



A Review on Buckling Analysis of thin walled structure for different boundary conditions

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Abstract: In this paper, buckling of the plates is investigated with different boundary conditions and alternating and axial compressive loadings by theoretical approach. In this review, the usage of Isotropic and heterogeneous materials in situations where large strength to weight ratio is required has been increased substantially over the world in all construction aspects. The behavior of the plate under each loading is different. Whenever the delamination is located at the middle plane of laminate, the panel exhibits only global buckling, i.e. there is no buckling of delaminated region. Accordingly, the minimum critical buckling loads are determined by two approaches such as analytical and numerical methods. Interestingly, good agreements are found between the analytical and numerical predictions for the critical buckling loads. The presented review article is a used to examine the issue of critically observed problems and opportunities in field of buckling analysis for structural engineering problems like airplane structure and its various components.

Key words: Buckling, thin plate, simply supported, hinged edge, clamped, fixed edge, rectangular plates, Aspect ratio, Buckling load, alternating and axial loadings

I. Introduction

Buckling is a big problem, particularly in thin-walled structures where the membrane stiffness is much greater than the bending stiffness. By definition, a thin-walled structure is a three-dimensional component where one dimension is very small in comparison to the other two and typical examples are plates, thin-walled cylinders, arches and shells. Unfortunately, these structures possess the property of being very sensitive to geometrical and mechanical imperfections. This, in combination with the restricted bending stiffness, makes buckling one of the most important failure criteria in these type of structures can be of varying types.

The buckling phenomena can be divided into two parts: global buckling and local buckling. Typical example of global buckling is when the whole structure buckle as one unit, while examples of local buckling is when the skin buckles in a stiffened panel or some minor buckling around an unstiffened hole.

Plate stability is considered by examining the slenderness of the individual elements that make up the Member and the potential for local buckling of those elements. Member Stability is considered by examining the slenderness of the cross-section and the potential for lateral-torsional buckling. Member stability Modes, such as lateral-torsional buckling, occur over the unbraced Length of the beam, which is typically much greater than the depth of the member ($L/d \gg 1$).

Different factors of major importance for the behaviour of thin walled structure or stiffened plates are:-

1. Length/width ratio of the panel
2. Stiffener geometry and spacing
3. Aspect ratio for plate between stiffener
4. Plate slenderness
5. Residual stresses
6. Initial distortions
7. Boundary conditions
8. Type of loading

The loading conditions or constrains are also an important factor that affects the buckling behavior of a structure. A structural member in service can be exposed to any variation of loads in a large number of cycles. During the normal service life of an airplane structure.

An aircraft structures in service are exposed to a large amount of cyclic loads, i.e. during take-off, landing and therefore also subjected to thousands of buckling cycles. Buckling is an instability phenomenon that can lead to very sudden loss of stiffness in a structure and therefore cause catastrophic results.

The buckling of rectangular plates with various plate boundary and load conditions has been studied extensively and there is an abundance of buckling results in the open literature. However, a new plate buckling problem where a rectangular plate is subjected to not only end loads, but also an intermediate uniaxial load remains to be studied. The aim of the study is to determine the buckling factors of rectangular plates under intermediate and end loads. The considered plates have two opposite simply supported edges that are parallel in direction to the applied uniaxial loads while the other two remaining edges may take any other combinations of clamped, simply supported and free edge.

II. Review

In the following, a literature review on the bucking of rectangular plates is presented to provide the background information for the present investigation. The review focuses on homogenous, isotropic, thin plates. Studies on sandwich, composite and orthotropic plates are not covered.

- a) Timoschenko (1936) utilized an energy method to study the buckling behavior of rectangular plates subjected to in-plane shear load. He only considered the symmetrical buckling mode, which caused an error in the prediction of the critical buckling load of the plate. Unlike Timoschenko (1936), Stein and Neff (1947) considered both symmetric and asymmetric buckling modes, which resulted in a correct evaluation.
- b) B. Johansson, R. Maquoi, G. Sedlacek, C. Müller, D. Beg (1993) suggest different levels in their book. Depending on the strain accepted for the extreme plate element in compression of a cross-section the reduced stress method provides different resistances with the following three resistance levels:

