Smith Predictive Control of Time-Delay Processes

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***Abstract*—The time-delay is the major problem in the most of industrial processes. It is very difficult to control such processes using Conventional controllers because time-delay introduces the oscillations in response. The effect of time delay on processes can be effectively controlled by the Model-based Predictive Control method. Smith predictive control is one of the possible approaches to control systems with time-delay. The Least Square Method (LSM) has been used to identify the parameters of process. This paper deals with a designing of the Digital Smith Predictor with PID controller in its structure. MATLAB/SIMULINK has been used for simulation of the Smith Predictor with PID.**

***Keywords*— Time-delay; MPC; Smith Predictor**

I. INTRODUCTION

Time-delay presents in many processes in industries and other fields, including economical and biological systems. They are caused by some of the following phenomena[2]:

- The time needed to transport mass, energy or information.
- The accumulation of time lags in a great numbers of loworder systems connected in series.
- The required processing time for sensors, such asanalyzers, controllers that need some time to implement a complicated control algorithms or process.

Consider a continuous time dynamical linear SISO system with time-delay T_d. The transfer function of a pure transportation lag is $e^{-T_d s}$. where s is complex variable. Overall transfer function withTime-delay is in the form

$$G_d(s) = G(s)e^{-T_d s} \qquad (1)$$

where G(s) is the transfer function without time-delay.

Processes with significant time-delay are difficult tocontrol using standard feedback controllers mainly because of thefollowing[2]:

- The effect of the disturbances is not felt until a considerabletime has elapsed.
- The effect of the control action requires some time to elapse.
- The control action that is applied based on the actual errortries to correct a situation that originated sometimebefore.

The problem of controlling time-delay processes can besolved by some control methods using [8]

- PID controllers.
- Time-delay compensators.
- Model predictive control techniques.

PID controllers are widely used in the controlling of majority industrial processes. When the process is having a time-delay,the tuning of the PID controller is very difficult. The most populartuning rules for tuning of PID for processes with small time-delay wereproposed by Ziegler and Nichols. Many methods for newtuning rules were proposed for stable and unstable processeswith time-delay.

Predictive strategy is very useful for large delay processes and it gives the tighter control. In the Predictive control the models are required in the structure. The first time-delay compensation algorithm wassuggested by Otto Smith in 1957. This control algorithm knownas the Smith predictor (SP) contained a dynamic model of thetime-delay process and it can be considered as the first modelpredictive algorithm.It was designed for continuous time algorithm but because of problems of implementation in the continuous algorithm only discrete versions are used in industries.

II. WORKING OF SMITH PREDICTOR

The working principle of the Smith Predictor is shown in Fig. 1. The Smith Predictor is divided into two parts;the first is $G_c(s)$ controller and second is predictor part. The Smith Predictor was firstly designed for continuous time PID controller. The $G_c(s)$ is continuous time controller and the predictor contains the of a models of the process without time delay (fast model) and $G_m(s)$ and a model of the time delay $e^{-T_d s}$ then the complete process model is [8].

