

Control of Twin Rotor MIMO System (TRMS) Using PID Controller

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Abstract – This paper presents the PID control for the Non-linear system. Here Twin rotor multi input-multi output (MIMO) system considered as a nonlinear system. The twin rotor mimo system is a laboratory setup to perform control experiments. The control objective is to make the TRMS more quickly and accurately on the desired position i.e. PITCH angle and YAW angle. The control of TRMS is challenging task due to coupling effect between two axes. Here controlling of TRMS is done by considering two SISO system. i.e. control of pitch angle and yaw angle separately using PID controller.

Keywords – Non-linear system, Twin rotor MIMO system, PID controller, tuning methods.

I. INTRODUCTION

The control of any aerodynamic vehicle like helicopter has always been a challenging task due to its nonlinearity. The TRMS is a laboratory set up which resembles the behavior of helicopter in certain aspects [1],[2]. The control effort is to make the beam of TRMS to move quickly and accurately to the desired attitudes in terms of YAW angle and PITCH angle under the coupling effect between two axes. It is difficult to design a controller for TRMS due to its nonlinear behavior between two axes [4]-[6].

Several model have been proposed for TRMS and various control techniques have been used to analyze the performance of TRMS. In [7], the author have developed a highly nonlinear one DOF mathematical model of TRMS and a nonlinear inverse model of is developed for pitch channel of system. In [8], the author have developed a PID control scheme with derivative filter for TRMS with two DOF. In [9], the author discuss the PID tuning methods.

The mathematical modelling of TRMS is simulated using MATLAB/Simulink for controlling it. In this paper, a dynamic

model for TRMS is developed and PID controller is designed. The performance of the system is analyzed for step input.

II. SYSTEM DESCRIPTION

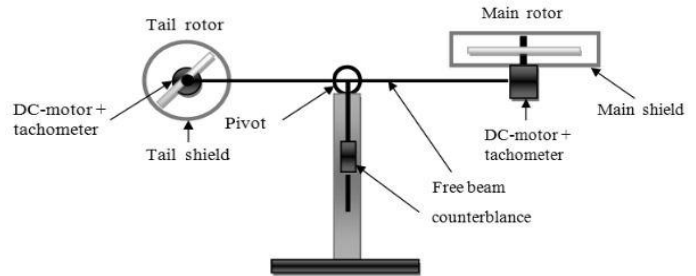


Fig. 1 Twin rotor MIMO system (TRMS)

Twin rotor MIMO system is highly nonlinear (small changes at input side, it leads to large change at output side), highly interactive and highly complex system. Controlling of such nonlinear system is very difficult task, because of cross-coupling between its parameters. In TRMS system, two DC motor connected via beam with tachometer to measure the speed of DC motor and two propellers are perpendicular to each other. This beam is placed on the base. This beam is joined at pivot point of the beam. Incremental encoder at pivot is used to measure angle of beam. The beam rotates freely in both the horizontal and vertical planes.

III. TRMS MODELLING

Mathematical Modelling can be done for one DOF and two DOF. In one DOF the whole system divided into two SISO (single input-single output) systems, one is YAW angle and another is PITCH angle. In one DOF both SISO system can be control separately. So coupling effect can't be consider in one DOF.

In two DOF, there is interaction between YAW angle and PITCH angle i.e. coupling occur between vertical plane and horizontal plane. In this paper, we are considering one DOF for controlling of TRMS.

A. Dc Motor Modelling

The TRMS is drive by two permanent magnet DC motors (PMDCs). One for the main propeller and other for tail propeller. The motors identical with different identical loads. The PMDC motor can be described by the following equations:

