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Estimation of Maximum and Minimum Temperature of One-Dimensional Steady Flow Heat Conduction Through Rectangle Plate By Using ANSYS

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Abstract - *This paper focuses on estimation of maximum minimum temperature using ANSYS. In this present paper, it is attempted to, derive, develop and apply finite element solution methodology to the governing differential equation for practical problems in conduction heat transfer. The first step in the determination of thermal stresses is to be to determine the temperature distribution of thermal stresses is to determine the temperature distribution within the body. Comparison is made between the experimental results, ANSYS with the help of numerical results heat conduction*

***Key words**: Finite element methods, Heat conduction, heat transfer temperature, stress.*

I. INTRODUCTION

Conduction takes place in a body by virtue of temperature differences. Conduction is the only mode of heat transfer that takes place in a solid body which is influenced by other modes of heat transfer at the boundary surfaces. If a heated body is not permitted to expand or contract freely in all directions, some stresses are developed with the body. The magnitude of thermal stresses will influence the design of various equipments like boilers, steam and gas turbines, nuclear reactions, Heat exchangers, jet engines & rocket motors for satisfactory operation of some of the equipments like steam & gas turbine rotors, high speed machine tools, it is essential to restrict the thermal deformations. This is also needs temperature distribution within the body.

1.Basic equation of heat transfer:

The basic equation of heat transfer are essentially the rate equations and the conservation of energy equation The rate equations describe the rate of energy flow with in a body as in conduction or between bodies as in convection or radiation.

A. Conduction: conduction is the mode of heat transfer with in a body without any net motion of the mass of the material.

Fourier‘s law of heat conduction gives

$$Q=-KA\frac{dt}{dl}$$

Q=rare of heat flow in, W
K= thermal conductivity of the material, W/m^{0}C
A= Area normal to the direction of heat flow, m2
T= Temperature, ^{0}C
l= length parameter, m (may be x. y, z)

In many situations, the rate of heat transfer is required per unit area which is termed heat flux denoted by q.

$$q = -k\ \partial T/\partial x$$

B. Convection:

Convection is the mode of heat transfer between a solid and a fluid surrounding it. The heat transfer between a solid and a fluid surrounding it. The rate of heat transfer by convection is given as

$$Q = hA(Tw - T\infty)$$

Where

h= surface heat transfer coefficient , W/m^{2}K
A= Surface area of the body from which heat flows,m^2
Tw = Surface (or wall) temperature,^{o}C
$T\infty$ = temperature of the surrounding medium, ^{o}C

$$q = h(Tw - T\infty)$$

C. Radiation :

Radiation hat transfer is the mode of heat transfer between two surfaces or bodies obeying laws of electromagnetic. This is the only mode of hat transfer which takes place in vacuum and also when two bodies are not in direct contact with each other.

The rate of heat flow by radiation between two surfaces is given by,

$$Q = \sigma\epsilon A(T_1^4 - T_2^4)$$

Where

σ = Stefan-Boltzmann Constant 5.669 x 10^{-8} W/m^{2}K^4

ϵ = emissivity of the surface
A= surface area of the body from which heat flows , m^2
T_1 = absolute temperature of the body 1, K
T_2 = absolute temperature of the body 2, K

2. Energy Balance Equations

The equation of conservation of energy is one of the key equations is

$$E_{in} + E_g = E_o + E_{int}$$

Where

E_{in} = energy inflow into the system, W
E_g = energy generated in the system, W
E_o = energy leaving the system, W
E_{int} = change of internal energy of the system, W

3. Governing Differential Equation for Heat Conduction:

The governing differential equation will be derived for a three-dimensional stationary system in Cartesian coordinated. Consider an elemental volume of a solid body as shown in fig., The energy balance for the elemental volume can be written as

$$\begin{bmatrix}\text{Heat in flow}\\ \text{in time dt}\end{bmatrix} + \begin{bmatrix}\text{Heat generated within}\\ \text{the body in time dt}\end{bmatrix} = \begin{bmatrix}\text{Heat leaving the}\\ \text{body in time dt}\end{bmatrix} + \begin{bmatrix}\text{change in internal}\\ \text{energy during dt}\end{bmatrix}$$