$$G_p(s) = G_m(s)e^{-T_d s} \qquad (2)$$

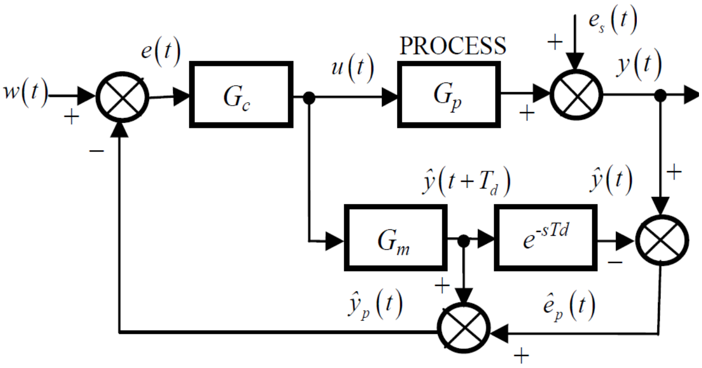

Fig. 1 Block diagram of an analog Smith Predictor

The model without delay G_m (s) is used to compute an open-loop prediction. The comparison between the output of the process $y(t)$ and the model including time delay $\hat{y}$ (t) is the predicted error $\hat{e}_p(s)$ as shown is in Fig. 1, where u (t), w (t), e (t) and $e_s(t)$ are the control signal, reference signal, the error and the noise. If there are no errors in the modeling and not any disturbances then the error between the current process output and the model output will be zero and the predictor output signal $\hat{y}_p$(t) will be the time-delay-free output of the process. If the process has no disturbances, the controller G_c (s) can be tuned in the nominal case,

The Smith Predictor structure for the without modeling errors has three fundamental properties: time-delay compensation, prediction and dynamic compensation [10].

III. WORKING OF DIGITAL SMITH PREDICTOR

Time-delay compensators were proposed in the 1957 butthe implementation of the Smith Predictor is very difficult with the analog techniques so they were not used in industrial practice. In the 1980s the digital algorithm of Smith Predictor can be implemented. Majority of Smith Predictor are implemented in digital version but they are analyzing the continuous work.

The discrete versions of the Smith Predictors are compensating the effect of time delay in majority industrial processes.The most of designed digital Smith Predictorare fixed parameters.

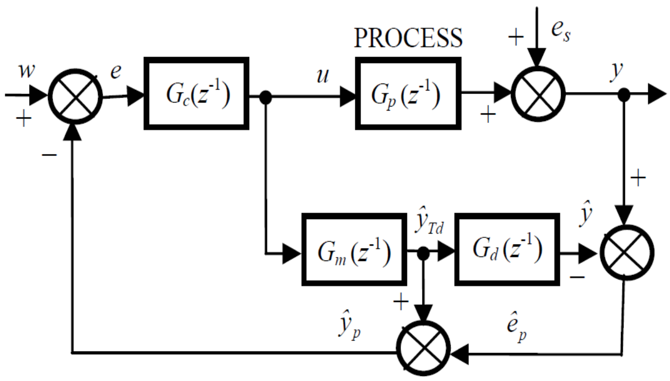

Fig. 2 Block diagram of a Digital Smith Predictor

A. *Digital Smith Predictor Structure*

The working of the digitalversion of Smith Predictor is similar to the continuous Smith predictor. The structure of digital Smith Predictor is shown in Fig. 2.The block named $G_m(z^{-1})$represents processdynamics without the timedelay (fast model) and is used to calculate an open-loop prediction output of plant. Thedifference between the output of the process y and the model including time-delay$\hat{y}$is the predicted error$\hat{e}_p$ as shown isin Fig.2,where u,w, e and$e_s$are the control signal, the reference signal, the error and the noise. If there are no errors in the modeling and no noise then the error between the current process output and the model output will be zero and thepredictor output signal$\hat{y}_p$will be the time delay- free outputof the process. Under these conditions, the controller G_c (z^{-1}) can be tuned in the nominal case, as if the process has no time-delay. The main controller$G_c(z^{-1})$isPID controller.The outward feedback-loop through the block$G_d(z^{-1})$ in Fig.2 is used to compensate for load disturbances and modeling errors.

Most industrial processes can be approximated by a reduced order model with some pure time-delay. Consider the following second order linear model with a time-delay [9]

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 z^{-1} + b_2 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-1}} z^{-d} \qquad (3)$$

for demonstration of the designof the Smith Predictor. The term z^{-d}represents the pure discrete time-delay. The time-delay is equal to$dT_0$where T_0*is* the sampling period.