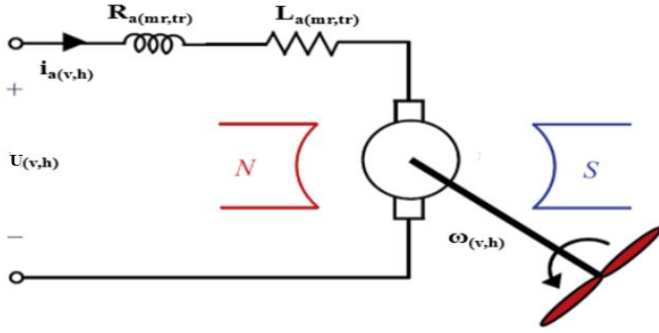


Fig 2. Permanent magnet DC motor model

$$U_{(v,h)} = R_{a(mr,tr)} \cdot i_{a(v,h)} + L_{a(mr,tr)} \cdot \frac{di_{a(v,h)}}{dt} \quad (1)$$

$$\frac{I(s)}{V(s)} = \frac{1}{Ls + R} \quad (2)$$

$$T_{e(v,h)} = J_{(mr,tr)} \cdot \frac{d\omega_{(v,h)}}{dt} + B_{(mr,tr)} \cdot \omega_{(v,h)} \quad (3)$$

Where, $U_{(v,h)}$ is the voltage control input for vertical and horizontal plane, $R_{a(mr,tr)}$, $i_{a(v,h)}$ and $L_{a(mr,tr)}$ are respectively armature resistance, armature current and armature inductance of main rotor and tail rotor, $T_{e(v,h)}$ is the electromagnetic torque, $J_{(mr,tr)}$ is the rotor moment of inertia, $B_{(mr,tr)}$ is the rotor damping coefficient and $\omega_{(v,h)}$ is the rotational velocity.

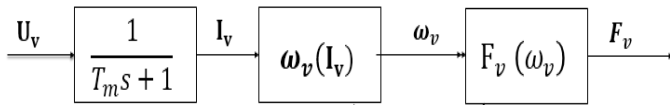


Fig 3. DC motor plus propeller subsystem for vertical plane

$$\omega_v(I_v) = \sum_{i=1}^6 \{p1(i) * I_v^i\} \quad (4)$$

$$F_v(\omega_m) = \sum_{i=1}^5 \{p2(i) * \omega_m^i\} \quad (5)$$

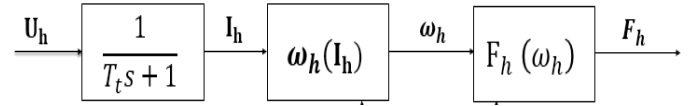


Fig 4. DC motor plus propeller subsystem for horizontal plane

$$\omega_h(I_h) = \sum_{i=1}^5 \{p3(i) * I_h^i\} \quad (6)$$

$$F_h(\omega_i) = \sum_{i=1}^5 \{p4(i) * \omega_i^i\} \quad (7)$$

B. Nonlinear model

The mathematical model of TRMS is developed under the following assumptions:

- The dynamics of the propeller subsystem can be expressed by first order differential equations.
- The friction in the system is of the viscous type.
- The propeller air subsystem can be described by flow theory of aerodynamics.

The mechanical system of TRMS is simplified using four point mass system as show in Fig 5. Includes main rotor, tail rotor, balance weight and counter weight.

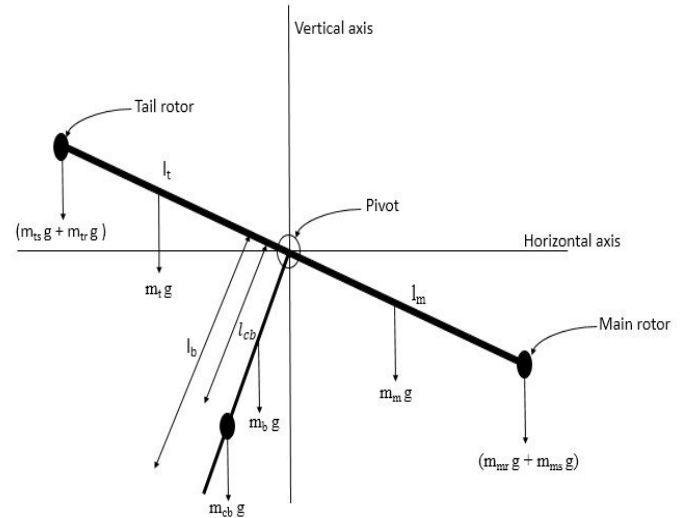


Fig 5. Free-body diagram of twin rotor MIMO system

Consider the rotation of the beam in vertical plane. The driving torque is produced by propellers. The vertical and horizontal parts of the system are derived via Newton's second law for rotational motion. First, we are considering dynamics of the vertical plane

$$\sum M_V = \frac{dS_V}{dt} = Mv_1 + Mv_2 + Mv_3 - Mv_4 \quad (8)$$

Where,
M = Torques,
S = Angular momentum

Where,
 M_{v1} = Total torque due to Gravity Force
= $g * [(A-B) \cos \alpha_v - C \sin \alpha_v]$ (9)