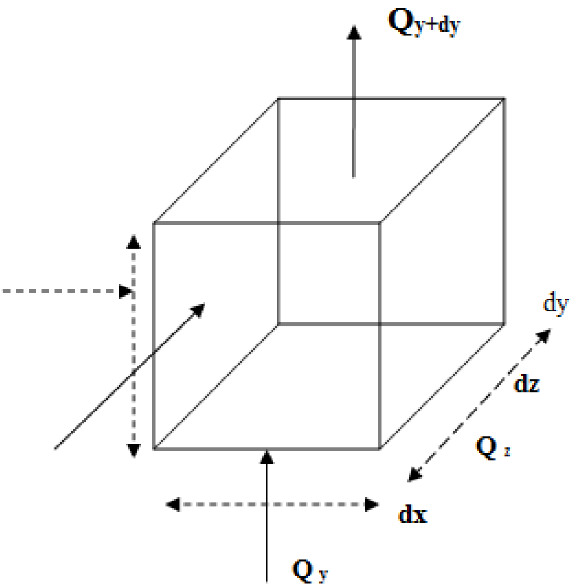

fig 1: Elemental volume

$$(Q_z + Q_y + Q_z)dt + G\,dxdydz\,dt = (Q_{x+dx} + Q_{y+dy} + Q_{z+dz})dt + \rho c dxdydz\,dT$$

Where

G = Rate of heat generated per unit volume, W/m^3
dT = rise in temperature during dt ^0c
ρ = density of the material, kg/m^3
C = specific heat of the material
$Q_x = -k_x A_x \frac{\partial T}{\partial x} \qquad = -k_x\,dy\,dz\,\frac{\partial T}{\partial x}$

Where

Q_x is the heat flow into the face dydz located at x.Similarly,
Q_{y+dy} = Heat flow from the face dydz located at x+dx

$$= Q_X + \frac{\partial T}{\partial X}\,dx$$
$$= -k_x\,dxdy\,\frac{\partial T}{\partial X} - \frac{\partial}{\partial X}\left(k_x A_x \frac{\partial T}{\partial X}\right) dx$$
$$= -k_x\,dxdy\,\frac{\partial T}{\partial X} - \frac{\partial}{\partial X}\left(k_x \frac{\partial T}{\partial X}\right) dxdydz$$

Where k_x is the thermal conductivity of the material in the x - direction. By writing down similar expression in the y and z directions and substituting in the energy balance yields (after dividing throughout by dx dy dz dt)

$$\frac{\partial}{\partial X}\left(k_x \frac{\partial T}{\partial X}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + G = \rho c \frac{\partial T}{\partial X} \quad - \boldsymbol{i}$$

The above differential equation governing heat conduction in a solid body in which k_x, k_y, k_z are different. if the thermal conductivities in x,y,z directions are assumed to be same $k_x = k_y = k_z = k = constant\ as\ in\ isotropic\ material, the\ eq.becomes$

$$\frac{\partial^2 T}{\partial_{x^2}} + \frac{\partial^2 T}{\partial_{y^2}} + \frac{\partial^2 T}{\partial_{z^2}} + \frac{G}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \quad - \boldsymbol{ii}$$

Where

$\frac{k}{\rho c}$ is denoted by α, thermal diffusivity whose units are m^2/s If there are no heat souces or sinks in a body the above eq., can be reduced as follows

$$\frac{\partial^2 T}{\partial_{x^2}} + \frac{\partial^2 T}{\partial_{y^2}} + \frac{\partial^2 T}{\partial_{z^2}} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

If the body is in a steady state, the temperature is independent of time in which case $\frac{\partial T}{\partial t}$ is zero. Hence the above Eq., can be reduced to Poisson's equations,

$$\frac{\partial^2 T}{\partial_{x^2}} + \frac{\partial^2 T}{\partial_{y^2}} + \frac{\partial^2 T}{\partial_{z^2}} = \frac{G}{\mathrm{k}} = 0$$

If the body is in a steady state without heat sources or sinks the above equation can be reduces to Laplace equation

$$\frac{\partial^2 T}{\partial_{x^2}} + \frac{\partial^2 T}{\partial_{y^2}} + \frac{\partial^2 T}{\partial_{z^2}} = 0$$

If the cylindrical coordinate system (with r, θ, z coordinates) is used instead of the Cartesian x,y,x system The $equation - \boldsymbol{i}$ will have the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_r\, r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_\theta\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + G = \rho c\frac{\partial T}{\partial t}$$

For an isotropic material this reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}) + \frac{\partial^2 T}{\partial z^2}) + \frac{G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

If a spherical coordinate system (r, θ, ϕ) is used in place x,y,z the equation ii reduces (for an isotropic material) to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} + \frac{\partial^2 T}{\partial \theta^2} + G = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