If the time-delay is not an exact multiple of the sampling periodT_0, then dT_0represents the largest integer multiple of the sampling period with remaining fractional deal absorbed into $B(z^{-1})$using the modified z-transformation

B. *Parameter Identification of process*

The time-delay of the process can be identified by using Least Square Method. The identified parameters of process aregiven as vector$\widehat{\Theta}$ [6].

$$\widehat{\Theta} = (F^T F)^{-1} F^T y \qquad (4)$$

Where the matrix $\boldsymbol{F}$ has dimension $(N\text{-}n\text{-}d,\ 2n)$, the vector $\boldsymbol{y}(N\text{-}n\text{-}d)$ and the vector of parameter model estimates$\widehat{\Theta}(2n)$. N is the number of samples of measured input and output data, nis the order of the model.

$$F = \begin{bmatrix} -y(n+d) & -y(n+d-1) & \cdots & -y(d+1) & u(n) & u(n-1) & \cdots & u(1) \\ -y(n+d+1) & -y(n+d) & \cdots & -y(d+2) & u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ -y(N-1) & -y(N-2) & \cdots & u(N-n) & u(N-d-1) & u(N-d-2) & \cdots & u(N-d-n) \end{bmatrix} \qquad (5)$$

Equation (4) gives one-off calculation of the vectorof parameter estimates$\widehat{\Theta}$ using N samples of measured data.The individual vectors and matrix in equation (4) have theform

$$y^T = [y(n+d+1) \quad y(n+d+2) \quad \dots. \quad y(N)] \quad (6)$$

$$\hat{\Theta}^T = [\hat{a}_1 \hat{a}_2 \ \dots \ \hat{a}_n \hat{b}_1 \hat{b}_2 \ \dots \ \hat{b}_n] \quad (7)$$

The quality of identified model can be judged by the prediction error.

$$\hat{e}(k) = y(k) - \hat{y}(k) \quad (8)$$

The prediction error has a main function in identification of model parameters derived from measured data. Prediction error is very important in selection of the order of the model and a sampling period. Here the prediction error was used for selection of the time-delaydT_0.

IV. SMITH PREDICTOR WITH PID CONTROLLER (PIDSP)

Main controller in the Smith Predictor is designed by Dahlin PID algorithm. The Dahlin PID algorithm isbased on the desired close-loop transfer function in the form [3]

$$G_e(z^{-1}) = \frac{1-e^{-\alpha}}{1-z^{-1}} \quad (9)$$

where $\alpha = T_0/T_m$ and T_0 is sampling period ,$T_m$is desired time constant of the firstorder closed-loop response. T_m should not be chosentoo small because of that it will demanda large control signal $u(k)$whichmaycause the saturation of the actuator. Then the individual parts of the controller are described by the transfer functions [3]

$$G_c(z^{-1}) = \frac{(1-e^{-\alpha})\hat{A}(z^{-1})}{(1-z^{-1})\hat{B}(1)} \quad (10)$$

$$G_m(z^{-1}) = \frac{z^{-1}\hat{B}(1)}{\hat{A}(z^{-1})} \quad (11)$$

$$G_d(z^{-1}) = \frac{z^{-d}\hat{B}(z^{-1})}{z^{-1}\hat{B}(1)} \quad (12)$$

where $B(1) = \hat{B}(z^{-1})\big|_{z=1} = \hat{b}_1 + \hat{b}_2$

Since $G_m(z^{-1})$ is the second order transfer function, the main controller $G_c(z^{-1})$ becomes a digital PID controller having the following form [9]:

$$G_c(z^{-1}) = \frac{U(z)}{E(z)} = \frac{q_0 + q_2 z^{-1} + q_2 z^{-2}}{1-z^{-1}} \quad (13)$$