M_{v2} = Moment of Propulsive forces (Thrust) applied to beam
= $[l_m * F_v (\omega_m)]$ (10)

M_{v3} = Moment of Centrifugal forces corresponding to motion of beam around vertical axis
= $[-\Omega_h^2 \{ (A+B+C) \sin \alpha_v \cos \alpha_v \}]$ (11)

M_{v4} = Moment of Friction depending on angular velocity of beam around horizontal axis
= $(\Omega_v k_v)$ (12)

Equivalent Torque for Vertical plane:

$$\frac{dS_v}{dt} = l_m f_v (\omega_v) - \Omega_v k_v + g[(A-B) \cos \alpha_v - C \sin \alpha_v] - \Omega_h^2 \{ (A+B+C) \sin \alpha_v \cos \alpha_v \} \quad (12)$$

$$A = [(m_{ts} + m_{tr} + \frac{m_t}{2}) l_t] \quad (13)$$

$$B = [(m_{nr} + m_{ns} + \frac{M_m}{2}) l_m] \quad (14)$$

$$C = (m_{cb} l_b + m_b \frac{l_{cb}}{2}) \quad (15)$$

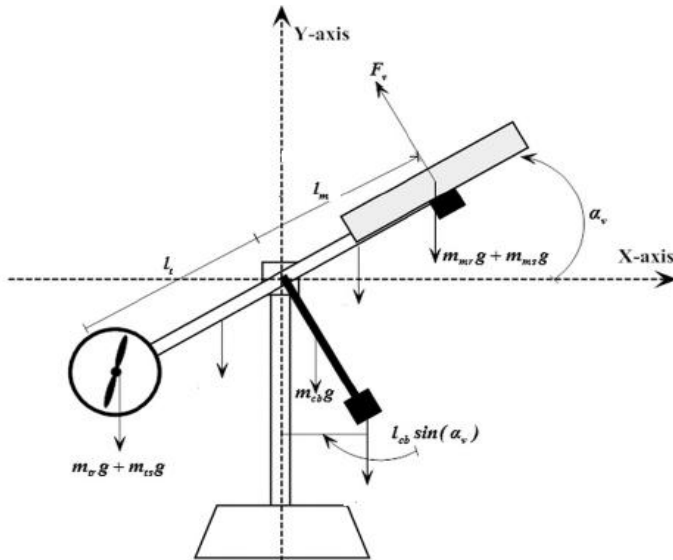


Fig 6. Forces on TRMS for vertical plane

Similarly, Fig. 7 and equation (14) describes the forces acting on TRMS in horizontal plane.

Total torque for Horizontal plane:

$$\frac{dS_h}{dt} = \sum M_h = M_{h1} - M_{h2} \quad (16)$$

Where,
 M_{h1} = Moment of Propulsive forces (Thrust) applied to beam
= $\{ F_h(w_t) * l_t * \cos \alpha_v \}$ (17)

M_{h2} = Moment of Friction depending on the angular velocity of beam around the vertical axis = $(\Omega_h k_h)$ (18)

$$\frac{dS_h}{dt} = [F_h(w_t) l_t \cos \alpha_v] - [\Omega_h k_h] \quad (19)$$

C. Calculation of inertia

The total inertia for vertical plane as follows:

$$J_v = \sum_{i=1}^8 J_{vi}$$

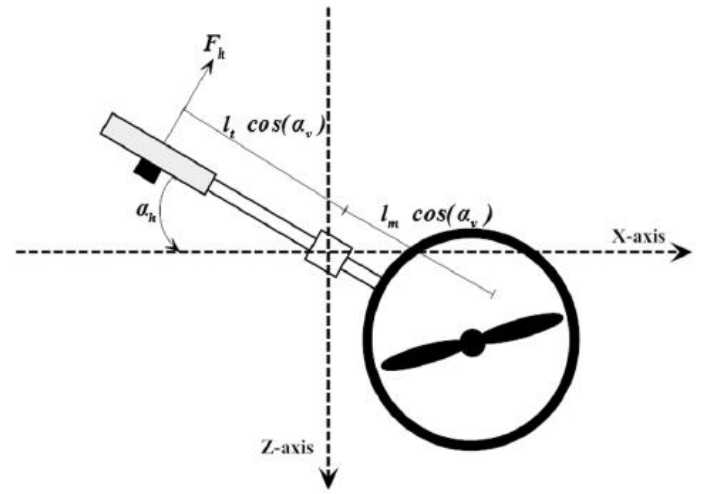


Fig 7. Forces on TRMS for horizontal plane

Where,

$$J_{v1} = \text{Moment of Inertia for main rotor with motor} = m_{nr} l_{nr}^2$$

$$J_{v2} = \text{Moment of Inertia for main rotor beam} = m_m \frac{l_m^2}{3}$$

$$J_{v3} = \text{Moment of Inertia for solid cylinder shaped counter weight} = m_{cb} l_{cb}^2$$

$$J_{v4} = \text{Moment of Inertia for rod of counter weight} = m_b \frac{l_b^2}{3}$$

$$J_{v5} = \text{Moment of Inertia for tail rotor with motor} = m_{tr} l_t^2$$

$$J_{v6} = \text{Moment of Inertia for tail beam} = m_t \frac{l_t^2}{3}$$

$$J_{v7} = \text{Moment of Inertia for shield of main rotor} = \{ \frac{m_{ms}}{2} r_{ms}^2 \} + \{ m_{ms} l_m^2 \}$$

$$J_{v8} = \text{Moment of Inertia for shield of tail rotor} = \{ m_{ts} r_{ts}^2 \} + \{ m_{ts} l_t^2 \}$$

$$J_v = \sum_{i=1}^8 J_{vi}$$

$$J_v = J_{v1} + J_{v2} + J_{v3} + J_{v4} + \dots + J_{v8}$$

$$J_v = (m_{nr} + \frac{m_m}{3} + m_{ms}) l_m^2 + (m_{tr} + \frac{m_t}{3} + m_{ts}) l_t^2 + m_{cb} l_{cb}^2 + m_b \frac{l_b^2}{3} + m_{ts} r_{ts}^2 + \frac{m_{ms}}{2} r_{ms}^2 \quad (20)$$

The total inertia for horizontal plane as follows:

$$J_h = \sum_{i=1}^8 J_{hi} ;$$

Where,

$$J_{h1} = \frac{m_m}{3} (l_m \cos \alpha_n)^2$$

$$J_{h2} = \frac{m_t}{3} (l_t \cos \alpha_v)^2$$

$$J_{h3} = \frac{m_b}{3} (l_b \sin \alpha_v)^2$$

$$J_{h4} = m_{tr} (l_t \cos \alpha_r)^2$$

$$J_{h5} = m_{nr} (l_m \cos \alpha_v)^2$$

$$J_{h6} = m_{cb} (l_{cb} \sin \alpha_v)^2$$

$$J_{h7} = \frac{m_{ts}}{2} r_{ts}^2 + m_{ts} (l_t \cos \alpha_v)^2$$

$$J_{h8} = m_{ms} r_{ms}^2 + m_{ms} (l_n \cos \alpha_v)^2$$

$$J_h = D \cos^2 \alpha_v + E \sin^2 \alpha_v + F \quad (21)$$

Where,

$$D = m_{cb} l_{cb}^2 + m_b \frac{l_b^2}{3} \quad (22)$$

$$E = (m_{nr} + m_{ms} + \frac{m_m}{3}) l_m^2 + (m_{ts} + m_{tr} + \frac{m_t}{3}) l_t^2 \quad (23)$$

$$F = m_{ms} r_{ms}^2 + \frac{m_{ts}}{2} r_{ts}^2 \quad (24)$$

IV. PID IMPLEMENTATION

The PID has the simple structure, easier to understand than that most other controllers. Here simple PID controller controls the vertical and horizontal plane separately. The effect of one rotor in other plane is not consider and not compensated by controller. Equation (1) to (19) of TRMS mathematical modelling is designed in MATLAB/Simulink. Here step input is given as reference angle. Output angle will be compared with step input using comparator and error is compensated using PID controller. The output of simple PID controller is given to both of the plants (subsystem of vertical and horizontal plane) to obtain the desired position response.

