Results

Beam Name	First Crack Load(KN)	Ultimate load(KN)	W_{exp} (mm)	W_{cal} (mm)	$\frac{W_{exp}}{W_{cal}}$
A2	48	135	3.12	3.10	1.008
B	51	178	2.52	2.28	1.107
C	52	193	2.39	2.25	1.060
D	55	219	2.1	2.39	0.877
E	85	258	1.8	1.85	0.974
F	27	71	3.25	3.14	1.034
A1	34	73	3.29	3.35	0.983
H	37	82	2.42	2.45	0.986
I	38	90	2.68	2.76	0.970
J	39	94	3.02	2.96	1.021

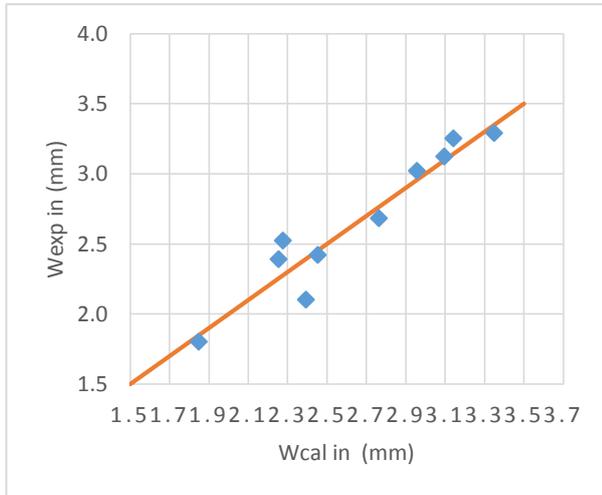


Figure 2 Comparison of Proposed Equation with the Experimental crack width

Obtained results shows scattered values with maximum error around $\pm 15\%$. Figure 2 shows that values are in a narrow range of unit slop and zero intercept line which expresses that W_{exp} values and W_{cal} values are almost equal. Above obtained formula for reinforced concrete moderate deep beam best fits with the experimental values. First crack load of specimens tested under two point loading system shows greater values compared to central point load test. A measurable difference in initial and ultimate load carrying capacity was observed for same specimen (A1 & A2) tested under pure bending case and central point load test.

V. CRACK PATTERN

Comparison of crack pattern was carried out in this investigation to get an idea about the effect of beam size and loading system on crack pattern. Amongst these 10 beams, 5 were tested under two point loading system while remaining's were tested under one point loading system. Flexure and diagonal shear cracks were developed in specimens same as in case of moderate deep beam it is observed previously in many researches papers.

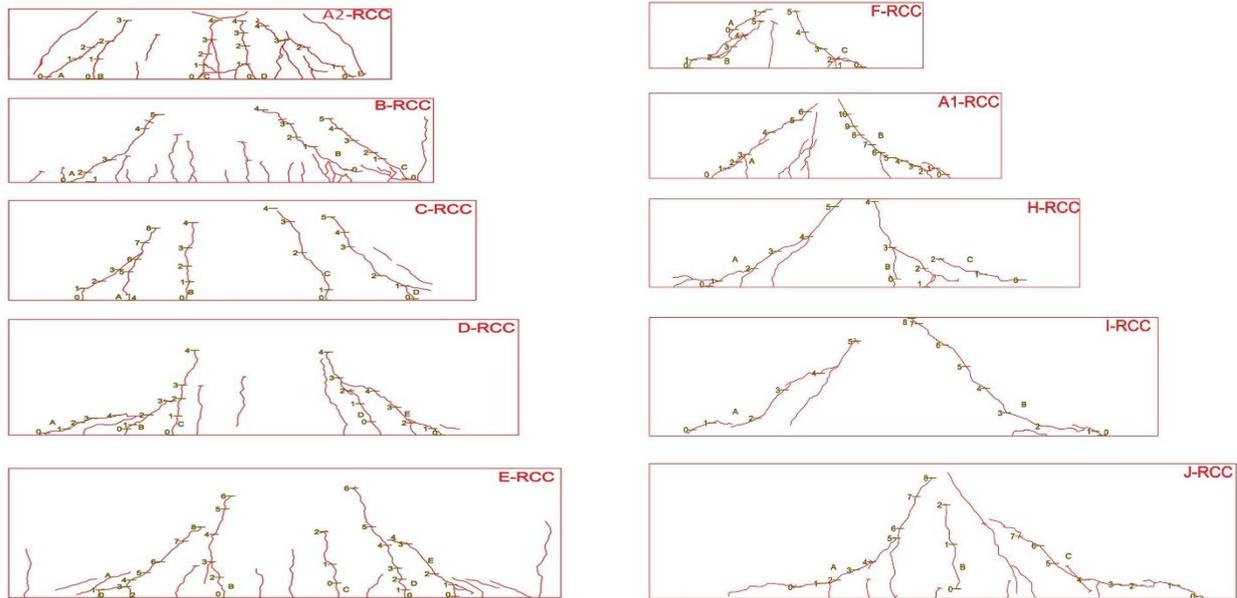


Figure 3 Crack Patterns of test specimens

Most common failure was shear failure in all of the beams as they were moderate deep beams. In A2 size specimens, first measurable crack developed in flexure zone at 8tn load as shown in Figure 3. Ultimate failure of A2 beams were observed in shear with the maximum crack width of 3.12mm. While same size of beam tested under 1 point loading system (A1) shows different results. First crack load and ultimate load decreases while on the other hand crack width increases. Specimens tested for pure bending case shows more number of minor cracks compared to other one. Crack widths decreases in case of two point loading system as moving from lower size of beams to the higher ones. Effect of bar diameter was clearly visible from crack pattern shown below which is as diameter increases, bonding of bars with concrete and thus effective area of concrete cylinder around bar increases. A vertical crack gets developed in side cover region though it cannot take a stand of major crack, causing failure of beams. Cluster of cracks initially develops in bottom phase of beam then get merged at middle one third portion of beam and finally propagate towards loading point.

VI. CONCLUSION

- A new formula for prediction of maximum crack width is obtained from statistical analysis and it gives satisfactory results within $\pm 15\%$ range when compared with experimental results.
- Initial load carrying capacity of beams increases as the size of beams increases.
- Ultimate load carrying capacity differs for a particular beam tested under two point and one point loading system. Same size of beam imparts higher value of ultimate load when tested under two point loading system while when tested under one point loading system, it gives lower value nearly half of the value obtained in above case.
- Bond between reinforcement and concrete plays a major role for deciding the width of crack as the whole stress transfer mechanism depends on it. Bond strength depends on the diameter of bar and longitudinal reinforcement ratio.
- The specimens remain elastic until flexural crack takes place. Diagonal cracks occur after flexural cracks have taken place and widen quickly under increasing load. As diagonal crack widths widen quickly, a few of them merge and develop into critical diagonal shear cracks that finally lead to beam failure.

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