Where$q_0 = \gamma$, $q_1 = \hat{a}_1\gamma$, $q_2 = \hat{a}_2\gamma$using by the substitution $\gamma = (1-e^{-\alpha})/\hat{B}(1)$. The PID controller output is given by

$$u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) + u(k-1) \quad (14)$$

V. SIMULATION OF DIGITAL SMITH PREDICTOR WITH PID CONTROLLER

Simulation is very useful tool for the analysis of control systems, it allows us to create and simulate mathematical models of a processes. It is used to design the virtual controllers in computer. The mathematical models provided are sufficiently close to a real object so by using simulation we can verify the results of processes for various controllers parameter settings. The models of the processes may also be excited by various random noise generators which can simulate the stochastic characteristics ofthe processes noise signals with similar properties to disturbance signals measured in the machinery. The simulation results are valuable for an implementation of a chosen control algorithm under laboratory and industrial conditions. The simulation and laboratory conditions can be very different from those in real plants, and therefore we must verify its practicability with regard to the process dynamics and the required standard of control quality.

The Smith Predictor with PID controller is not suitable for the control of unstable processes. Therefore, three types of processes were chosen for simulation of digital PIDSP.

Consider the following continuous-time transfer functions:

1) Stable and without oscillations

$$G_1(s) = \frac{1}{s^2+6s+1}e^{-4s} \quad (15)$$

2) Stable with oscillations

$$G_2(s) = \frac{1}{s^2+s+1}e^{-4s} \quad (16)$$

3) Non-minimum phase Process

$$G_3(s) = \frac{-4s+1}{s^2+6s+1}e^{-4s} \quad (17)$$

Let us now discretize them a sampling period T_0= 2 s. The discrete forms of these transfer functions are

$$G_1(z^{-1}) = \frac{0.2689z^{-1}+0.02152z^{-2}}{1-0.7095z^{-1}+0.0000061z^{-2}}z^{-2} \quad (18)$$

$$G_2(z^{-1}) = \frac{0.8494z^{-1}+0.404z^{-2}}{1+0.11819z^{-1}+0.1353z^{-2}}z^{-2} \quad (19)$$

$$G_3(z^{-1}) = \frac{-0.8277z^{-1}+2.081z^{-2}}{1+0.1181z^{-1}+0.1353z^{-2}}z^{-2} \quad (20)$$

A simulation of proposed design wasperformed in MATLAB/SIMULINK. A typicalcontrol schemes used is shown in Fig. 3 and Fig. 4. The internal structure of PID is shownin Fig.5.

These schemes are used for systems with time-delay. Individual blocks of the Simulink scheme match to blocks of the general control scheme presented in Fig. 2. The Process block represents continuous-time system with delay. Blocks Fast Model and Delay Model are parts of the Smith Predictor and they correspond to $G_m(z^{-1})$ and $G_d(z^{-1})$ blocks of Fig. 2 respectively. The control algorithm is compressed in PID Controller which corresponds to $G_c(z^{-1})$ Fig. 2 block. The Dead-time is entered in the no. of samples.