A. Vertical Plane of TRMS

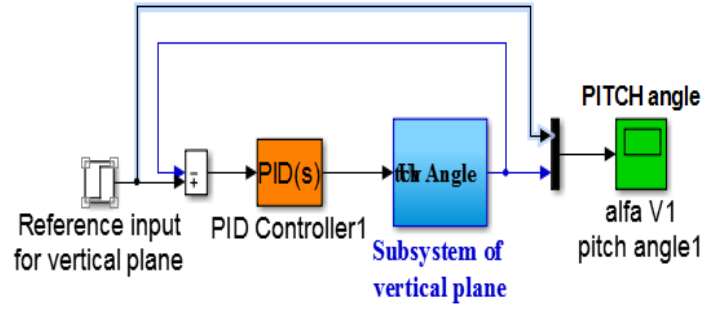


Fig 8. PID control for vertical plane

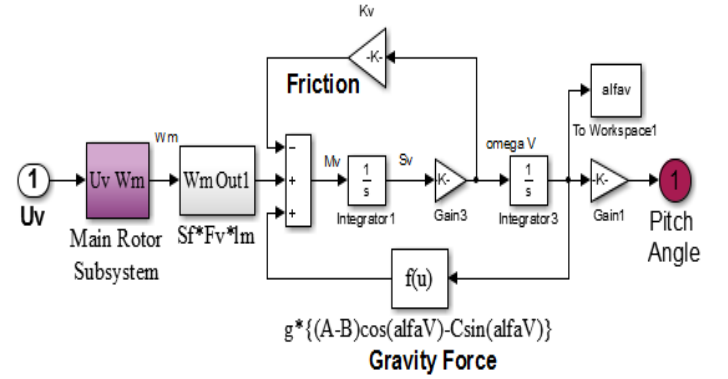


Fig 9. Subsystem of vertical plane model

B. Horizontal Plane of TRMS

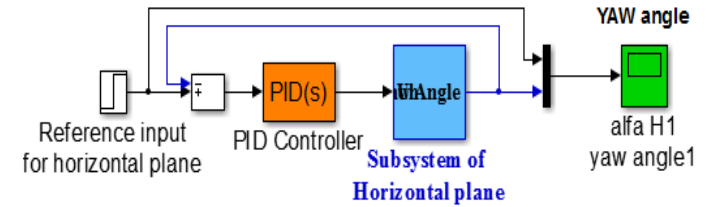


Fig 10. PID control for horizontal plane

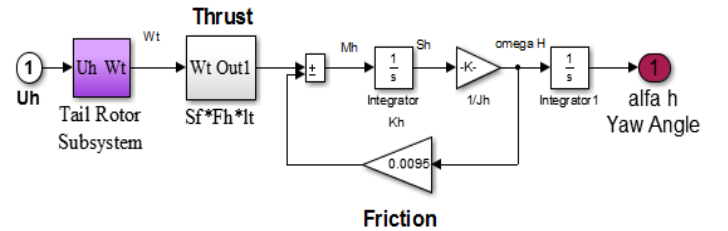


Fig 11. Subsystem of horizontal plane model

Here PID controller tuning is done by Ziegler-Nichols tuning method.

Table 1.
Ziegler-Nichols table for tuning of PID

Controller	k_c	k_c	k_c
P	$0.5 * k_{cu}$	-	-
PI	$0.45 * k_{cu}$	$P_u / 1.2$	-
PID	$0.6 * k_{cu}$	$P_u / 2$	$P_u / 8$

Where k_{cu} is ultimate gain and P_u is the ultimate period.