4. Boundary And Initial Conditions: let T(x,y,z,t) = T_0 for t > 0 on the surface S_1

$$k_x\frac{\partial T}{\partial x}l + k_y\frac{\partial T}{\partial y}m + \frac{\partial T}{\partial z}n + q = 0 \; on \; s_2 \text{ for } t > 0$$

$$k_x\frac{\partial T}{\partial x}l + k_y\frac{\partial T}{\partial y}m + \frac{\partial T}{\partial z}n + h(T - T_\infty) = 0 \; on \; s_3 \text{ for } t > 0$$

the above differential equations is first order in time t. hence is requires one initial condition

$$\mathrm{T}(x,y,z,0) = T_i(x,y,z) \quad \text{in V}$$

the general energy equation for heat conduction taking into account the motion of the body in space is given by

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + G = \rho c\left(\frac{\partial T}{\partial t} + u_x\frac{\partial T}{\partial x} + u_y\frac{\partial T}{\partial y} + u_z\frac{\partial T}{\partial z}\right)$$

5. Solution Approach Finite Element Approach:

One dimensional heat conduction:

Element Characteristics

A fin dissipating heat to the ambient is an example of one dimensional heat conduction. assume for simplicity, a linear variation of temperature in the element

$$T = c_0 + mx$$

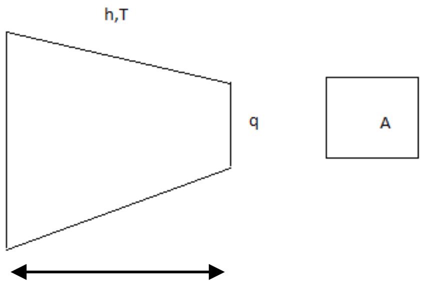

Fig2: Tapered Element

$$T_i = C_O$$

$$T_j = C_o + ml$$

$$C_o = T_i \; and \; m = \frac{T_j - T_i}{l}$$

$$\mathrm{T} = T_i + (T_j - T_i)\frac{x}{l}, \quad \mathrm{T} = T_i\left(1 - \frac{x}{l}\right) + T_j\left(\frac{x}{l}\right)$$

$$\mathrm{T}=T_i N_i + T_j N_j = [N_i\ N_j]\begin{Bmatrix}T_i\\T_j\end{Bmatrix} = [N]\{T\}$$

where $N_i = (1-\frac{x}{l})$ and $N_j = \frac{x}{l}$

$N_i N_j$ are called shape functions or interpolation function or basis functions.

TABLE 1:

Consider the property of the above shape functions

X	N_i	N_j	$N_i + N_j$
0 = i	1	0	1
1 = j	0	1	1
any x	N_i	N_j	1

element characteristics for 1-D problem to derive the expression for [K] and {f}

$$\mathrm{T}=T_i(N_i) + T_j(N_j) = [N_i\ N_j]\begin{Bmatrix}T_i\\T_j\end{Bmatrix}$$

$$\frac{dT}{dx}=T_i\frac{dN_i}{dx} + T_j\frac{dN_j}{dx} = [\tfrac{-1}{l}\ \ \tfrac{1}{l}]\begin{Bmatrix}T_i\\T_j\end{Bmatrix}$$

$$\frac{dN_i}{dx} = -\frac{1}{l}\ and\ \frac{d(N_i)}{dx} = \frac{1}{l}$$

$$[\mathrm{B}]= [-\tfrac{1}{l}\ \ \tfrac{1}{l}]$$

$$[\mathrm{k}]= \int_V [B]^T[D][B]\mathrm{dV}+\int_s [N]^T[N]\mathrm{dS}$$

$$= \int_l \begin{bmatrix}{-1}/{l}\\ {1}/{l}\end{bmatrix} [k_x]\begin{bmatrix}{-1}/{l} & {1}/{l}\end{bmatrix} Adx + \int_l h \begin{bmatrix}N_i\\N_j\end{bmatrix} \begin{bmatrix}N_i & N_j\end{bmatrix} Pdx$$

Where A= area of cross section and P = perimeter of fin

$$= [k] = \int_l \frac{Ak_x}{l^2}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}dx + \int_l hp\begin{bmatrix}N_i N_i & N_i N_j\\N_i N_j & N_j N_j\end{bmatrix}dx$$

In case of linear problem $N_i\ and\ N_j$ are also local coordinates, i.e., L_i , L_j. As indicated in equation explicit integration can be carried out as ,$\int_l L_i^{\,a}\ \ L_j^{\,a}\ dl = \frac{a!b!}{(a+b+1)!}\ l$