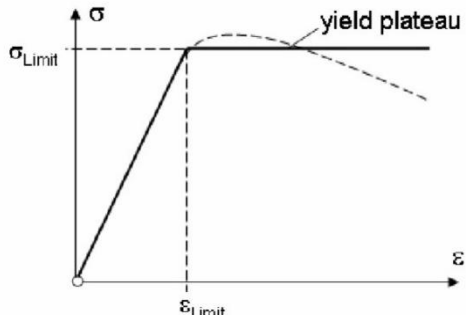


Figure 1, stress strain curve for buckling in biaxial loading

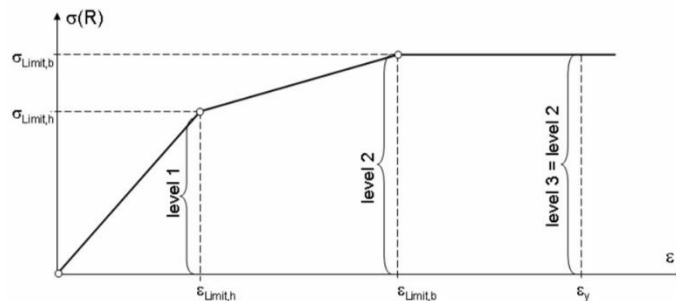


Figure 2: stress strain function for cross section

(Source: Commentary and worked examples Report, b. Johansson, r. Maquoi, g. Sedlacek, c. Müller, d. Beg)

1. level 1 limits the exploitation of the cross-section to the plate buckling resistance of the weakest plate element
2. level 2 allows for stress redistribution up to the plate buckling resistance of the strongest plate element
3. level 3 allows to straining the extreme plate elements in compression to the yield strain (equivalent to the yield strength of the material) with the possibility of exploiting further reserves

Elastic buckling of rectangular plates

This part is concerned with the research done for the elastic buckling of rectangular plates under various in-plane loads and boundary conditions for the plate edges. Navier (1822) derived the basic stability equation for rectangular plates under lateral load by including the twisting action. The inclusion of the 'twisting' term is very important because the resistance of the plate to twisting can considerably reducedeflections under lateral load. Saint-Venant (1883) modified the equation by including axial edge forces and shearing forces. The modified equation formed the basis for much of the work on plate stability of plates with various loads and boundary conditions

I. Buckling of plates under uniaxial compression

The most basic form of plate buckling problem is a simply supported plate under uniaxial compression.

- a) Soh et al. (2000) have performed elasto-plastic buckling analysis of simply supported plates subjected to uniaxial compression load. The aim of the work was to determine the buckling load of three different types of plates made out of composite material. The obtained numerical results showed good correlation when compared to the experiments.
- b) Bryan (1891) gave the first solution for the problem by using the energy method to obtain the values of the critical loads. He assumed that the deflection surface of the buckled plate could be represented by a double Fourier series. Timoshenko (1925) used another method to solve the problem. He assumed that the plate buckled into several sinusoidal half waves in the direction of compression. When satisfying the boundary conditions, the equations formed a matrix problem which upon solving yields the critical load. The problem was discussed in many standard textbooks such as Timoshenko and Gere (1961) and Bulson (1970). Apart from simply supported plates, Timoshenko (1925) explored the buckling of uniformly compressed rectangular plates that are simply supported along two opposite sides perpendicular to the direction of compression and having various edges along the other two sides. The various boundary conditions considered include SSSS, CSCS, FSSS, FSCS, CSES (S - simply supported edge, F - free edge, C - clamped or built-in edge and E - elastically restrained edge). The theoretical results were in good agreement with experimental results obtained by Bridget *et al.* (1934).
- c) Bleich (1952) obtained the critical load for the ESES plates with loaded edges elastically restrained. The results are for the symmetric mode only and values of aspect ratio are less than 1.0. For the elastic buckling of rectangular plates with linearly varying axial compression there is no exact analytical solution. The best-known analysis for simply supported plates is due to Timoshenko and Gere (1961), who employed the principle of conservation of energy and assumed the buckled form of the plate consisted of several half-waves in the loading direction.
- d) Kollbrunner and Hermann (1948) examined the CSSS plates. They found when the clamped edge is on the tension side of the plate, the critical load factors do not differ greatly from those with both edges simplysupported.
- e) Schuette and McCulloch (1947) employed theLagrangian multiplier to solve the buckling problem of ESSS plates. Walker (1967) used the Galerkin's method to give accurate values of critical load for a number of the edge conditions as mentioned before. He also studied the case of ESFS plates. Xiang *et al.* (2001) considered the elastic buckling of a uniaxially loaded rectangular plate with an internal line hinge. Using the Levy's method, they

succeeded in presenting the exact solution for many different boundary conditions such as SSSS, FSFS, CSCS, FSSS and SSCS plates.