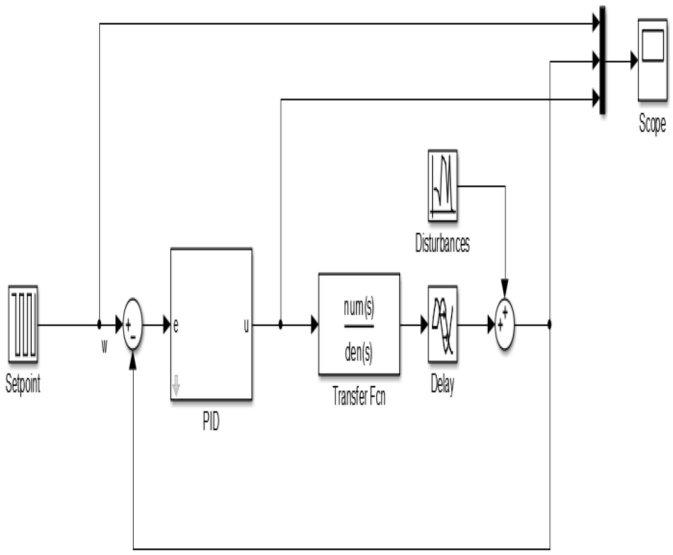

Fig. 3 Simulink scheme for process control by Dahlin PID

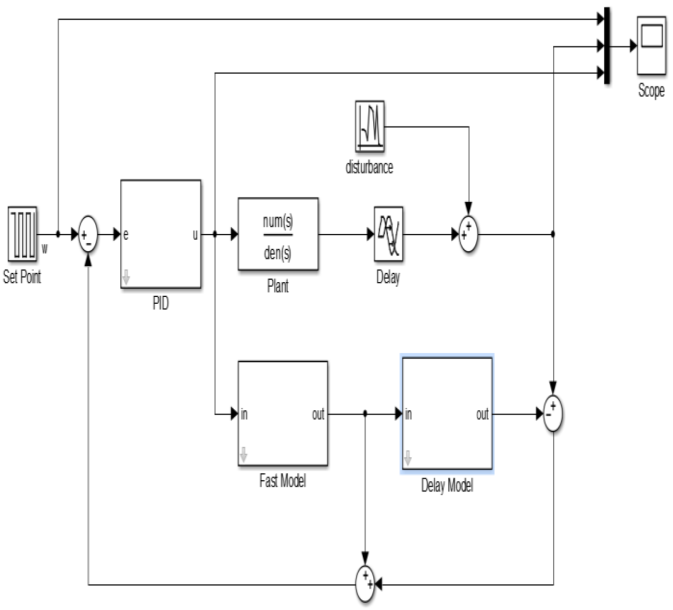

Fig. 4 Simulink scheme for process control by PIDSP

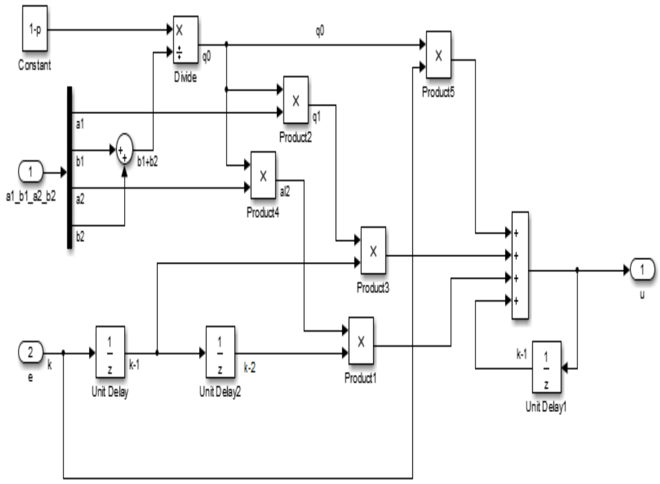

Fig. 5 Internal structure of the controller

VI. Simulation Results for PID and PIDSP

The configuration for simulation of the PID and PIDSP was chosen as follows:

- A suitable time constant $T_m$was chosen for the controlusing the PID and PIDSP controller.
- The outputs of the process models were influenced by White Noise Generator with mean value E = 0 and covariance $R = 10^{-4}$.

Figs. 6-11 illustrate the simulation control performance using PID and PIDSP controller.