Using the above equation can be written as $\int_l N_i\ \ N_j\ \ dl = \frac{1!1!}{(1+1+1)!}\ l = {l}/{6}$

$$\int_l N_i\ \ N_j\ \ dl = \frac{2!\,0!}{(2+0+1)!}\ l = {l}/{3}$$

$$[k] = \frac{Ak_x}{l}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + hp\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$$

$$\{f_x\} = \int_v G\,[N^T]\,ds + \int_s q[N^T]ds + \int_s hT\infty\,\{N\}^T ds$$

$$= \int_l G\begin{bmatrix}N_i\\N_j\end{bmatrix}Adx - \int_s q\begin{bmatrix}N_i\\N_j\end{bmatrix}dA + \int_s hT\infty\,\frac{N_i}{N_j}\ Pdx$$

Either we can substitute the value of $N_i\ and\ N_j$ and integrate or use relation above we get

$$\int_l N_i\ \ dx = \frac{1!\,0!}{(1+0+1)!}\ l = {l}/{2}$$

Therefore,

$$\{f_x\} = \frac{GAL}{2}\begin{bmatrix}1\\1\end{bmatrix} - q\begin{bmatrix}0\\1\end{bmatrix}A + \frac{hT\infty P}{2}\begin{bmatrix}0\\1\end{bmatrix}$$

Finally,

$$\underset{2x1}{[k]}\ \underset{2x1}{\begin{bmatrix}T_i\\T_j\end{bmatrix}} = \underset{2x1}{\{f_x\}}$$

CAESSTUDY 1:

Calculate the temperature distribution in stainless steel fin in a fig. the region is discredited into 5elements and 6 nodes. the properties of an element are compute. the results from finite element solution of {T} and exact solution ANSYS are compared for closeness.

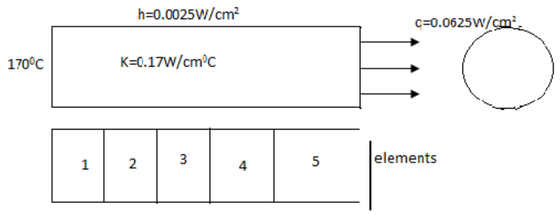

fig3: one dimensional example with linear element

$$K=\frac{KA}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\frac{hpl}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

$$K=\pi\begin{bmatrix}0.086 & -0.0834\\ -0.0834 & 0.086\end{bmatrix}$$

$$\{f_1\}=\frac{hT_\propto pl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

$$\{f_1\}=\pi\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

$$\{f_1\}=\{f_2\}=\{f_3\}=\{f_4\}$$

$$\{f_5\}=\{f_1\}-qA\begin{Bmatrix}0\\ 1\end{Bmatrix}$$

$$\{f_5\}=\pi\begin{Bmatrix}0.125\\ 0.0625\end{Bmatrix}$$

$$\begin{pmatrix}0.0866 & -0.0834 & 0 & 0 & 0 & 0\\ -0.0834 & 0.0866 & -0.0834 & 0 & 0 & 0\\ 0 & -0.0834 & 0 & 0 & -0.0834 & 0\\ 0 & 0 & 0 & -0.0834 & 0 & 0\\ 0 & 0 & -0.0834 & 0 & 0.0866 & 0\\ 0 & 0 & 0 & 0.0866 & 0 & 0.0866\end{pmatrix}\begin{pmatrix}T_1\\ T_2\\ T_3\\ T_4\\ T_5\\ T_6\end{pmatrix}=\begin{Bmatrix}0.125\\ 14.428\\ 0.25\\ 0.25\\ 0.25\\ 0.065\end{Bmatrix}$$

the exact solution

$$\frac{T-T_\infty}{T-T_1}=\frac{\cos hm(1-x)+\left(\frac{h}{mk}\right)\sinh\ m(1-x)}{\cos ml+\left(\frac{h}{mk}\right)\sinh\ ml}$$

heat dissipated from the fin Q_d=10.06 watts

solving the above system of equations we get the values for {T}. the exact solution of {T} is

Exact ^{o}c maximum	170	135.28	105.43	75.85	41.13
minimum	153	120	90	57	24

Compared With Ansys :

preferences → thermal → ok
preprocessor → element type → ADD/EDIT/DELETE → add → solid → quad 4 node 55 → ok
material properties → material models → thermal conductivity → isotropic KXX values 0.17 → ok → close
modeling → create → areas → rectangle → by 2 corners →
up x 0
up y 0
width 10
height 1
meshing → size controls → manual size → areas → all areas → 2 → ok
mesh → areas → free → pick all
solution → analysis type → new analysis → steady state → ok
define loads → apply → thermal → temperature → on lines → pick left line → apply → select all → DOF → temperature as 170^{0}c → apply
select top right sides → lines → apply → select all → DOF temperature 25^{0}C → ok
solve → current LS → ok → close
general post processor → plot results → contour plot → nodal solution → DOF → solution → DOF → solution → nodal temperature → ok