- f) According to the research presented by Alinia et al. (2012) the buckling of stocky plates are followed by an unstable postbuckling path meaning no further load carrying capacity. In the other hand, slender plates show a load carrying capacity up to the ultimate load after the buckling load is reached.
- g) Simply supported plates under in-plane shear load with landings (or reinforcements) have been investigated earlier at Saab Aeronautics (LN-017497, 2014) and one of the main conclusions was that the placement of the landing (symmetric or asymmetric) did not affect the buckling load of the plate.

II. **Buckling of plates under biaxial compression**

- a) Bryan (1891) first considered the SSSS plates under biaxial compressions by assuming that the deflection could be written as a double Fourier series. Wang (1953) solved the equation of equilibrium. More recently, Reddy (1999) applied the Rayleigh-Ritz approximation to solve the CCCC plates under shear forces.

III. **Buckling of plates under combined loads**

- a) Batdorf and Stein (1947) evaluated the buckling problem under combined shear and compression combinations for simply supported plates by adopting the deflection function in the form of infinite series. Batdorf and Houbolt (1946) gave a solution to the equation of equilibrium for infinitely long plates with restrained edges under shear and uniform transverse compression. Johnson and Buchert (1951) used the energy method to explore the buckling behavior of rectangular plates with compression edge simply supported or elastically restrained, tension edge simply supported. Researchers who are interested in this field of research may refer to Bulson (1970), in which many research papers were cited. More recently, Kang and Leissa (2001) presented exact solutions for the buckling of rectangular plates having two opposite, simply supported edges subjected to linearly varying normal stresses causing pure in-plane moments, the other two edges being free.

IV. **Buckling of plates under body forces**

- a) Farvre (1948) is probably the first researcher to work out approximate buckling solutions of rectangular plates under self-weight and uniform in-plane compressive forces. However, he treated only plates with all four edges simply supported. Wang and Sussman (1967) solved the same problem using the Rayleigh-Ritz method and concluded that the average stress in the plate at buckling is less than that for a plate with uniform compression at buckling. Both Favre (1948) and Wang and Sussman (1967) did not give numerical values in their papers. Using the conjugate load-displacement method, Brown (1991) investigated the buckling of rectangular plates under (a) a uniformly distributed load, (b) a linearly increasing distributed load and (c) a varying sinusoidal load across the plate width. The second type of load is equivalent to the plate self-weight. In his study, Brown treated a number of combinations of boundary conditions. More recently, Wang *et al.* (2002) considered the buckling problem of vertical plates under body forces/self-weight. The vertical plate is either clamped or simply supported at its bottom edge while its top edge is free. The two sides of the plate may either be free, simply supported or clamped. Xiang *et al.* (2003) treated yet another new elastic buckling problem where the buckling capacities of cantilevered, vertical, rectangular plates under body forces are computed.

V. **Buckling of plates under other forms of loads**

- a) Bulson cited Yamaki's buckling studies on SSSS, CSCS and CCCC plates under equal and opposite point loads. Bulson (1970) also cited Yamaki's research on buckling problems of CSCS and SSSS plates under partially distributed loads which are acted upon the simply supported edges. Lee *et al.* (2001) considered the elastic buckling problem of square EEEE and ESES plates subjected to in-plane loads of different configurations acting on opposite sides of plates. The effects of Kinney's fixity factor (introduced to describe the support conditions at the edges covering the boundary conditions of simply supported and fixed edges) and the width factor on critical load factors were treated.

III. **Conclusion/ Observation**

Critical Buckling load for homogeneous plate is more compare to laminated heterogeneous plate

- a) Buckling load per unit length of the plate decreases with increase in aspect ratio. It is noted that the presence of crack lowers the buckling load. The study is useful in selecting the above said parameters in designing the laminates in buckling point of view.
- b) The study also revealed that plate deforms along the transverse direction, leading to the stretching of the longitudinal fibers of the plate, when uniaxially loaded. In this way, the longitudinal fibers of the plate would undergo stress redistribution, as well as develop transverse tensile stresses. These tensile stresses provide the postbuckling reserve load.
- c) Like columns, plates also undergo instability and buckle under compressive and shear stresses. The critical buckling load for a plate depends upon its width-thickness ratio and support conditions. However, unlike columns, the post-

buckling behaviour of plates is stable and plates will continue to carry higher loads beyond their elastic critical loads. This post-buckling range is substantial in the case of plates with high width-thickness ratios (slender plates).

- d) In the post-buckling range, the stress distribution is non-uniform and the plate fails when the maximum stress at the supported edge in the post-buckling range reaches the yield stress. So the concept of effective width is used to calculate the strength of the plate.

IV. References

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