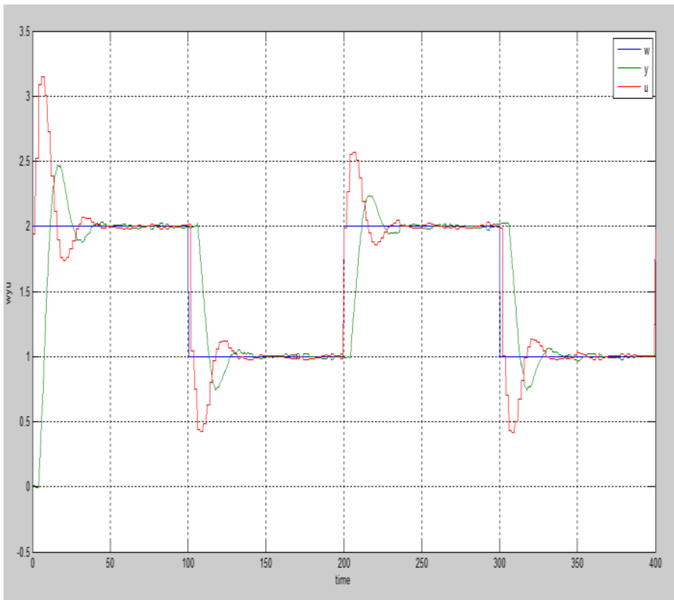

Fig. 6 Control of the model $G_1(z^{-1})$ using only PID

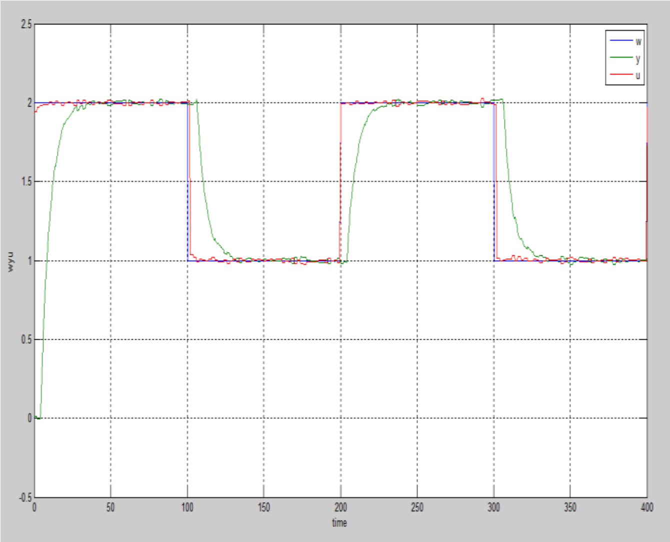

Fig. 7 Control of the model $G_1(z^{-1})$ using PIDSP

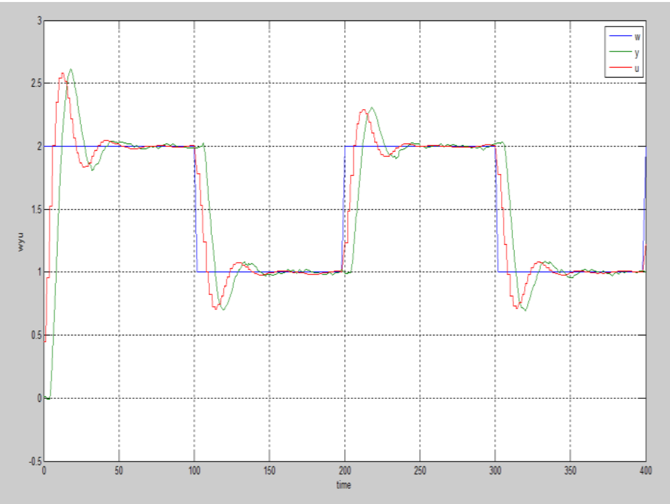

Fig. 8 Control of the model $G_2(z^{-1})$ using PID

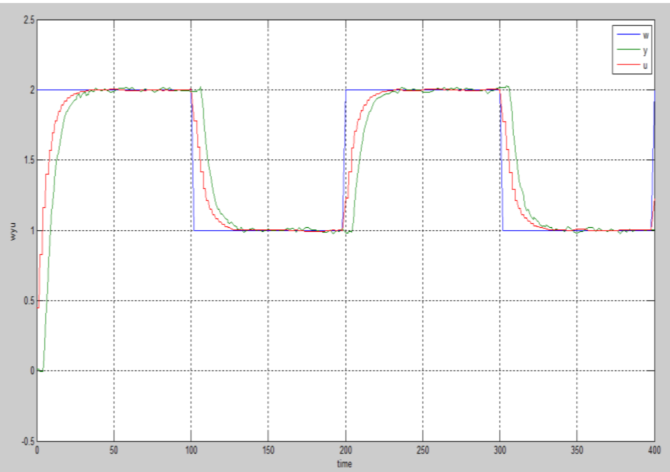

Fig. 9 Control of the model $G_2(z^{-1})$using PIDSP

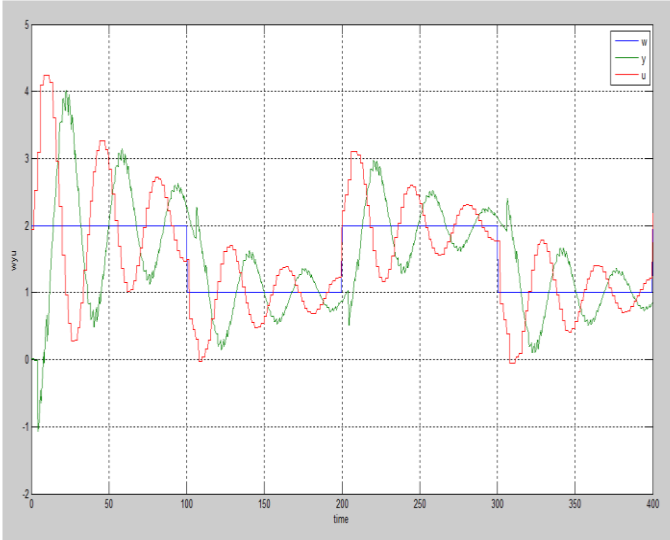

Fig. 10 Control of the model $G_3(z^{-1})$, using PID

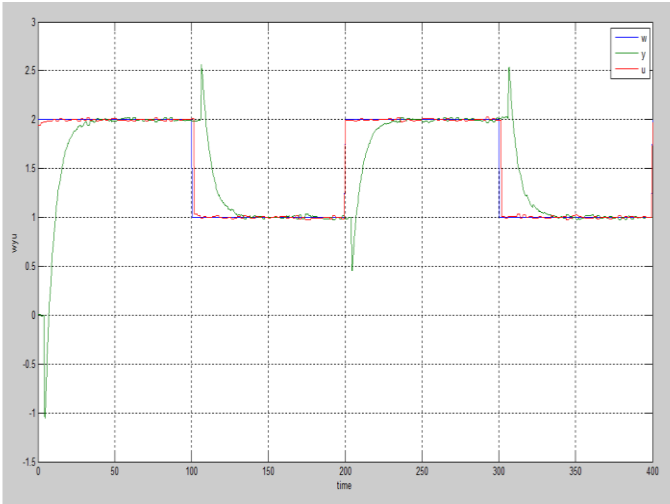

Fig. 11 Control of the model $G_3(z^{-1})$, using PIDSP

Table1Characteristic of Responses

Model	**Characteristics of Responses**			
	Peak Overshoot (%)	**Rise time(s)**	**Peak time(s)**	**Settling time(s)**
G1 with PID	24.55	6	18	38.50
G1 with PIDSP	1.40	12.50	38.39	42.05
G2 with PID	31.70	5.54	18	42.08
G2 with PIDSP	1.30	13	39.28	42.35
G3 with PID	101.90	4.60	22.49	214
G3 with PIDSP	1.60	12.41	39.35	41.51

The characteristics of responses for various process controlled by PID and PIDSP is shown in table 1.

Thesecharacteristics were obtained by the from the simulation result. From the Table 1 we can easily observe that the time delay introduces the oscillations in the response but we can compensate the effect of time delay by using the Smith Predictive algorithm.

VII. CONCLUSION

PID Smith Predictor algorithm was proposed. The simulation of PIDSP was done by using MATLAB/SIMULINK. Three models of control processes were used forsimulation (the stable non-oscillatory, the stableoscillatory and the non-minimum phase).In the simulation results we can see that there are some oscillations in responses of only PID control because of time delay but we can remove the oscillations by the implementing Smith Predictor.Results of the simulation demonstrates the advantages anddisadvantages of PID Smith Predictor.

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