V. SIMULATION RESULTS

In this paper, the TRMS with one DOF has been considered. Two PID controllers applied to two subsystems to control the horizontal and vertical movements separately. The response of step inputs of the PID controller are presented in this section. Fig. 12 (a) and (b) shows the step response of the twin rotor MIMO system in vertical plane for two reference input 1 & 3 respectively using conventional PID controller, whereas Fig. 13 (a) and (b) shows the step response of the twin rotor MIMO system in horizontal plane for two reference input 1 & 3 respectively using PID controller.

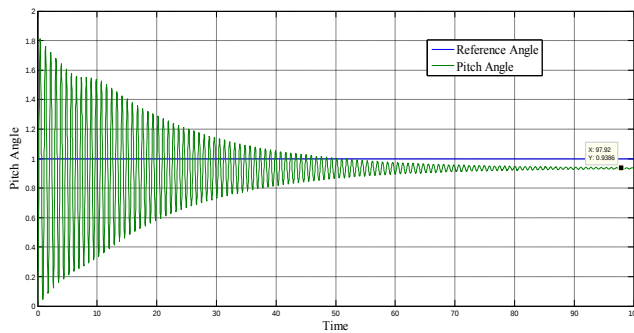


Fig. 12 (a) Step response of vertical plane

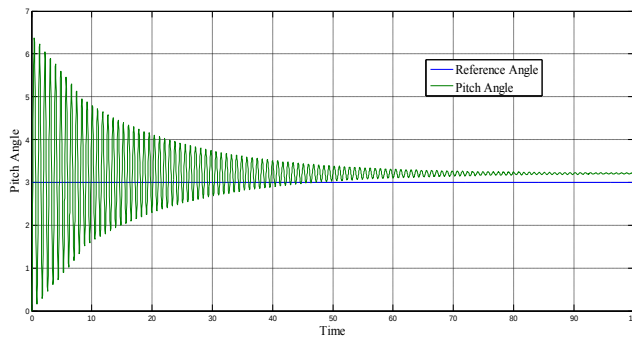


Fig. 12 (b) Step response of vertical plane

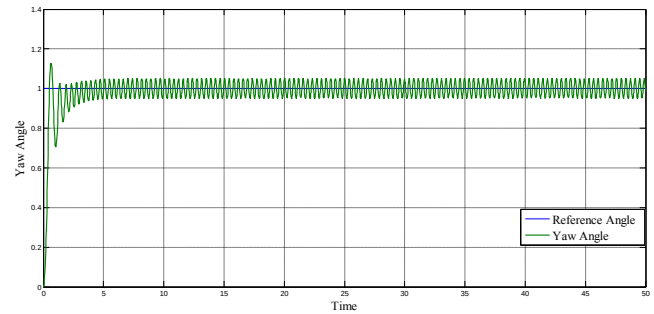


Fig. 13(a) Step response of horizontal plane

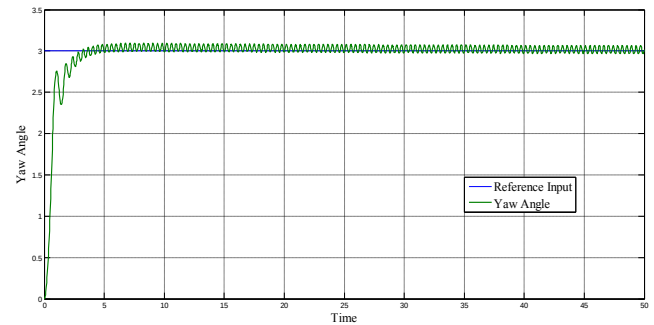


Fig. 13(b) Step response of horizontal plane

VI. CONCLUSION

In this paper, the TRMS with one degrees of freedom was considered. The mathematical modeling of TRMS in twoDOF is designed using MATLAB/SIMULINK. Two PID controllers were designed to control the horizontal and vertical movements of the system separately. The performance of the designed controllers has been evaluated with step reference inputs. It has been shown that TRMS achieves the desired trajectory. But there is more oscillation in the system which is not desired. We can tune PID controller for specific desired angle but we cannot control the system continuously. So far better control one can go for other control.

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