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Comparison of Fractional order proportional integral controller (FO-PI) and Integer order proportional integral controller (IO-PI)

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Abstract: *-This paper includes the comparative study of fractional order proportional integral control system (FO-PI) and Integer order proportional integral control system (IO-PI). In this paper both controllers are used for the water level control of the coupled tank system. The FO-PI tuning rules based on fractional Ms constrained integral gain optimization (F-MIGO) algorithm. The FO-PI controller is compared with both Ziegler Nichol's (ZN) tuned and Modified Ziegler Nichol's (MZN) tuned conventional integer order proportional integral (PI) controllers in terms of load disturbance rejection, changes in plant dynamics, and set point tracking. By Simulation results compare the FO-PI and IO-PI in terms of percentage overshoot and system response in liquid level and flow control. The result confirmed that fractional order PI controller is better than integer order PI controller.*

Keywords: *-Fractional calculus, fractional order controller,Integer ordercontroller, proportional and integral control, controller tuning, liquidlevel control.*

I. INTRODUCTION

PID controller is still dominating the feedback control applications until today especially for PI control [1], [2]. In process control applications, more than 90% of thecontrollers are of PI type [1]. Its application is adequate forwide control problems with modest performance requirements.PI is normally used for a system that can be approximated by afirst-order system. Otherwise, PID will be more appropriate.PID control is not suitable for all processes compared to PIwhich is more universal[3].

The performance of the closed-loop system mainly depends on the value of P, I, and D gain. The most popular tuningtechnique is the Ziegler-Nichols that had been proposed since1942 [4] but was still largely applied in its original form orwith some modifications. The rules were simple because notrequiring process transfer function. The rules only requireinformation on the process gain, dead-time and lag-time whichcan be obtained from an s-shaped step response. However, therule often produced poor robustness since it uses very littleinformation about the plant to be controlled [5]. Ziegler andNichols presented two methods, a process reaction curve method and frequency response method. The rules weredeveloped based on simulation performed on a large numberof different processes to formulate the general PID tuningrules. Modified versions of Ziegler-Nichols were proposed byCohen-Coon and Chien, Hrones and Reswick [6] where moreprocess parameters were considered.The Ziegler-Nichols method which had two major drawbacks [13]:

1. The controller parameters were designed with just the S-shaped step response and for complicated systems this information was not adequate enough to design a goodcontroller.
2. The controllers obtained from this method showed very poor robustness and damping properties.

Fractional calculus is a more than 300 years old topic. The number of applications where fractional calculus has beenused rapidly grows. These mathematical phenomena allowto describe a real object more accurately than the classical"integer-order" methods.About a decade ago, the PID controller had been generalized with the implementation of non-integer integraland differentiation proposed by Podlubny in 1999 [7]. ThePID was generalized in the form of $PI^{\lambda}D^{D}$ involving anintegrator of order λ and differentiator of order D of less than1. The new structure known as fractional-order PID wasacknowledged to improve the performance of the feedbackcontrol loop [8]. The concept of fractional-order control(FOC) is represented by fractional-order differential equations.Theoretical framework regarding fractional derivative andintegral had been established by Liouville, Riemann, Euler,and Lagrange since the 19th century [9]. The knowledge hadbeen transferred into control engineering by Tustin [10] tocontrol the position of massive object in 1958. This wasfollowed by Manabe [11] around 1960. However, the FOCapplication was not widely incorporated in control engineeringthen, due to lack of theory and computational limitation [12].At present time there are lots of methods for approximation of fractional derivative and integral and fractional calculus can be easily used in wide areas ofapplications.

II. FRACTIONAL ORDER PI CONTROLLER AND ITS PRACTICAL TUNING RULE (F-MIGO)

Lots of work on fractional controllers can be found in literature [14], [15]. Expressing in time-domain, if r (t) is the set pointsignal, and u (t) is the control input, y (t) is the output, thefractional order PI controller is represented by:

$$u(t) = K_p(r(t) - y(t)) + K_i D_t^{-\alpha}(r(t) - y(t))$$

Here, $u(t)$ is control input;

$r(t)$ is set-point signal:

$y(t)$ is output:

$D_t^{-\alpha}$ is fractional operator.