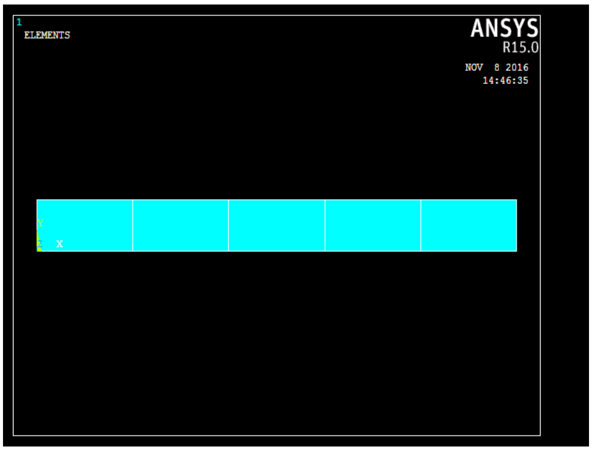

fig4: load at left side

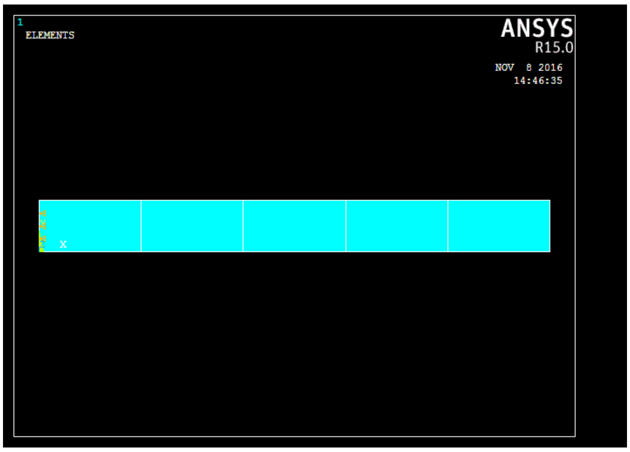

fig5: meshing

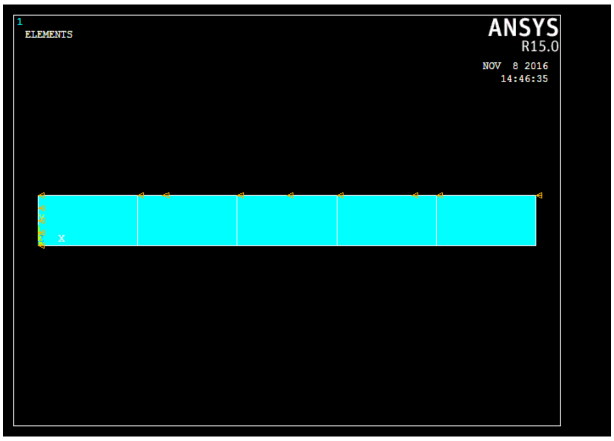

fig6 : load at top side

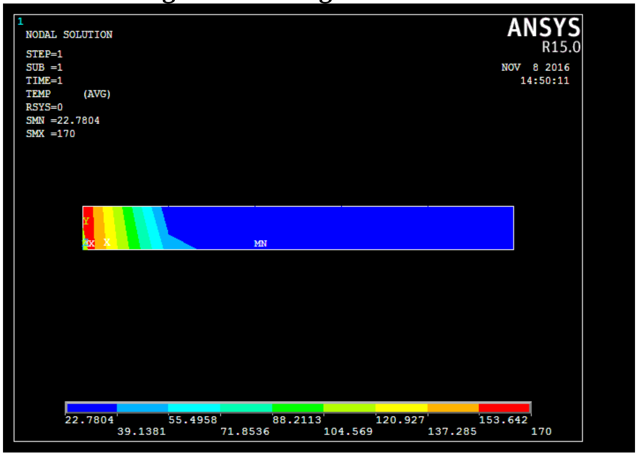

fig7: temperature distribution

6. Conclusion:

from this work the following conclusion are drawn. the results obtained from exact values and ANSYS values compared with the results :

	T_1	T_2	T_3	T_4	T_5
Exact ^{o}c	170	135.28	105.43	75.85	41.13
ANSYS	170	137.285	104.569	71.853	39.1381

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