Where $D_t^{-\alpha}$ the fractional isdiffer integral operator. The followingdefinition is used for the fractional derivative of order α of functionf (t) [8], [9]:

$$D_t^{-\alpha} = \frac{d^\alpha}{dt^\alpha} f(t) = \begin{cases} f^n(t) & if \quad \alpha = n \in N \\ \frac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} * f^n(t) & if \quad n-1 < \alpha < n \end{cases}$$

In frequency domain, FOPI controller C(s) is given as:

$$C(s) = K_p + \frac{K_i}{s^\alpha}$$

Where Kp and Ki are the proportional and integral gain valuesof the fractional controller and αis the noninteger order of thefractional integrator.

- **FOPDT Model:**

FOPDT model is necessary for finding the tuning parameters of the FO-PI controller.The First Order plus Delay Time (FOPDT) model is also known as the KLT model and is given by Figure 1.Figure 1 illustrates the FOPDT parameters given the S-shapedstep response of a FOPDT class system.

$$G = \frac{K}{Ts+1} e^{-Ls}$$

$$\tau = \frac{L}{T+L}$$

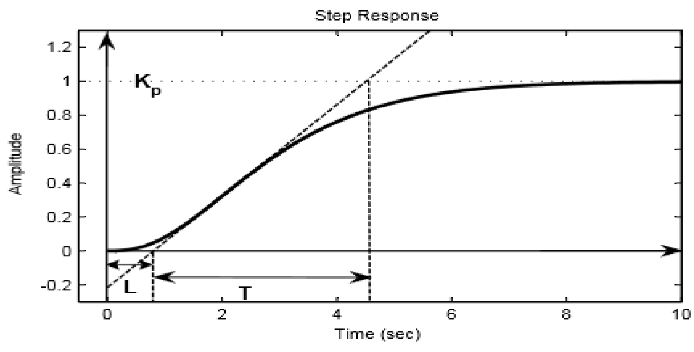


Figure 1

Where,

K is steady state gain

L is apparent delay

T is time constant

$\tau$is relative time delay The fact that the ratio L/T is important has been noticed before. Cohen and Coon called L/T the self-regulating index. The ratio of L/T is called the controllability index. The use of τinstead of L/T has the advantage that the parameter is bounded to the region [0, 1].Systems with $\tau > 0.6$, are called delay dominated systems and systems with $\tau < 0.1$ are called lag dominated systems.

F-MIGO Tuning Rule

The mainidea was to come up with simple rules that are robust to loaddisturbance. The design looked into maximization of theintegral gain with a constraint on maximum load disturbance to-output sensitivity, *Ms* The tuning rules developed are restated as:

$$\alpha = \begin{vmatrix} 1.1 if \tau \geq 0.6 \\ 1.0 if\ 0.4 \leq \tau \prec 0.6 \\ 0.9 if\ 0.1 \leq \tau \prec 0.4 \\ 0.7 if \tau \prec 0.1 \end{vmatrix}$$

$$K_p = \frac{1}{K}\left(\frac{0.2978}{\tau + 0.000307}\right)$$

$$K_i = T\left(\frac{0.8578}{\tau^2 - 3.402\tau + 2.405}\right)$$

The tuning rule is only dependent on the τ parameter. By finding a value of τ we can find a value of the proportional (K_p), integral gain (K_i)and the non-integer order (α) of thefractional integrator.The tuning rules can be summarized as follows:

1. Find the FOPDT model of the system and define the values K, L, T.
2. Find the relative dead time of the system τ.
3. From the value of τ, calculate the fractional order α.
4. Find the controller gains (Kp and Ki).

III. INTEGER ORDER PI CONTROLLER AND ITS PRACTICAL TUNING RULE (ZN AND MZN)

Ziegler Nichols Tuning Method:

This traditional method, alsoknown as the closed-loop method (or) on-line tuning method was proposed by Ziegler and Nichols .To tune a controller using the Z-N method the integral and derivative elements of the PID controller are ignored. The proportional element is used to find a K_c that will sustain oscillation. This value is considered as K_c, or the ultimate gain. The period of oscillation is the T_c, or ultimate period. Z-N Method consists of two steps:

 Determination of the dynamic characteristics of the control loop.

 Estimation of the controller tuning parameters that produce a desired response for the dynamic characteristicdetermined in the first step, in other words, matching the characteristics of the controller to that of the otherelements in the loop.

The tuning formula for Z-N method is shown in table.1.

Controller Type	Kp	Ki	Kd
P	0.5 K_c		
PI	0.45 K_c	0.833 T_c	
PID	0.6 K_c	0.5 T_c	0.125T_c

Table 1

Modified Ziegler Nichols Tuning Method

This design method is based on Nyquist loop shaping method [34]. The controller, if designed properly, can be used to move a point A on the Nyquist curve of the uncontrolled point to an arbitrary position B on the Nyquist plot of the controlled plant. Suppose that we have a define a point A on the complex plane defined by $G_A(j\omega_0) = r_a e^{j(\pi+\phi_a)}$ and we want to move this point to B defined by $G_B(j\omega_0) = r_b e^{j(\pi+\phi_b)}$. If the controller is defined at ω_0 as $G_C(j\omega_0) = r_c e^{j(\pi+\phi_c)}$, we have,

$$r_b e^{j(\pi+\phi_b)} = r_a r_c e^{j(\pi+\phi_a+\phi_c)}$$

Therefore, $r_c = r_b/r_a$ and $\phi_c = \phi_b - \phi_a$. Based on this relationship, the PI controller gains are given by:

$$K = \frac{r_b \cos(\phi_b - \phi_a)}{r_a}$$

$$K_i = \frac{K}{\omega_0 \tan(\phi_b - \phi_a)}$$

As a special case, when $r_a = 1/K_c$ and $\phi_a = 0$ we obtain the Modified Ziegler Nichols tuning method. Here, K_c is defined as the critical gain at the cross over frequency ω_c. To ensure T_i is positive, $\cos(\phi_b) < 0$. If r_b and ϕ_b are chosen properly we can have a good control over the overshoot and rise time of the controlled system, giving a definite advantage over the ZN method

IV. COMPARISION

Ex: 1 first order SISO coupled tank system.

The system model for the first order SISO coupled tank system (CTS) is shown in fig. 2. Consider an open water tank with cross-sectional area *A*. Water is pumped into the tank at the top at rate of flow of *in* $Q_i$cubic metres per second. Water is flowing out of the tank through a hole in the bottom is Q_o. Assume that the first order SISO Coupled tank system is:

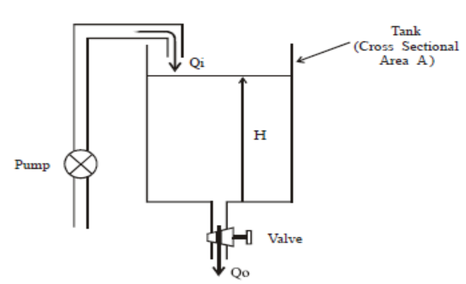


Figure 2

$$G = \frac{2.2972}{41.667s + 1} e^{-1.75s}$$

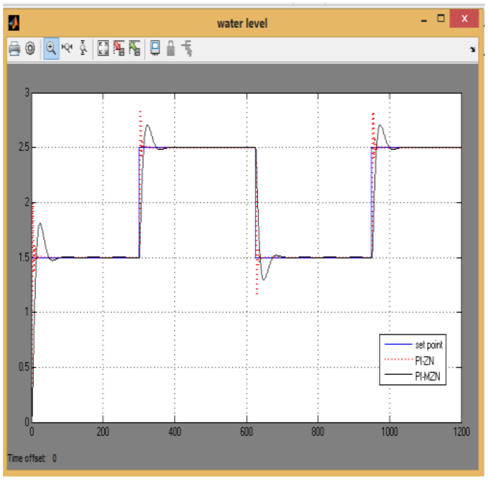
By using the F-MIGO tuning method for FO-PI and ZN and MZN tuning methods for the IO-PI found a tuning parameters for the controllers. And by the MATLAB simulation compare the output of the different controllers.

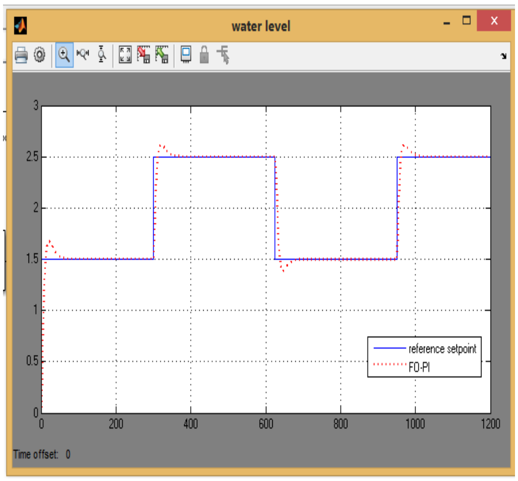
Tuning parameters:

	Kp	Ki	α
PI-ZN	6.6245	5.5071	
PI-MZN	4.641	3.313	
FO-PI F-MIGO	3.1924	15.7487	0.7

Table 2

Simulated control of 1st order SISO system:





Ex: 2 second order SISO coupled tank system.

The system model for the first order SISO coupled tank system (CTS) is shown in fig.3. The aim of this experiment is to design a controller that maintains a fixed water-level in tank 2 by varying input water flow of tank 1.Assume that the second order SISO Coupled tank system:

$$G = \frac{2.06 * 0.04^2}{s^2 + 2(0.824)0.04 + 0.04^2}$$

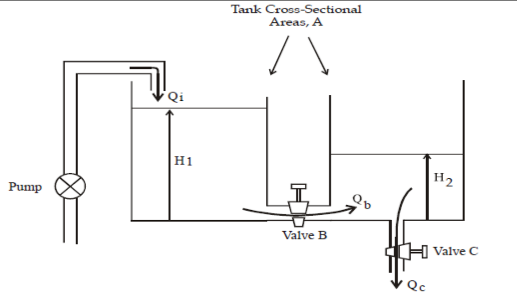


Figure 3

By using MATLAB simulation to approximate a second order system by first order plant system.The approximated FOPDT model obtained is given:

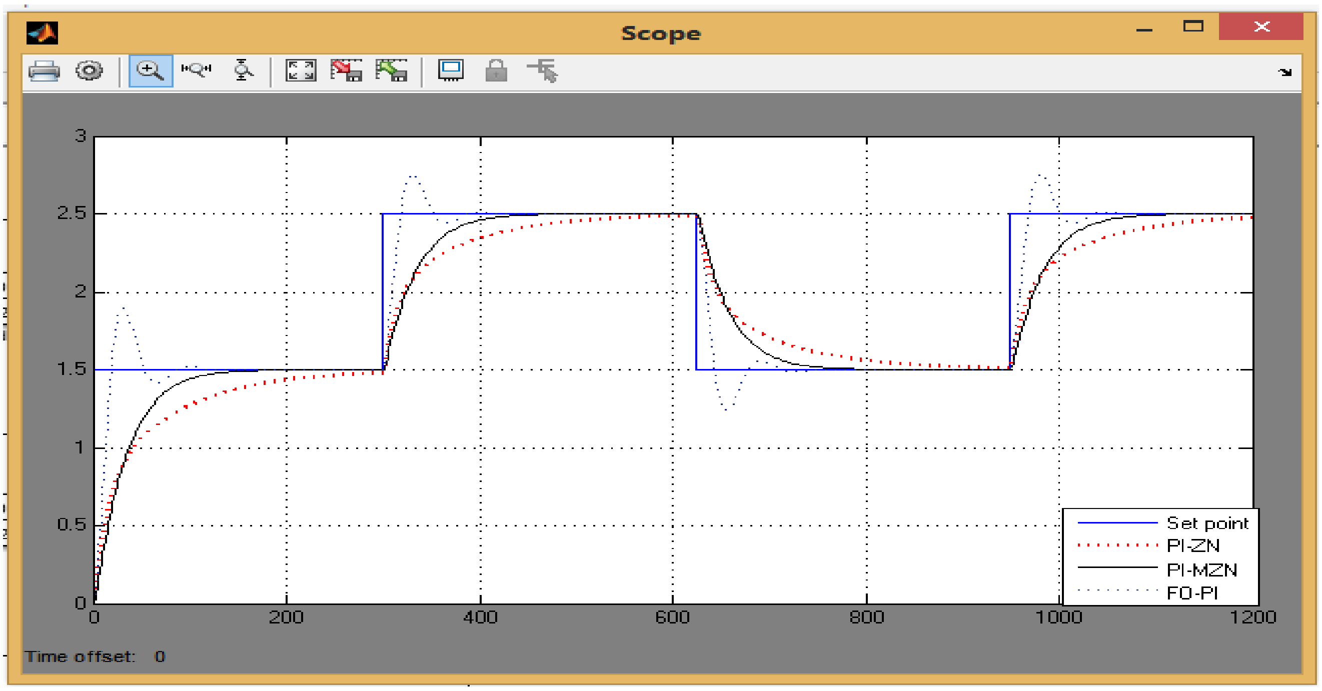
$$G\_approx = \frac{2.06}{21.1528s + 1} e^{-20.0472s}$$

Tuning parameters:

	Kp	Ki	α
PI-ZN	0.4556	50.0966	
PI-MZN	0.4692	63.171	
FO-PI F-MIGO	0.2969	18.3949	1

Table 3

Simulated control of 2nd order SISO system:



Ex: 2The Cascaded Control CTS Configuration

The system model for the first order SISO coupled tank system (CTS) is shown in fig.4. This type of control system has two cascadedcontrollers namely primary and secondary controllers. The master controller decides the set point of the slave controller, while the slave controller attempts to track the set point. The master controller uses the water-levelin tank 2 as the process variable by varying water-levels in tank 1.Assume that the Cascaded control Coupled tank system is:

$$G_1 = \frac{1.57}{35s+1}e^{-2s} \qquad G_2 = \frac{1.03}{25s+1}e^{-2s}$$

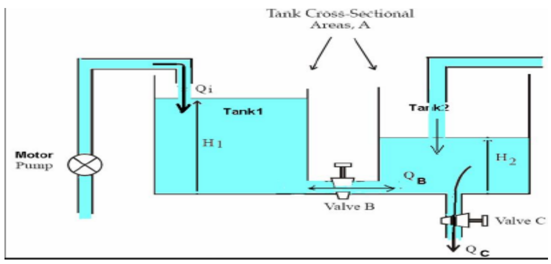


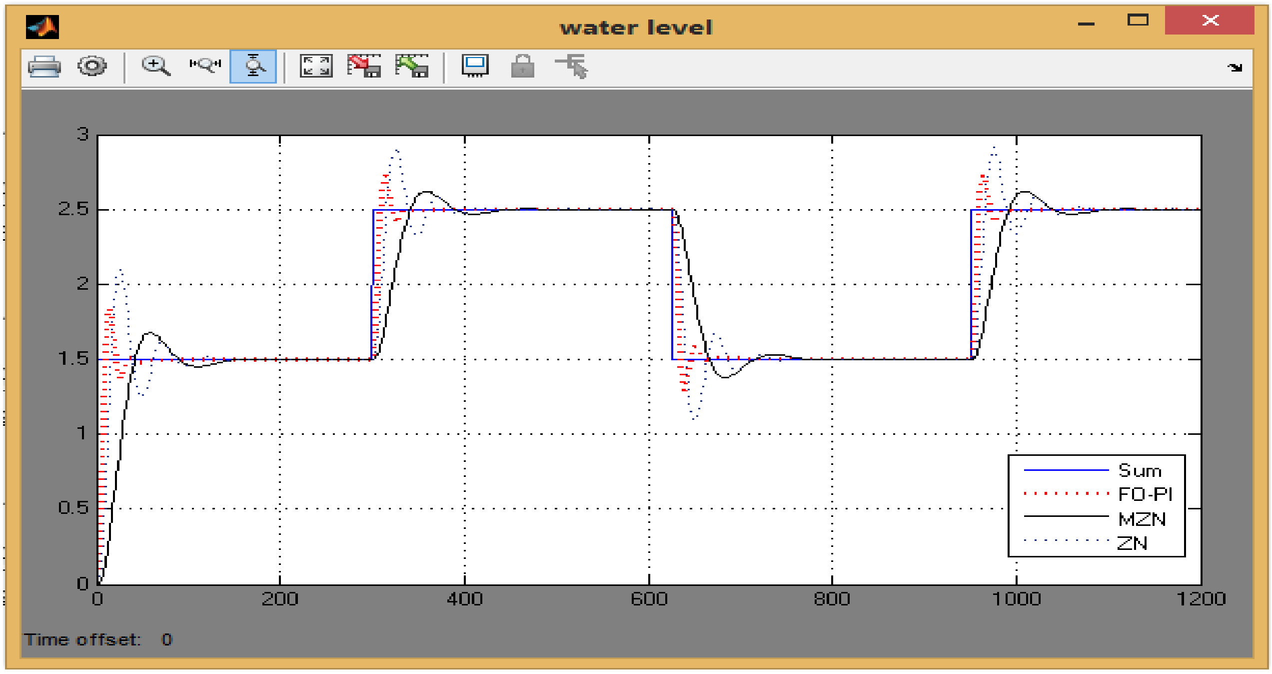
Figure 4

Tuning parameter:

	Kp	Ki	α
PI-ZN	2.945	18.2240	
PI-MZN	3.4485	6.1765	
FO-PI F-MIGO	1.5677	12.1543	0.9

Table 4

Simulated control of The Cascaded Control system:



V. CONCLUSION

In 1st order SISO Coupled tank system, PI/ZN accounts for the largest amount of overshoot, but has a fast response. PI/MZN has a relatively less percentage overshoot. Whereas the new controller, i.e., the FO-PI/F-MIGO controller, results in a much less percentage overshoot. Quick response and small overshoot are desirable in most of the target-tracking control problems. In 2nd order SISO coupled tank system, PI/ZN accounts for relatively less overshoot as compared to the fractional controller, but this is on the account of slow system performance. PI/MZN has minimum overshoot and slowest response among the three controller designs. FOPI (FMIGO) results in large overshoot when compared with integer order PI-ZN and PI-MZN method but then the response time is much less when compared to other controllers. In cascade control system, PI/ZN accounts for the largest amount of overshoot, but has a quick response. PI/MZN has a relatively less percentage overshoot whereas the new controller, i.e. the FO-PI/F-MIGO controller, results in a much smaller percentage overshoot. From this comparison,it can be concluded that FO-PI tuned by F-MIGO method can be a promising controller in process industries and can even perform better than its integer-order counterpart.

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